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Comparing theoretical and practical solution of the first order first degree ordinary differential equation of Population model

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Abstract

Population dynamics is the branch of mathematics that studies the size and age composition of populations as dynamical systems, the biological and environmental processes driving them such as birth and death rates and by immigration and emigration. In this paper, We are discussed how to read mathematical models and how to analyze them with the ultimate aim that we can critically judge the assumptions and the contributions of such models whenever we encounter them in your future biological research. Mathematical models are used in all areas of biology. All models in this paper are formulated in ordinary differential equations (ODEs). These will be analyzed by computing steady states. We developed the differential equations by ourselves following a simple graphical procedure, depicting each biological process separately. Experience with an approach for writing models will help us to evaluate models proposed by others.

Keywords

Introduction, General equation of population growth, Logistic Equation, Logistic Model for given data, Solution of Logistic model, comparing logistic model with actual data and conclusion.

1. Introduction

In 1798, English economist Thomas Malthus was stated that population would grow at a geometric rate while the food supply grows at an arithmetic rate. The theory has been seen as flawed because of the limited factors observed when he developed the Law. It does not include factors, such as technology, disease, poverty, international conflict and natural disasters.

Malthusian models have the form $P(t) = P_0 e^{kt}$ where P_0 is the initial number of population, k is population growth rate (Malthusian parameter) and t is the time. Sometimes this model is called simple exponential growth model.

General equation of population growth:

The rate of change of quantity = The rate of births - The rate of deaths

Suppose P(t) is the population, α is the per capital births rate and β is the per capital number of deaths population.

$$\begin{aligned}\frac{dP(t)}{dt} &= \alpha P(t) - \beta P(t) \\ &= P(t)(\alpha - \beta) \\ &= P(t)K \qquad \text{where } K = \alpha - \beta\end{aligned}$$

This is the first order first degree ordinary differential equation [1]. The solution of (1) is

$$P(t) = ce^{kt}. \text{ If } t = 0,\ P = P_0 \quad \therefore\ P_0 = C \ \ and \ \ P = P_0 e^{Kt}$$

Birth rate is constant and death rate is linearly increasing:

If $\alpha = \alpha_0$ and $\beta = \beta_0 + \beta_1 P(t)$ then we have

$$\frac{dP}{dt} = \alpha_0 P(t) - \{\beta_0 + \beta_1 P(t)\} P(t)$$

$$= (\alpha_0 + \beta_0) P(t) - \beta_1 P^2(t)$$

Birth rate constant and death rate is exponentially increasing:

If $\beta = \beta_1 e^{kt}$ and $\alpha = \alpha_0$ then

$$\frac{dp}{dt} = \alpha_0 P(t) - \beta_1 e^{kt} P(t)$$

Birth rate constant and death rate is sine function:

If $\beta = \beta_1 \sin t$ and $\alpha = \alpha_0$ then

$$\frac{dP}{dt} = \alpha_0 P(t) - \beta_1 Sin(t) P(t)$$

Death rate constant and birth rate linearly increasing:

If $\alpha = \alpha_0 + \alpha_1 P(t)$ and $\beta = \beta$ then

$$\frac{dP(t)}{dt} = (\alpha_0 + \alpha_1 P(t)) P(t) - \beta P(t)$$

Death rate constant and birth rate exponentially increasing:

If $\alpha = \alpha_1 e^{kt}$ and $\beta = \beta_0$ then

$$\frac{dP(t)}{dt} = \alpha_1 e^{kt} P(t) - \beta_0 P(t)$$

Death rate and birth rate are linearly increasing:

If $\alpha = \alpha_0 + \alpha_1 P(t)$ and $\beta = \beta_0 + \beta_1 P(t)$ then

$$\frac{dP(t)}{dt} = (\alpha_0 + \alpha_1 P(t)) P - (\beta_0 + \beta_1 P) P$$

$$= (\alpha_0 - \beta_0) P + (\alpha_1 - \beta_1) P^2$$

Logistic Equation:

In real population growth is not always unlimited but may have an upper limit L where population can no longer be sustained as time increase. The logistic ODE is

$$\frac{dP}{dt} = KP(1 - \frac{P}{L}) \quad [2]. \tag{1}$$

Logistic Model for given data:

Since we have discrete data, then we describe the model using a difference equation. The equation (A) can be written as

$$P(t+1)-P(t)=KP\left(1-\frac{P}{L}\right)$$
$$\Rightarrow \frac{\Delta P}{P}=K\left(1-\frac{P}{L}\right) \qquad (2)$$

The equation says that the ratio of ΔP to P is linear function of P. First of all, let's consider the left hand side (LHS) of equation (2). We calculate the difference of the populations for two consecutive years, and then use those differences against the corresponding function values [3].

Year	Bangladesh		India		Pakistan		Canada	
	P(t)	A	P(t)	a	P(t)	a	P(t)	a
1950	2.859358061	0.000424596	2.982949128	0.000264194	2.858822666	0.000230285	2.799449912	0.000566412
1951	2.860572649	0.000402042	2.983737412	0.000276397	2.859481009	0.000266827	2.801035555	0.000578528
1952	2.861723181	0.000404666	2.984562336	0.000286109	2.860243995	0.000299569	2.802656031	0.000589931
1953	2.862881692	0.000424609	2.985416491	0.000293914	2.861100835	0.00032888	2.804309406	0.000598178
1954	2.864097814	0.000454662	2.986294205	0.000300358	2.862041794	0.000355147	2.805986882	0.000601193
1955	2.865400601	0.00048799	2.987191432	0.000305969	2.86305824	0.000378781	2.807673823	0.000597077
1956	2.866799571	0.00051868	2.988105699	0.000311211	2.864142711	0.000400171	2.809350219	0.000584414
1957	2.868287295	0.00054222	2.989035921	0.000316456	2.865288859	0.000419657	2.810992042	0.000562513
1958	2.869843381	0.000555854	2.989982119	0.00032195	2.866491297	0.000437496	2.812573262	0.000531553
1959	2.871439481	0.000560382	2.990945053	0.000327651	2.867745376	0.00045375	2.814068294	0.000493925
1960	2.873049487	0.000559736	2.99192536	0.000333249	2.869046615	0.000468289	2.815458233	0.000454019
1961	2.874658536	0.000559995	2.992922747	0.00033823	2.870390158	0.000480852	2.816736505	0.000417697
1962	2.876269232	0.000567471	2.993935387	0.00034204	2.871770392	0.000491159	2.817913048	0.00039106
1963	2.877902359	0.000584453	2.994959785	0.000344458	2.873180889	0.000499178	2.819015019	0.000377284
1964	2.87958534	0.000604142	2.995991777	0.00034607	2.874615119	0.000505449	2.820078588	0.000372449
1965	2.88132607	0.000630036	2.997028959	0.000346831	2.87606809	0.000511509	2.821128922	0.000368925
1966	2.883142553	0.000644271	2.998068783	0.000348137	2.877539225	0.00051718	2.822169707	0.000363079
1967	2.885001276	0.000625292	2.999112886	0.000351589	2.87902743	0.000521135	2.823194376	0.000357597
1968	2.886806373	0.000566892	3.000167712	0.000357809	2.880527792	9.55365E-09	2.824203942	0.000351751
1969	2.888443809	0.000487288	3.001241582	0.000365534	2.880527819	0.001048221	2.825197357	0.000345275
1970	2.889852	0.000400136	3.002339039	0.000373561	2.883547249	0.000525409	2.826172828	0.00034036
1971	2.891008798	0.000332539	3.003461014	0.000379932	2.885062292	0.000527711	2.827134743	0.000334978
1972	2.891970492	0.000303864	3.004602559	0.000383482	2.886584773	0.000534129	2.828081772	0.000324717
1973	2.892849524	0.000326043	3.005755211	0.000383488	2.888126582	0.000545568	2.829000097	0.000308188
1974	2.893793024	0.000381858	3.006908323	0.000381117	2.889702253	0.000560021	2.829871961	0.000287872
1975	2.894898465	0.000445559	3.008054743	0.000377636	2.891320548	0.000573427	2.830686601	0.000267655
1976	2.896188887	0.000494536	3.009191121	0.00037627	2.89297851	0.000584771	2.831444249	0.000250432
1977	2.897621866	0.000525985	3.010323816	0.000371791	2.894670239	0.000596151	2.832153333	0.000236211
1978	2.899146775	0.000533228	3.011443443	0.000373848	2.896395899	0.000607419	2.832822319	0.000226072
1979	2.900693505	0.000524252	3.012569687	0.000375436	2.898155226	0.000617481	2.833462741	0.000219678
1980	2.902214996	0.000512075	3.013701139	0.00037694	2.899944781	0.000625689	2.834085192	0.000212629
1981	2.903701909	0.000504875	3.014837553	0.00037699	2.901759245	0.000630553	2.834687801	0.000207187
1982	2.905168656	0.000533649	3.015974545	0.000375043	2.903588959	0.000630867	2.83527511	0.000210032
1983	2.906719823	0.000466425	3.017106089	0.000370646	2.905420736	0.000626145	2.835870609	0.000223077
1984	2.908076221	0.000501541	3.018224782	0.000364544	2.907239952	0.00061742	2.836503227	0.000242382
1985	2.909535471	0.000503177	3.01932546	0.000358043	2.909034941	0.000608164	2.837190746	0.000264377
1986	2.911000219	0.000500997	3.020406896	0.000351815	2.910804112	0.000597616	2.837940833	0.000282116

1987	2.912459352	0.000492845	3.021469895	0.000345509	2.912543654	0.000582279	2.838741461	0.000290569
1988	2.91389545	0.000477571	3.022514199	0.000339293	2.914239568	0.000561727	2.839566312	0.000286531
1989	2.915287708	0.000458033	3.023540065	0.000333202	2.915876575	0.000538397	2.840379936	0.000273704
1990	2.916623617	0.00043666	3.02454785	0.00032684	2.917446474	0.000513734	2.841157359	0.000259379
1991	2.917897747	0.000417772	3.025536716	0.000320526	2.918945265	0.00049156	2.841894296	0.00024727
1992	2.919117272	0.000404166	3.026506791	0.000314997	2.9203801	0.000475015	2.842597011	0.000235051
1993	2.920297557	0.000397513	3.027460432	0.000310469	2.921767324	0.000465605	2.843265166	0.000223583
1994	2.921458873	0.000395164	3.028400655	0.000306539	2.923127714	0.00046072	2.84390087	0.000213131
1995	2.922613784	0.000393151	3.029329264	0.000302753	2.924474457	0.000457853	2.844506992	0.00020235
1996	2.923763264	0.000388437	3.03024668	0.000298531	2.925813437	0.000453035	2.845082578	0.000192465
1997	2.924899403	0.00038105	3.031151573	0.000293611	2.927138933	0.000444081	2.845630156	0.00018641
1998	2.92601436	0.000370072	3.032041814	0.00028777	2.92843882	0.000429626	2.846160609	0.000185269
1999	2.927097596	0.000494887	3.032914597	0.000281285	2.929696953	0.000412182	2.846687914	0.000187864
2000	2.928546894	0.00020515	3.033767949	0.000274638	2.930904522	0.0003946	2.847222706	0.00019111
2001	2.92914781	0.000331171	3.034601367	0.00026813	2.932061056	0.000380334	2.847766838	0.000194259
2002	2.93011818	0.000314258	3.035415252	0.000261656	2.933176218	0.000370803	2.848320042	0.000199225
2003	2.931039282	0.000351751	3.036209693	0.000255302	2.934263849	0.000367248	2.848887499	0.000206034
2004	2.93207064	0.000143743	3.036985041	0.000249019	2.935341451	0.000356123	2.849474466	0.000213726
2005	2.932492166	0.000370522	3.037741495	0.000242942	2.936386794	0.000381009	2.850083472	0.000222121
2006	2.933579121	0.000152102	3.038479669	0.000236826	2.937505584	0.000369838	2.850716535	0.000229198
2007	2.934025391	0.000203976	3.03919943	0.000230301	2.938591986	0.000371355	2.851369915	0.000232592
2008	2.934623984	0.000200769	3.039899522	0.000223245	2.939683246	0.000373543	2.85203312	0.000231138
2009	2.935213284	0.000205278	3.040578317	0.000215973	2.940781345	0.000375849	2.852692333	0.00022614
2010	2.935815944	0.000211987	3.04123514	0.000208574	2.941886635	0.000378209	2.853337441	0.000220002
2011	2.93643843	0.000216674	3.041869594	0.000201723	2.942999284	0.000379619	2.853965182	0.000214274
2012	2.937074817	0.000219459	3.042483333	0.000196131	2.944116504	0.000378854	2.854576713	0.000208539
2013	2.937719526	0.000218914	3.043080176	0.000192146	2.945231895	0.000375381	2.855172003	0.000203251
2014	2.938362775	0.00021588	3.043665005	0.00018927	2.946337478	0.000369837	2.855752319	0.000198238
2015	2.938997245	0.000212822	3.044241188	0.000186773	2.947427142	0.000363733	2.856318437	0.000193006
2016	2.939622861	0.000210338	3.044809875	0.000183966	2.948499219	0.000357406	2.856869722	0.000187329

Determining the value of K and L: In the Least Square Approximation graph, we know the equation for the line, which is,

$$y = a + bx \tag{3}$$

Substituting the point P(1950) and P(1951) in (3) we have

Variable/Country	Bangladesh	India	Pakistan	Canada
$y = a + bx.$	$y = 0.471146818 - 0.16228x$	$y = 0.471146818 - 0.16228x$	$y = 0.471146818 - 0.16228x$	$y = 0.471146818 - 0.16228x$
P_1	2.859358064	2.982949128	2.858822666	2.799449912
P_1	2.86057649	2.983737412	2.859481009	2.80103555
y_1	0.045499	0.033148173	0.045835	0.034733
y_2	0.044285	0.03235856	0.045177	0.033148

Equation (2) can be written as

$$K(1 - P_1 / L) = y_1 \tag{4}$$

$$and \;\; K(1 - P_2 / L) = y_2 \tag{5}$$

Solving (4) and (5) we have $L = \dfrac{P_1 y_2 - P_2 y_1}{y_2 - y_1}$ and $K = \dfrac{y_1}{1 - P_1 / L}$

We have

Variable/ Country	Bangladesh	India	Pakistan	Canada
L(Caring Capacity)	2.904878989	3.016041509	2.90468156	2.834196893
Exp(Exp(L))	85415102.72	731302266	85107708.4	24562428.95
K (Constant)	2.903479866	3.021126376	2.90316821	2.833056521

Solution of Logistic model:

Equation (1) is Bernoulli equation [4], we have

$$\begin{aligned} &\frac{dP}{dt} = KP(1-\frac{P}{L}) \\ &\Rightarrow \frac{dP}{dt} = KP - \frac{K}{L}P^2 \\ &\Rightarrow \frac{dP}{dt} - KP = -\frac{K}{L}P^2 \\ &\Rightarrow -\frac{1}{P^2}\frac{dP}{dt} + \frac{K}{P} = \frac{K}{L} \end{aligned} \tag{6}$$

$$\text{Put } \frac{1}{P} = V$$

$$\therefore -\frac{1}{P^2}\frac{dP}{dt} = \frac{dV}{dt}$$

From (6) we have

$$\frac{dV}{dt} + KV = \frac{K}{L}$$

Now this equation is exact. Hence integrating factor

$$\begin{aligned} IF &= e^{\int K dt} \\ &= e^{Kt} \end{aligned}$$

Hence the solution is

$$\begin{aligned} &V.e^{Kt} = \int \frac{K}{L}e^{Kt}dt \\ &\Rightarrow V.e^{Kt} = \frac{K}{L}\frac{e^{Kt}}{K} + c \\ &\Rightarrow \frac{1}{P}e^{Kt} = \frac{1}{L}e^{Kt} + c \\ &\Rightarrow \frac{1}{P} = \frac{1}{L} + ce^{-Kt} \\ &\Rightarrow P = \frac{L}{1+Lce^{-Kt}} \end{aligned} \tag{7}$$

If $t \to \infty$, then P=L.

Comparing logistic model with actual data:

$$\frac{dP}{dt} = P(K - \frac{PK}{L})$$

$$\Rightarrow \frac{dp}{dt} = KP\left(\frac{L-P}{L}\right)$$

$$\Rightarrow \frac{1}{K}\frac{L}{P(L-P)}dP = dt$$

Integrating we have,

$$\frac{1}{K}Ln\left(\frac{P}{L-P}\right) = t + c$$

If t = 0 then find the value of c.

Country	Proportionality (K)	Caring Capacity (L)	Integral Constant (C)	No. of Population of Caring Capacity
Bangladesh	2.903479866	2.904878989	1.425937326	85415102.72
India	3.021126376	3.016041509	1.489962381	731302266
Pakistan	2.90316821	2.90468156	1.42347799	85107708.4
Canada	2.833056521	2.834196893	1.549240369	24562428.95

Putting the values of c in (7), we have

Year	time	Bangladesh		India		Pakistan		Canada	
		Theoretical data	Original data	Theoretical data	Original data	Theoretical data	Original data	Theoretical data	Original data
1950	0	0.564912469	2.859358061	0.548991205	2.982949128	0.565690912	2.858822666	0.525741899	2.799449912
1951	1	0.338815486	2.860572649	2.474080616	2.983737412	2.367713453	2.859481009	2.252355146	2.801035555
1952	2	0.507007543	2.861723181	2.984176061	2.984562336	2.868993821	2.860243995	2.791767536	2.802656031
1953	3	0.734770217	2.862881692	3.014472417	2.985416491	2.902701119	2.861100835	2.831664983	2.804309406
1954	4	1.021245416	2.864097814	3.015964984	2.986294205	2.904572864	2.862041794	2.834047808	2.805986882
1955	5	1.349886163	2.865400601	3.016037779	2.987191432	2.904675598	2.86305824	2.834188122	2.807673823
1956	6	1.689356054	2.866799571	3.016041327	2.988105699	2.904681233	2.864142711	2.834196377	2.809350219
1957	7	2.004061225	2.868287295	3.0160415	2.989035921	2.904681542	2.865288859	2.834196863	2.810992042
1958	8	2.267870264	2.869843381	3.016041509	2.989982119	2.904681559	2.866491297	2.834196891	2.812573262
1959	9	2.470983034	2.871439481	3.016041509	2.990945053	2.90468156	2.867745376	2.834196893	2.814068294
1960	10	2.617359471	2.873049487	3.016041509	2.99192536	2.90468156	2.869046615	2.834196893	2.815458233
1961	11	2.717895793	2.874658536	3.016041509	2.992922747	2.90468156	2.870390158	2.834196893	2.816736505
1962	12	2.784687865	2.876269232	3.016041509	2.993935387	2.90468156	2.871770392	2.834196893	2.817913048
1963	13	2.828086247	2.877902359	3.016041509	2.994959785	2.90468156	2.873180889	2.834196893	2.819015019
1964	14	2.855878615	2.87958534	3.016041509	2.995991777	2.90468156	2.874615119	2.834196893	2.820078588
1965	15	2.873511959	2.88132607	3.016041509	2.997028959	2.90468156	2.87606809	2.834196893	2.821128922
1966	16	2.884633741	2.883142553	3.016041509	2.998068783	2.90468156	2.877539225	2.834196893	2.822169707
1967	17	2.891622366	2.885001276	3.016041509	2.999112886	2.90468156	2.87902743	2.834196893	2.823194376

1968	18	2.896003525	2.886806373	3.016041509	3.000167712	2.90468156	2.880527792	2.834196893	2.824203942
1969	19	2.898746022	2.888443809	3.016041509	3.001241582	2.90468156	2.880527819	2.834196893	2.825197357
1970	20	2.900461176	2.889852	3.016041509	3.002339039	2.90468156	2.883547249	2.834196893	2.826172828
1971	21	2.901533212	2.891008798	3.016041509	3.003461014	2.90468156	2.885062292	2.834196893	2.827134743
1972	22	2.902203033	2.891970492	3.016041509	3.004602559	2.90468156	2.886584773	2.834196893	2.828081772
1973	23	2.902621451	2.892849524	3.016041509	3.005755211	2.90468156	2.888126582	2.834196893	2.829000097
1974	24	2.902882788	2.893793024	3.016041509	3.006908323	2.90468156	2.889702253	2.834196893	2.829871961
1975	25	2.903046	2.894898465	3.016041509	3.008054743	2.90468156	2.891320548	2.834196893	2.830686601
1976	26	2.903147925	2.896188887	3.016041509	3.009191121	2.90468156	2.89297851	2.834196893	2.831444249
1977	27	2.903211575	2.897621866	3.016041509	3.010323816	2.90468156	2.894670239	2.834196893	2.832153333
1978	28	2.903251321	2.899146775	3.016041509	3.011443443	2.90468156	2.896395899	2.834196893	2.832822319
1979	29	2.903276141	2.900693505	3.016041509	3.012569687	2.90468156	2.898155226	2.834196893	2.833462741
1980	30	2.90329164	2.902214996	3.016041509	3.013701139	2.90468156	2.899944781	2.834196893	2.834085192
1981	31	2.903301318	2.903701909	3.016041509	3.014837553	2.90468156	2.901759245	2.834196893	2.834687801
1982	32	2.903307361	2.905168656	3.016041509	3.015974545	2.90468156	2.903588959	2.834196893	2.83527511
1983	33	2.903311135	2.906719823	3.016041509	3.017106089	2.90468156	2.905420736	2.834196893	2.835870609
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1985	35	2.903314962	2.909535471	3.016041509	3.01932546	2.90468156	2.909034941	2.834196893	2.837190746
1986	36	2.903315881	2.911000219	3.016041509	3.020406896	2.90468156	2.910804112	2.834196893	2.837940833
1987	37	2.903316455	2.912459352	3.016041509	3.021469895	2.90468156	2.912543654	2.834196893	2.838741461
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1990	40	2.903317177	2.916623617	3.016041509	3.02454785	2.90468156	2.917446474	2.834196893	2.841157359
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1992	42	2.903317318	2.919117272	3.016041509	3.026506791	2.90468156	2.9203801	2.834196893	2.842597011
1993	43	2.903317352	2.920297557	3.016041509	3.027460432	2.90468156	2.921767324	2.834196893	2.843265166
1994	44	2.903317374	2.921458873	3.016041509	3.028400655	2.90468156	2.923127714	2.834196893	2.84390087
1995	45	2.903317387	2.922613784	3.016041509	3.029329264	2.90468156	2.924474457	2.834196893	2.844506992
1996	46	2.903317395	2.923763264	3.016041509	3.03024668	2.90468156	2.925813437	2.834196893	2.845082578
1997	47	2.9033174	2.924899403	3.016041509	3.031151573	2.90468156	2.927138933	2.834196893	2.845630156
1998	48	2.903317404	2.92601436	3.016041509	3.032041814	2.90468156	2.92843882	2.834196893	2.846160609
1999	49	2.903317406	2.927097596	3.016041509	3.032914597	2.90468156	2.929696953	2.834196893	2.846687914
2000	50	2.903317407	2.928546894	3.016041509	3.033767949	2.90468156	2.930904522	2.834196893	2.847222706
2001	51	2.903317408	2.92914781	3.016041509	3.034601367	2.90468156	2.932061056	2.834196893	2.847766838
2002	52	2.903317408	2.93011818	3.016041509	3.035415252	2.90468156	2.933176218	2.834196893	2.848320042
2003	53	2.903317408	2.931039282	3.016041509	3.036209693	2.90468156	2.934263849	2.834196893	2.848887499
2004	54	2.903317409	2.93207064	3.016041509	3.036985041	2.90468156	2.935341451	2.834196893	2.849474466
2005	55	2.903317409	2.932492166	3.016041509	3.037741495	2.90468156	2.936386794	2.834196893	2.850083472
2006	56	2.903317409	2.933579121	3.016041509	3.038479669	2.90468156	2.937505584	2.834196893	2.850716535
2007	57	2.903317409	2.934025391	3.016041509	3.03919943	2.90468156	2.938591986	2.834196893	2.851369915

2008	58	2.903317409	2.934623984	3.016041509	3.039899522	2.90468156	2.939683246	2.834196893	2.85203312
2009	59	2.903317409	2.935213284	3.016041509	3.040578317	2.90468156	2.940781345	2.834196893	2.852692333
2010	60	2.903317409	2.935815944	3.016041509	3.04123514	2.90468156	2.941886635	2.834196893	2.853337441
2011	61	2.903317409	2.93643843	3.016041509	3.041869594	2.90468156	2.942999284	2.834196893	2.853965182
2012	62	2.903317409	2.937074817	3.016041509	3.042483333	2.90468156	2.944116504	2.834196893	2.854576713
2013	63	2.903317409	2.937719526	3.016041509	3.043080176	2.90468156	2.945231895	2.834196893	2.855172003
2014	64	2.903317409	2.938362775	3.016041509	3.043665005	2.90468156	2.946337478	2.834196893	2.855752319
2015	65	2.903317409	2.938997245	3.016041509	3.044241188	2.90468156	2.947427142	2.834196893	2.856318437
2016	66	2.903317409	2.939622861	3.016041509	3.044809875	2.90468156	2.948499219	2.834196893	2.856869722

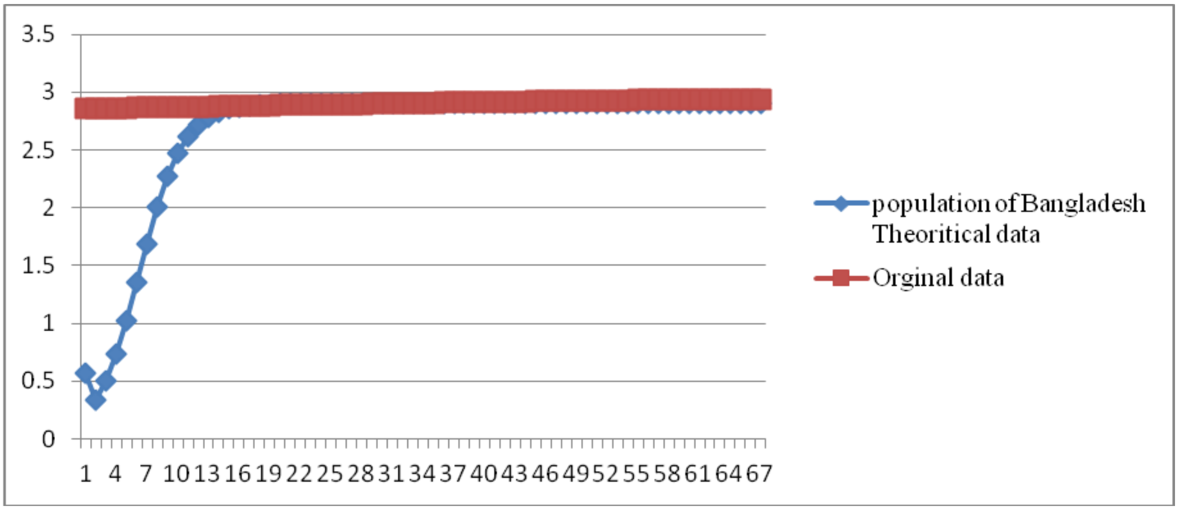

Fig 1. Comparing graph of theoretical data with original data of Bangladesh

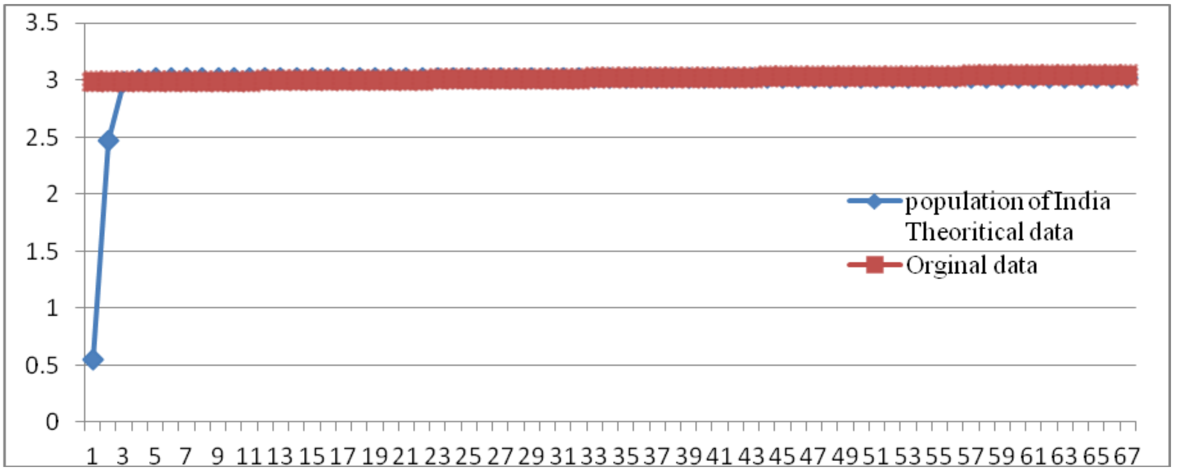

Fig 2. Comparing graph of theoretical data with original data of India

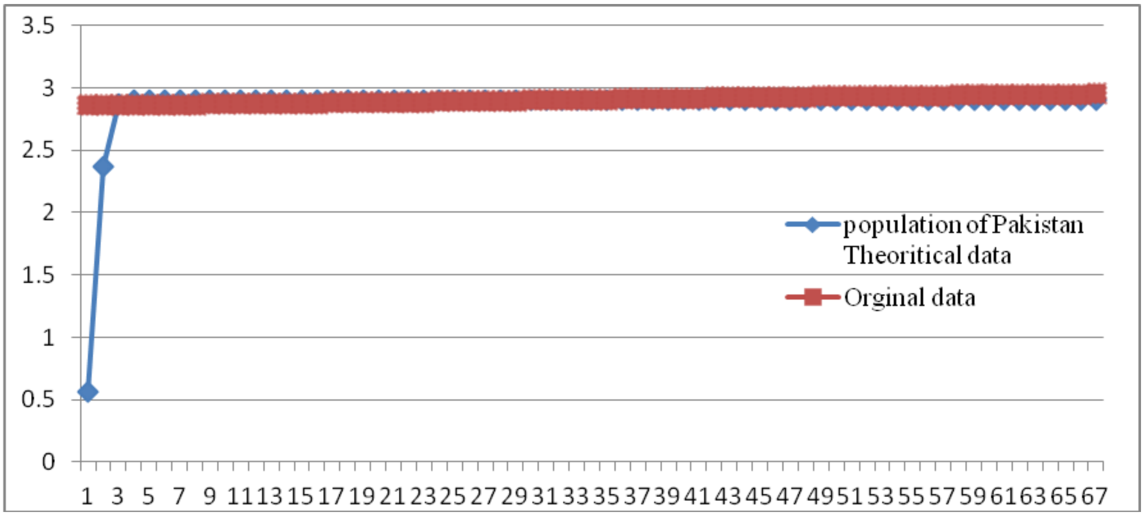

Fig 3. Comparing graph of theoretical data with original data of Pakistan

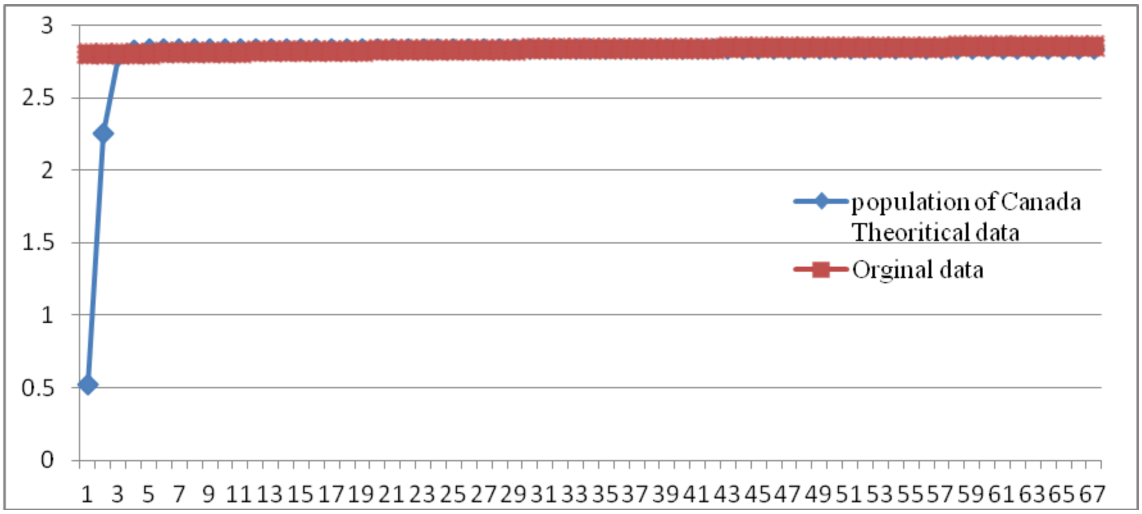

Fig 4. Comparing graph of theoretical data with original data of Canada

2. Conclusion

The carrying capacity of Bangladesh is 85415102.72 but at this moment total number of population is 164827718. It is the biggest problem. The government of Bangladesh needs to take necessary step otherwise socio economic system is breakdown. Every country of Subcontinent, the total population of these countries is grater twice of carrying capacity. In Canada, total number of population is greater the carrying capacity.

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