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On Computational Assessment of Perturbation Parameter (Ɛ) on Third Order Singularly Perturbed Convection-Diffusion Boundary Value Problems

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Abstract

Two point boundary value problems for third order singularly perturbed ordinary differential equation in which the highest order derivative is multiplied by perturbation parameter ε was examined. Exponentially Fitted Collocation Approximate Method was proposed and employed to assess the effect of perturbation parameter (ε) on the highest derivation of the singularly perturbed convection-diffusion equation. The perturbation parameter (ε) was varied from 0.5 to 0.1 and obtained the corresponding solutions of $y(x)$. The results demonstrate that this method is very convenient for solving boundary value problems and also can be successfully apply to a lot of practical engineering and physical problems.

Keywords

Third order singularly perturbed convection-diffusion equation, exponentially fitted collocation approximate method, effect of perturbed parameter (ε) and numerical results.

1. Introduction

Singularly perturbed third order two-points boundary value problems have been received a significant attention in past and recent years. It arises in many areas of both physical and biological sciences such as in fluid mechanics, particle physics, combustion processes and decay of specimen in biology sciences. These problems depend on a small positive parameter (ε) in such a way that the solution varies rapidly in some parts and varies slowly in other parts. An extensive study and analysis have been carried on for more than two decades. A good amount of research works is reported on the qualitative and quantitative analysis for singularly perturbed of boundary value problems [1-4].

In this study, we consider Third order singularly perturbed convection-diffusion perturbedboundary value problem of the form:

$$\begin{cases} -\varepsilon \dfrac{dy^3}{dt^3} + a(t)\dfrac{dy^2}{dt^2} + b(t)\dfrac{dy}{dt} + c(t)y(t) = \\ 4\left(1 + \dfrac{(1-e^{\frac{-2t}{\varepsilon}})}{2(1-e^{\frac{-2}{\varepsilon}})} - \dfrac{t}{2}\right) + \varepsilon\dfrac{(1-e^{\frac{-2t}{\varepsilon}})}{4(1-e^{\frac{-2}{\varepsilon}})} + \dfrac{t^2}{4} - t\left(1 + \dfrac{1}{2(1-e^{\frac{-2}{\varepsilon}})}\right) \qquad t \in [0,1] \end{cases} \tag{1}$$

With boundary conditions

$$\begin{cases} y(0) = \alpha \\ y/(0) = \beta \\ y/(1) = \gamma \end{cases} \tag{2}$$

Where a(t), b(t) and c(t) are smooth function, α, β, γ are known constants and ε is perturbation parameter.

In recent years, several studies concerning computational analysis are resisted to second order boundary value problems where only few results are available for higher order equation [5,6]. The behaviors of singularly perturbed boundary value problems depend on value of perturbed parameter ε. If we set $\varepsilon = 0$ the equation (1) reduce to second order differential equation which is also well- posed and $\varepsilon > 0$ equation (1) is remain third order singularly perturbed ordinary differential of convection-diffusion type.Singularly perturbed ordinary differential are usually difficult to solve analytically so it is required to obtain the approximate solution. So the present work will serve as an alternate technique to solve the SPBVP.

The first contribution of this work is to assess the effect of perturbation parameter ε from 0.5 to 0.1 on the highest derivative of the equation (1). Second contribution is to demonstrate the applicability of the present method to solve linear problems with left end boundary layer are considered. Table 1 computational results of absolute error is presented and it is observed that the present method has very low error when epsilon is higher whilecomparing to analytical solution.

1.1 Definition of Chebyshev Polynomials

The Chebyshev polynomials of first kind can be defined by the recurrence relation given by

$$T0\ (t) = 1$$
$$T1\ (t) = 2t - 1$$

Thus, we have

$$T_{N+1}(t) = 2(2t - 1)\, T_N(t) - T_{N-1}(t) \qquad N \geq 1 \tag{3}$$

Table 1. The First Ten (10) Chebyshev Polynomials

$T_N(t)$	Chebyshev Polynomials
$T_0(t)$	1
$T_1(t)$	$2t - 1$
$T_2(t)$	$8t^2 - 8t + 1$
$T_3(t)$	$128t^4 - 258t^3 + 160t^2 - 32t + 1$
$T_4(t)$	$128t^4 - 258t^3 + 160t^2 - 32t + 1$
$T_5(t)$	$512t^5 - 1280t^4 + 1120t^3 - 400t^2 + 50t - 1$
$T_6(t)$	$2048t^6 - 6144t^5 + 6912t^4 - 3584t^3 + 640t^2 - 72t + 1$
$T_7(t)$	$8172t^7 - 20672t^6 + 39424t^5 - 26880t^4 + 9408t^3 - 1568t^2 + 98t - 1$
$T_8(t)$	$32768t^8 - 131072t^7 + 212992t^6 - 180224t^5 + 84480t^4 - 21504t^3 + 2688t^2 - 128t + 1$
$T_9(t)$	$131072t^9 - 589824t^8 + 1105920t^7 - 1118208t^6 + 658944t^5 - 228096t^4 + 44352t^3 - 4320t^2 - 1$
$T_{10}(t)$	$52488t^{10} - 2621440t^9 - 5570560t^8 + 6553600t^7 - 4659200t^6 + 2050048t^5 - 549120t^4 + 84480t^3 - 6600t^2 - 200t + 1$

2 Descriptionof Numerical Technique

In this section, we present and employ exponentially fitted collocation approximate method in [7] to examine the effect of epsilon (ε) on third order singularly perturbed convection-diffusion equation (1).

Approximate solutions used are of the form:

i. The Power series of the form

$$y_N(t) = \sum_{q=0}^{N} s_q\, t^q \tag{4}$$

ii. The Exponentially fitted approximate solution of the form:

$$y_N(t) \approx \sum_{q=0}^{N} s_q\, t^q + \tau_3\, e^t \tag{5}$$

where t represents the dependent variable and τ_3 represents free tau parameter of the order 3rd of the dependent variable in equation (1), s_q y_N (t) ($q \geq 0$) are the unknown constants to be determined and is the length of computational length of degree of Chebysev polynomials.

Obtaining the third derivative of equation (4) and Substitute into equation (1), we have

$$-\varepsilon\left[\sum_{q=3}^{N} q(q-1)(q-2)s_q t^{q-3}\right]+a(t)\left[\sum_{q=2}^{N} q(q-1)s_q t^{q-2}\right]+b(t)\left[\sum_{q=1}^{N} q s_q t^{q-1}\right] + c(t)\left[\sum_{q=0}^{N} s_q t^{q}\right] = \left[\mathbf{4}\left(\mathbf{1}+\frac{\left(\mathbf{1}-e^{\frac{-2t}{\varepsilon}}\right)}{\mathbf{2}\left(\mathbf{1}-e^{\frac{-2}{\varepsilon}}\right)}-\frac{t}{\mathbf{2}}\right)+\varepsilon\frac{\left(\mathbf{1}-e^{\frac{-2t}{\varepsilon}}\right)}{\mathbf{4}\left(\mathbf{1}-e^{\frac{-2}{\varepsilon}}\right)}+\frac{t^2}{\mathbf{4}}-t\left(\mathbf{1}+\frac{\mathbf{1}}{\mathbf{2}\left(\mathbf{1}-e^{\frac{-2}{\varepsilon}}\right)}\right)\right] \tag{6}$$

$$\begin{cases} -\varepsilon\,[6s_3+24ts_4+60t^2s_5+\cdots+N(N-1)(N-2)s_Nt^{N-3}]+ \\ a(t)[2s_2+6ts_3+12t^2s_4+20t^3s_5+\cdots+N(N-1)s_Nt^{N-2}]+ \\ b(t)[s_1+2ts_2+3t^2s_3+4t^3s_4+5t^4s_5+\cdots+Ns_Nt^{N-1}]+ \\ c(t)[s_0+ts_1+t^2s_2+t^3s_3+t^4s_4+t^5s_5+\cdots+s_Nt^N]+ \\ =\left[\mathbf{4}\left(\mathbf{1}+\frac{\left(1-e^{\frac{-2t}{\varepsilon}}\right)}{2\left(1-e^{\frac{-2}{\varepsilon}}\right)}-\frac{t}{2}\right)+\varepsilon\frac{\left(1-e^{\frac{-2t}{\varepsilon}}\right)}{4\left(1-e^{\frac{-2}{\varepsilon}}\right)}+\frac{t^2}{4}-t\left(\mathbf{1}+\frac{1}{2\left(1-e^{\frac{-2}{\varepsilon}}\right)}\right)\right] \end{cases}$$

Collect the likes terms of (6)

$$\begin{cases} c(t)s_0 \\ [b(t)+c(t)t]s_1 \\ [2a(t)+2tb(t)+c(t)t^2]s_2 \\ [-6\varepsilon+6ta(t)+3t^2b(t)+c(t)t^3]s_3 \\ [-24t\varepsilon+12t^2a(t)+4t^3b(t)+c(t)t^4]s_5 \\ [-60t^2\varepsilon+20t^3a(t)+5t^4b(t)+c(t)t^5]s_6 \\ + \\ \vdots \\ + \\ (-\varepsilon N(N-1)(N-2)t^{N-3}+a(t)N(N-1)t^{N-2}+b(t)Nt^{N-1}+c(t)t^N)s_N \\ =\left[\mathbf{4}\left(\mathbf{1}+\frac{\left(1-e^{\frac{-2t}{\varepsilon}}\right)}{2\left(1-e^{\frac{-2}{\varepsilon}}\right)}-\frac{t}{2}\right)+\boldsymbol{\varepsilon}\frac{\left(1-e^{\frac{-2t}{\varepsilon}}\right)}{4\left(1-e^{\frac{-2}{\varepsilon}}\right)}+\frac{t^2}{4}-t\left(\mathbf{1}+\frac{1}{2\left(1-e^{\frac{-2}{\varepsilon}}\right)}\right)\right] \end{cases} \tag{7}$$

Slightly perturbed and collocate equation (7), we have

$$\begin{cases} c(t_i)s_0 \\ [b(t_i)+c(t_i)t_i]s_1 \\ [2a(t_i)+2t_ib(t_i)+c(t_i)t^2]s_2 \\ [-6\varepsilon+6t_ia(t)+3t_i{}^2b(t_i)+c(t_i)t_i{}^3]s_3 \\ [-24t_i\varepsilon+12t_i{}^2a(t_i)+4t_i{}^3b(t_i)+c(t_i)t_i{}^4]s_5 \\ [-60t_i{}^2\varepsilon+20t_i{}^3a(t_i)+5t_i{}^4b(t_i)+c(t_i)t_i{}^5]s_6 \\ + \\ \vdots \\ + \\ (-\varepsilon N(N-1)(N-2)t_i{}^{N-3}+a(t_i)N(N-1)t_i{}^{N-2}+b(t_i)Nt_i{}^{N-1}+c(t_i)t_i{}^N)s_N \\ =\left[\mathbf{4}\left(\mathbf{1}+\frac{\left(1-e^{\frac{-2t_i}{\varepsilon}}\right)}{2\left(1-e^{\frac{-2}{\varepsilon}}\right)}-\frac{t_i}{2}\right)+\boldsymbol{\varepsilon}\frac{\left(1-e^{\frac{-2t_i}{\varepsilon}}\right)}{4\left(1-e^{\frac{-2}{\varepsilon}}\right)}+\frac{t_i{}^2}{4}-t_i\left(\mathbf{1}+\frac{1}{2\left(1-e^{\frac{-2}{\varepsilon}}\right)}\right)\right] \\ +H(t_i) \end{cases} \tag{8}$$

Where $H(t) = -\tau_1 T_N(t_i) - \tau_2 T_{N-1}(t_i) - \tau_3 T_{N-2}(t_i)$ and $t_i = a + (b-a)^i / N+2$; $i = 1,2,\dots\dots N+1$

$$\left\{\begin{array}{c} c(t_i)s_0 \\ [b(t_i) + c(t_i)t_i]s_1 \\ [2a(t_i) + 2t_i b(t_i) + c(t_i)t^2]s_2 \\ [-6\varepsilon + 6t_i a(t) + 3{t_i}^2 b(t_i) + c(t_i){t_i}^3]s_3 \\ [-24t_i\varepsilon + 12{t_i}^2 a(t_i) + 4{t_i}^3 b(t_i) + c(t_i){t_i}^4]s_5 \\ [-60{t_i}^2\varepsilon + 20{t_i}^3 a(t_i) + 5{t_i}^4 b(t_i) + c(t_i){t_i}^5]s_6 \\ + \\ \vdots \\ + \\ (-\varepsilon N(N-1)(N-2){t_i}^{N-3} + a(t_i)N(N-1){t_i}^{N-2} + b(t_i)N{t_i}^{N-1} + c(t_i){t_i}^N)s_N \\ -\tau_1 T_N(t_i) - \tau_2 T_{N-1}(t_i) - \tau_3 T_{N-2}(t_i) \\ = \left[\mathbf{4}\left(\mathbf{1} + \frac{\left(\mathbf{1}-e^{\frac{-2t_i}{\varepsilon}}\right)}{\mathbf{2}\left(\mathbf{1}-e^{\frac{-2}{\varepsilon}}\right)} - \frac{t_i}{\mathbf{2}}\right) + \boldsymbol{\varepsilon}\frac{\left(\mathbf{1}-e^{\frac{-2t_i}{\varepsilon}}\right)}{\mathbf{4}\left(\mathbf{1}-e^{\frac{-2}{\varepsilon}}\right)} + \frac{{t_i}^2}{\mathbf{4}} - t_i\left(\mathbf{1} + \frac{\mathbf{1}}{\mathbf{2}\left(\mathbf{1}-e^{\frac{-2}{\varepsilon}}\right)}\right)\right] \end{array}\right. \tag{9}$$

where τ_1, τ_2 and τ_3 are free tau parameters to be determined and $T_N(t)$, $T_{N-1}(t)$ and $T_{N-2}(t)$ are the Chebyshev Polynomials defined in table (1).

Couple with boundary conditions (2) and using approximate solution (5), we have

$$\left\{\begin{array}{l} y(0) = s_0 + \tau_3 e^0 = \alpha \\ y/(0) = s_1 + \tau_3 e^0 = \beta \\ y(1) = s_1 + 2s_2 + 3s_3 + \dots + Ns_N + \tau_3 e^1 = \gamma \end{array}\right. \tag{10}$$

Altogether, we obtained (N+4) algebraic linear equations in (N+4) unknown constants. Thus, we put the (N+4) algebraic equations in Matrix form as:

$$SQ = G \tag{11}$$

Here

S = The system of equations of N+4

$Q = (s_0, s_1, s_2, s_3, \dots\dots s_N, \tau_1, \tau_2, \tau_3)^T$

$G = (g(t_0), g(t_1), g(t_2), g(t_3), .. g(t_N), \alpha, \beta, \gamma)^T$

Theunknown constants $s_0, s_1, s_2, s_3, s_4, \dots\dots, s_N, \tau_1, \tau_2$ and τ_3 are obtain using MAPLE 18 software which are then substituted into the approximate solution given (5)

3. Computational Experiment

In this section, we consider the singularly perturbed BVP (1) with discountinous source terms $a(t) = -2$. $b(t) = 4$, $c(t) = -1$ andepisolons $\varepsilon = 0.5, 0.4, 0.3, 0.2, 0.1$

Subject to boundary conditions:

$$y(0) = 0,\ \ y/(0) = 1,\ \ y/(1) = -1 \tag{12}$$

EFCAM Technique

Consider equation (8) and taking computational length N=18 (see table 1), we obtained the following:

$$\begin{cases} -s_0+(-t_i+4)s_1+(-t_i^2+8t_i-4)s_2+(-t_i^3+12t_i^2-3)s_3+ \\ (-t_i^4+16t_i^3-24t_i^2-3t_i)s_4+(-t_i^5+20t_i^4-40t_i^3-30t_i^2)s_5+ \\ (-t_i^6+24t_i^5-60t_i^4-60t_i^3)s_6+(-t_i^7+28t_i^6-84t_i^5-105t_i^4)s_7+ \\ (-t_i^8+32t_i^7-112t_i^6-168t_i^5)s_8+(-t_i^9+36t_i^8-144t_i^6-252t_i^5)s_9 \\ (-t_i^{10}+40t_i^9-180t_i^8-360t_i^7)s_{10}+(-t_i^{11}+44t_i^{10}-220t_i^9-495t_i^{10})s_{11}+ \\ (-t_i^{12}+48t_i^{11}-264t_i^{10}-660t_i^9)s_{12}+(-t_i^{13}+52t_i^{12}-312t_i^{11}-858t_i^{10})s_{13}+ \\ (-t_i^{14}+56t_i^{13}-364t_i^{12}-1092t_i^{11})s_{14}+(-t_i^{15}+60t_i^{14}-420t_i^{13}-1365t_i^{12})s_{15}+ \\ (-t_i^{16}+64t_i^{15}-480t_i^{14}-1680t_i^{13})s_{16}+(-t_i^{17}+68t_i^{16}-544t_i^{15}-2040t_i^{14})s_{17}+ \\ (-t_i^{18}+72t_i^{17}-612t_i^{16}-2448t_i^{15})s_{18} \\ -\left\{\begin{matrix} 34359738368t_i^{18}-313532612608t_i^{17}+1314259992576t_i^{16}- \\ 3354637893632t_i^{15}+5827062661120t_i^{14}-7291747172352t_i^{13}+ \\ 6787709272064t_i^{12}-4785513168896t_i^{11}+2577283940352t_i^{10}- \\ 1061606522880t_i^9+332657131520t_i^8-78324719616t_i^7+ \\ 135719034888t_i^6-1675631856t_i^5+140229120t_i^4-7343888t_i^3- \\ 209576t_i^2-2460t_i+5 \end{matrix}\right\}\tau_1 \\ -\left\{\begin{matrix} 8589934593t_i^{17}-74088185856t_i^{16}+292057776128t_i^{15}- \\ 696925552640t_i^{14}+1123872145408t_i^{13}-1294823587840t_i^{12}+ \\ 1098588880896t_i^{11}-697247276160t_i^{10}+332837683200t_i^9- \\ 119155808448t_i^8+31625183232t_i^7-6097082368t_i^6+ \\ 826009600t_i^5-74702080t_i^4+4155904t_i^3-123588t_i^2+1480t_i-3 \end{matrix}\right\}\tau_2 \\ -\left\{\begin{matrix} 2147483648t_i^{16}-17179869184t_i^{15}+62277025792t_i^{14}- \\ 135291469824t_i^{13}+196293427200t_i^{12}-200655503360t_i^{11}+ \\ 148562247680t_i^{10}-80648077312t_i^9+32133218304t_i^8- \\ 9313976320t_i^7+1926299648t_i^6-275185664t_i^5+25798656t_i^4- \\ 1462272t_i^3+43520t_i^2-512t_i+1 \end{matrix}\right\}\tau_3 \\ =\left[\mathbf{4}\left(\mathbf{1}+\frac{\left(1-e^{\frac{-2t_i}{0.5}}\right)}{2\left(1-e^{\frac{-2}{0.5}}\right)}-\frac{t_i}{2}\right)+\mathbf{0.5}\frac{\left(1-e^{\frac{-2t_i}{0.5}}\right)}{4\left(1-e^{\frac{-2}{0.5}}\right)}+\frac{t_i^2}{4}-t_i\left(\mathbf{1}+\frac{1}{2\left(1-e^{\frac{-2}{0.5}}\right)}\right)\right] \end{cases} \tag{13}$$

Collocate equation (13) as follows:

$t_i = a + \frac{(b-a)i}{N+2}$; $i = 1,2,3 \ldots\ldots N+1$ Where $a = 0$, $b = 1$, $N = 18$

$$\begin{cases} t_1=\frac{1}{20},\ t_2=\frac{2}{20},\ t_3=\frac{3}{20},t_4=\frac{4}{20},t_5=\frac{5}{20},t_6=\frac{6}{20},t_7=\frac{7}{20} \\ t_8=\frac{8}{20},\ t_9=\frac{9}{20},t_{10}=\frac{10}{20},t_{11}=\frac{11}{20},t_{12}=\frac{12}{20},t_{13}=\frac{13}{20} \\ t_{14}=\frac{14}{20},\ t_{15}=\frac{15}{20},t_{16}=\frac{16}{20},t_{17}=\frac{17}{20},t_{18}=\frac{18}{20},t_{19}=\frac{19}{20} \end{cases}$$

The above process was repeated for ε = 0.5, 0.4, 0.3, 0.2, 0.1. Moreover, we consider boundary conditions (12), Matrix equation (11) and using MAPLE 18 software to obtain twenty two (22) unkown constants of equations (13), thus, we obtained the following constants:

	When $\varepsilon = 0.5$	When $\varepsilon = 0.4$	When $\varepsilon = 0.3$	When $\varepsilon = 0.2$	When $\varepsilon = 0.1$
s_0	1.1636E − 06	−8.6902E − 06	−1.4397E − 05	−5.1755E − 06	−1.0143E − 03
s_1	1.0000011640	0.9999913098	0.9999856030	0.9999482443	0.9989856901
s_2	−1.427924825	−1.473625807	−1.532970439	−1.628450371	−1.872325586
s_3	1.9038319800	2.4565268000	3.4073206240	5.4342509300	12.443812400
s_4	−3.367072146	−5.147235927	−8.971161939	−20.15086862	−83.75434007
s_5	4.0591698980	7.6154198540	17.006860320	55.462891600	421.20495020
s_6	−3.927342307	−9.670754574	−26.8395936	−128.0048971	−1681.041317
s_7	2.78220809800	12.681284380	38.939385850	268.14262460	5413.7936780
s_8	−0.2644793304	−21.69756187	−59.34876860	−538.9897119	−14003.61527
s_9	−4.4282897030	43.13865343	97.177711350	018.57357400	28428.896300
s_{10}	11.2772908600	−73.8589032	−146.213342	−1663.45892	−43352.2737
s_{11}	−17.45664855	89.80408131	165.1682500	2133.584930	45341.19378
s_{12}	18.160624210	−60.7867198	−103.853302	−1940.86518	−23537.9384
s_{13}	−11.75122987	−8.05615254	−24.3834126	1020.015636	−13272.9781
s_{14}	2.9314504890	66.38315959	128.0032498	8.897697004	39055.91037
s_{15}	2.1085852740	−73.1862047	−136.180727	−474.572418	−38043.1721
s_{16}	−2.326818278	42.43409148	77.69522227	369.0598589	20789.34569
s_{17}	0.9136036384	−13.3976911	−24.2691245	−131.551422	−6327.03945
s_{18}	−0.139261008	1.824251728	3.276140493	19.15547632	840.0284046
τ_1	−1.7093E − 07	1.0130E − 06	2.2130E − 06	3.6247E − 06	1.0510E − 04
τ_2	−1.8030E − 07	1.802E − 06	4.2680E − 06	8.0987E − 06	4.4926E − 04
τ_3	−1.1636E − 06	8.690E − 06	1.4397E − 06	5.1755E − 06	1.0143E − 03

Substitute the above values into approximation solution (5), we obtained the following approximate solutions for equation (1),

$$y_{\varepsilon=0.5}(t) \approx \begin{cases} 0.000001636 + 1.0000011640\text{t} - 1.427924825t^2 \\ +1.9038319800t^3 - 3.367072146t^4 + 4.0591698980t^5 \\ -3.927342307t^6 + 2.7822080980t^7 - 0.2644793304t^8 \\ -4.42828970309t^9 + 11.2772908600t^{10} - 17.45664855t^{11} \\ +18.160624210t^{12} - 11.75122987t^{13} + 2.9314504890t^{14} \\ +2.1085852740t^{15} - 2.326818278t^{16} + 0.9136036384t^{17} \\ -0.139261008t^{18} - 0.000001163e^t \end{cases} \tag{14}$$

$$y_{\varepsilon=0.4}(t) \approx \begin{cases} 0.0000086902 + 0.9999913098\text{t} - 1.473625807t^2 \\ +2.4565268000t^3 - 5.147235927t^4 + 7.6154198540t^5 \\ -9.670754574t^6 + 12.681284380t^7 - 21.69756187t^8 \\ +43.13865343t^9 - 73.8589032t^{10} + 89.80408131t^{11} \\ -60.7867198t^{12} - 8.05615254t^{13} + 66.38315959t^{14} \\ -73.1862047t^{15} + 42.43409148t^{16} - 13.3976911t^{17} \\ +1.824251728t^{18} - 0.000008690e^t \end{cases} \tag{15}$$

$$y_{\varepsilon=0.3}(t) \approx \begin{cases} -0.000014397 + 0.9999856030\text{t} - 1.532970439t^2 \\ +3.4073206240t^3 - 8.971161939t^4 + 17.006860320t^5 \\ -26.8395936t^6 + 38.939385850t^7 - 59.34876860t^8 \\ 97.177711350t^9 - 146.213342t^{10} + 165.1682500t^{11} \\ -103.853302t^{12} - 24.3834126t^{13} + 128.0032498t^{14} \\ -136.180727t^{15} + 77.69522227t^{16} - 24.2691245t^{17} \\ +3.276140493t^{18} + 0.0000014397e^t \end{cases} \tag{16}$$

$$y_{\varepsilon=0.2}(t) \approx \begin{cases} 0.0000051755 + 0.9999482443\text{t} - 1.628450371t^2 \\ +5.4342509300t^3 - 20.15086862t^4 + 55.462891600t^5 \\ -128.0048971t^6 + 268.14262460t^7 - 538.9897119t^8 \\ +018.57357400t^9 - 1663.45892t^{10} + 2133.584930t^{11} \\ -1940.86518t^{12} + 1020.015636t^{13} + 8.897697004t^{14} \\ -474.572418t^{15} + 369.0598589t^{16} - 131.551422t^{17} \\ +19.15547632t^{18} + 0.0000051755e^t \end{cases} \tag{17}$$

$$y_{\varepsilon=0.1}(t) \approx \begin{cases} 0.000001636 + 1.0000011640\text{t} - 1.427924825t^2 \\ +1.9038319800t^3 - 3.367072146t^4 + 4.0591698980t^5 \\ -3.927342307t^6 + 2.7822080980t^7 - 0.2644793304t^8 \\ -4.42828970309t^9 + 11.2772908600t^{10} - 17.45664855t^{11} \\ 18.160624210t^{12} - 11.75122987t^{13} + 2.9314504890t^{14} \\ +2.1085852740t^{15} - 2.326818278t^{16} + 0.9136036384t^{17} \\ -0.139261008t^{18} - 0.000001163e^t \end{cases} \tag{18}$$

Table 2.Computational Results

t	**Solution**	$\varepsilon = 0.5$	$\varepsilon = 0.4$	$\varepsilon = 0.3$	$\varepsilon = 0.2$	$\varepsilon = 0.1$
0.0	Analytical	0.00000000000	0.00000000000	0.00000000000	0.00000000000	0.00000000000
	EFCAM	0.00000000000	0.00000000000	0.00000000000	0.00000000000	0.00000000000
	E_t	0.00000000000	0.00000000000	0.00000000000	0.00000000000	0.00000000000
0.1	Analytical	0.08732480667	0.08727316226	0.08732714050	0.08758380987	0.08829670039
	EFCAM	0.08732480698	0.08727316127	0.08732718805	0.08758384808	0.08830724140
	E_t	3.1E-10	1.0E-09	4.96E-08	3.82E-08	1.05E-06

0.2	Analytical	0.15380743913	0.15441264766	0.15569467519	0.15809116007	0.16223677044
	EFCAM	0.1538074376	0.15441268760	0.15569488610	0.15809167550	0.16226139000
	E_t	1.5E-09	3.99E-08	2.11E-07	5.15E-07	2.46E-05
0.3	Analytical	0.20316520477	0.20535369210	0.20903433839	0.21481446918	0.22301986773
	EFCAM	0.20316519850	0.20535378940	0.20903475900	0.21481553700	0.22305676040
	E_t	6.3E-09	9.73E-08	4.21E-07	1.07E-06	3.69E-05
0.4	Analytical	0.23656925886	0.24096907270	0.24764237571	0.25711561974	0.26922564119
	EFCAM	0.23656924710	0.24096922920	0.24764301850	0.25711716080	0.26927090950
	E_t	1.18E-08	1.56E-07	6.43E-07	1.15E-06	4.53E-05
0.5	Analytical	0.25379653544	0.26067454790	0.27045536396	0.28346995642	0.29926141059
	EFCAM	0.25379652020	0.26067475660	0.27045622260	0.2834718191	0.29931059480
	E_t	1.52E-08	2.08E-07	8.59E-07	1.86E-06	4.91E-05
0.6	Analytical	0.25383754315	0.26316488178	0.27586535944	0.29208901917	0.31132785630
	EFCAM	0.25383752960	0.26316512550	0.27586640700	0.29209095250	0.31137341060
	E_t	1.38E-08	2.44E-07	1.04E-06	1.93E-06	4.55E-05
0.7	Analytical	0.23520499291	0.24673102237	0.26197491407	0.28097879930	0.30333897621
	EFCAM	0.23520498380	0.24673127400	0.26197610460	0.28098051720	0.30337324640
	E_t	9.1E-09	2.52E-07	1.19E-06	1.77E-06	3.42E-05
0.8	Analytical	0.19607909489	0.20937859564	0.22664570698	0.24788853076	0.27286270989
	EFCAM	0.19607909350	0.20937881560	0.22664696210	0.24788959650	0.27287182820
	E_t	1.4E-09	2.20E-07	1.25E-06	1.06E-06	9.11E-05
0.9	Analytical	0.13436423441	0.14885362783	0.16747495371	0.19024648020	0.21706062723
	EFCAM	0.13436424650	0.14885378840	0.16747614340	0.19024631040	0.21704379980
	E_t	1.21E-08	1.61E-07	1.19E-06	1.67E-06	5.68E-05
1.0	Analytical	0.04769756447	0.06262527802	0.081747561244	0.1050958218	0.13261918283
	EFCAM	0.04769759489	0.06262531054	0.081748338150	0.10509460650	0.13254278020
	E_t	3.041E-08	3.22E-08	7.77E-07	1.22E-06	7.64E-05

Note: E_t = |Analytical - EFCAM|

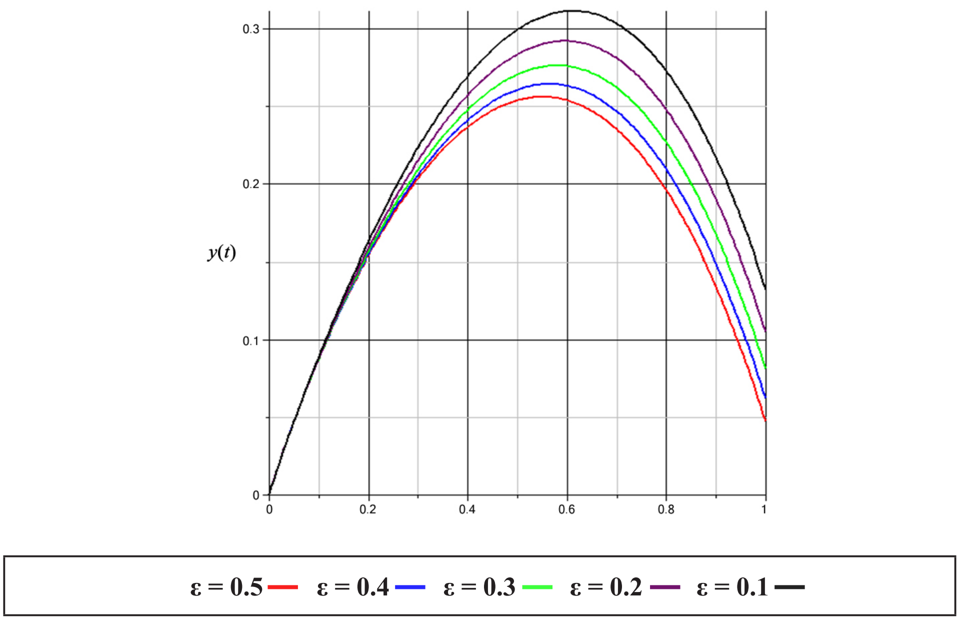

Figure 1. On computational assessment of epsilon (Ɛ) on third order singularly perturbed boundary value problem

4. Conclusion

In this study, third order convection-diffusion singularly perturbed boundary equation was examined and perturbation parameter (ε) was varied from highest to lowest ε = 0.5 to 0.1. The effect of epsilon was investigated at the highest derivation of the equation (1) and the result showed that higher the epsilon, given rises to lesser $y(t)$ (see table 2). Figure 1 shows the effect of epsilon parameters on third order convection-diffusion equation under study.The method is straightforward in application and a powerful mathematical tool for finding approximate solutions of initial value problems (IVP), boundary value problems (BVP), singular value problems (SVP) and Integro-differential equations (IDE) in terms of accuracy, computational length and efficiency.

Reference

[1] Abrahamsson, L. R., Keller, H. B., & Kreiss, H. O. (1974). Difference approximations for singular perturbations of systems of ordinary differential equations. *Numerische Mathematik*, 22(5), 367-391..

[2] Jayakumar, J., & Ramanujam, N. (1994). A numerical method for singular perturbation problems arising in chemical reactor theory. *Computers & Mathematics with Applications*, 27(5), 83-99.

[3] Natesan, S., & Ramanujam, N. (1998). A computational method for solving singularly perturbed turning point problems exhibiting twin boundary layers. *Applied Mathematics and Computation*, 93(2-3), 259-275.

[4] Roos, H. G., Stynes, M., & Tobiska, L. (2008). *Robust numerical methods for singularly perturbed differential equations: convection-diffusion-reaction and flow problems* (Vol. 24). Springer Science & Business Media.

[5] Weili, Z. (1990). Singular perturbations of boundary value problems for a class of third order nonlinear ordinary differential equations. *Journal of Differential equations*, 88(2), 265-278.

[6] Kadalbajoo, M. K., & Reddy, Y. N. (1987). Approximate method for the numerical solution of singular perturbation problems. *Applied Mathematics and Computation*, 21(3), 185-199.

[7] Falade K.I (2015) *Exponentially fitted collocation approximation method for singular initial value problems and integro-differential equations*. (Doctoral dissertation, University of Ilorin).