The Educational Review, USA, 2018, 2(7), 371-378
http://www.hillpublisher.com/journals/er

Riemann Integral: Didactical Mediation with Geogebra Software Articulated with Usual Practices with 1st Year Graduate[1] Students in Mathematics Teaching

Pedro Mateus

Pedagogical University, Beira branch, Moçambique

How to cite this paper: Mateus, P. (2018). Riemann Integral: Didactical Mediation with Geogebra Software Articulated with Usual Practices with 1st Year Graduate Students in Mathematics Teaching. *The Educational Review, USA*, 2(7), 371-378.
http://dx.doi.org/10.26855/er.2018.07.001

Corresponding author: Pedro Mateus, Pedagogical University, Beira branch, Moçambique.

Abstract

In the article we present the results of an experimental teaching aiming to answer the question: how effective is a didactic mediation of concepts on Riemann integral, using Geogebra software, articulated with usual practices, by students, based on their mobilized and available retrospective knowledge? The goal was to experiment a teaching and learning modes of the Riemann integral of real functions of a real variable, using Geogebra software as instrument, articulated with usual practices. The study was based in Anthropological Theory of the Didactic—TAD by Chevallard and theory of instrumentation by Rabardel. It was a qualitative study in the form of case study, having appealed to some aspects of didactic engineering: design and a priori analysis of tasks; a posteriori analysis and internal validation. The experiment showed that the blended computer and usual practices processes promote construction of knowledge by students to the Riemann integral.

Keywords

Didactical Mediation, Riemann Integral, Anthropological Theory of the Didactic, Theory of Instrumentation/Instrumentalization

1. Introduction

The Riemann integral is one of the fundamental concepts of Calculus and Mathematical Analysis, with wide application in important sectors of social activity, industrial and economic as well as in many sciences and mathematics itself. Furthermore, this concept causes some problems in the teaching and learning in high school and / or college.

The studies by Araujo (2002), Viana (1998), Leng (2011), just to mention a small part, show a diversity of problems on teaching and learning Calculus. Reported problems highlight the lack of understanding of the basic concepts on the subject, often causing failures of students.

We conducted this study to answer the following question: how effective is a didactical mediation for building and learning the

[1] Direct translation from Portuguese word "Licenciatura", a university degree between bachelor and master degrees. It seems that the most appropriate translation should be "licentiate degree", instead of "graduate". We preferred "graduate" to maintain the meaning most dictionaries suggest. In Brazilian system licenciatura's courses are only designed for teaching courses. In Portugal, however, the case of Mozambique, we find licentiate degrees in many others areas. The participants come from math for teachers' course. In the context of Mozambique, the courses of this type are said teaching courses. That why we say "students on Mathematics teaching".

concept of Riemann integral using the Geogebra software articulated with the usual practices? And we aim to experiment a modality of teaching and learning of the Riemann integral concept of real functions of a real variable, incorporating in the process Geogebra software articulated with usual media and practices.

From this objective, the theoretical framework of the research, which helped us in formulation and delimitation of the problematic is presented in the two following sections.

2. Anthropological Theory of the Didactic

The Anthropological Theory of the Didactic—TAD—describes the mathematical activity in the set of human activities regularly developed, describing the mathematical knowledge in terms of praxeological organizations or praxeologies whose basic notions are the notions of types of tasks T, techniques (ways of solving the tasks), technology (a rational discourse aimed to justify, explain and to produce techniques) and theories (aimed to justify, explain and to produce technologies) that allow to model the social practices in general and the mathematics activity, in particular (Chevallard, 1999, 2014). From this point of view, according to the author, the praxeology consists of a practical-technical block (praxis) [type of task/technique], which corresponds to a know-how, and a technological-theoretical block (logos) [technology/theory] which corresponds to a knowing. The notion of task presupposes a relatively precise object, for which there is some available technique with a technological-theoretical surrounding more or less explicit. In most cases, a task (and the type of associated task) is expressed by a verb evoking an action, what exists to do, for example, integrating the function $f(x) = \ln x$ between x = 1 and x = 2 is a task that can be justified with technology of integration by parts and as theory, the Fundamental Theorem of Calculus.

The following definition is the most important result on Riemann integral:

> Let f be a real function of a real variable in interval I = [a, b]. We say f is *integrable in I* (in Riemann sense) if and only if there is a number, denoted by $\int_a^b f(x)dx$ and called *integral* of f over I, such that, for any $\varepsilon > 0$, there is a partition
>
> P_ε (P_ε generally depends on ε) so that, $P \supset P_\varepsilon \Rightarrow$ $<\varepsilon$, understood that the inequality holds always an arbitrary number from the set of numbers { $\sum(P, f)$ } is replaced by number. (Labarre, 2008, p. 147).

We recall that the partition P of real interval I = [a, b], with more than one point, is a finite subset of points $P = \{t_0, t_1, t_2, \ldots, t_n\}$, such that $a \in P$ e $b \in P$, and $a = t_0 < t_1 < t_2 < t_3 < \ldots < t_{n-1} < t_n = b$. The relation $P \supset P_\varepsilon$ implies that the partition P refines the partition P_ε. In Labarre's (2008) view, the refinement process of I is the key idea on Riemann integral. We completely subscribe with this view and we believe that the geogebra software is particularly good to perform such a process. We also recall that the definition above does not distinguishes the type of partition P, whether regular or irregular. And, because of our limited capacity on computer programming, we used regular partition, and we think there was no loss of generality.

According to Chevallard (2002), the set of conditions and restrictions that allow the mathematical development (ecology of a mathematical praxeology), i.e., conditions and restrictions that allow the production and use of tasks in institutions, depends on the ostensible objects (perceptible to the human senses and capable to be handled, such as sounds, graphics and gestures) and non-ostensive objects. We assume the geogebra software as an ostensible object that allows materialize and handle a mathematical knowledge.

The non ostensive objects are those, such as ideas, intuitions or concepts institutionally existing, but cannot be seen, perceived or shown by themselves. The non-ostensive objects can only be evoked by an appropriate manipulation of certain associated ostensive objects. For example, to find the integral , we have to evoke some ideas, principles and laws of integration that can't be seen, but they drive the solver's actions to get the result. In the next section we present the theory of instrumentation.

3. Instrumentation Theory of Rabardel

We chose to study some aspects of Instrumentation/Instrumentation Theory from the perspective of Rabardel (1995), considering it important for the analysis of the relationship between researcher and students in their effort to interact with the tasks and with computer during the discussions in experimental work sessions.

Rabardel (1995) states that instrument replaces some functions from others, rebuilds and reconstructs the whole structure of behaviour. We feel that using geogebra software, changes the practices of teaching and learning Riemann integral, and the possibilities of abstraction and generalization of this concept.

The intermediary position of the instrument makes it as a mediator of the relationship between the subject and the object. It is an intermediary world, whose main characteristic is to be adapted both to the subject, as to the object. This adaptation occurs in material terms and in terms of cognitive and semiotic properties according to the type of activity in which the instrument is inserted or is intended to be inserted. Thus, two types of mediation are identified:

-A mediation from object to the subject, described as an epistemic mediation, in which the instrument is a means enabling the user to know the intended object;

-A pragmatic mediation, from subject to the object, in which the instrument is a means for transforming action (in a broader sense, including control and regulation) directed to the object.

The instrumental elaboration by the user is thus, addressed, both for himself (this is the dimension of instrumental genesis called instrumentation), and for the artefact (the instrumentalization dimension).

Instrumentation processes are related to the emergence and evolution of the use of schemes and action mediated by the instrument: its constitution, functioning, and its evolution by accommodation, combination, coordination, inclusion and mutual assimilation, the assimilation of new artefacts to the set of the schemes already existing.

The instrumentalization processes are related to the emergence and evolution of artefactual components of the instrument: selection, consolidation, production and institution of functions, deviations and catachresis, assignment of property, artefactual transformation (structure, function, etc.) that prolong the creations and achievements of the artefacts whose limits are difficult to determine. In the sequence, we highlight the research method.

4. Research Methods

Based on the established problematic and built theoretical framework, we developed the methodological framework precisely based on the two previous theoretical assumptions and limited to some elements of didactic engineering (Artigue, 2010), with regard to the design and a priori analysis of tasks for experimentation; the experimentation; the a posteriori analysis and internal validation.

The experimental teaching was carried out in two sessions. The 1st session took place on 15th June 2013, and the second one, on 19th June 2013. Because of space available for this article, we present only some parts of experimental discussion of the session 2, and some descriptive general indications on what did happen in the session 1. In this respect, in the first session we discussed, using the function $f\colon I = [-2, 2] \to IR$, with the following formation rule: $f(x) = 2\sin(2x) + 3$, several basic ideas conducting to the integrability of a function f in the Riemann sense: partition P of I in n subintervals; the refinement of P, continuity of f in I and in subinterval $I_i = [t_{i-1}, t_i]$ of I, with i = 1, 2, …, n; upper and lower bounds, the existence of supremum (maximum) and the infimum (minimum) of f in I and in each subinterval $I_i = [t_{i-1}, t_i]$, and monotonicity of f in I and in each subinterval I_i.

Eight first year graduate prospect teachers attended the both sessions. The teaching experiment took place before the students formally studied the topic. Note that we assume the geogebra software as an ostensive object which materialize the mathematical ideas in classroom (Chavallard, 2002), allowing the process of constructing of the techniques and technologies (properties and theorems) of target notion, and as an instrument which mediates actions and structures the behaviour of teacher and learner (Rabardel, 1995, 2005). Also, as we stressed before, the geogebra software is integrated into the usual practices.

Group: **19/06/2013**

Task 1: Let us consider the function f: IR → IR, defined by $f(x) = x^2$. Consider the interval I = [0, 1] of the domain of f. Using Geogebra, let's approximate the area under the graph of f in the interval I, for underestimating and overestimating. As we obtain these areas we fill the table below.

Table caption:

n – number of subdivisions (subintervals);

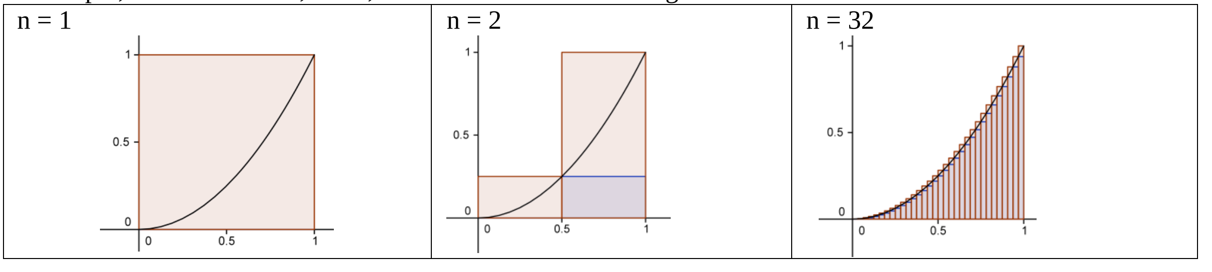
$\Delta x = \frac{b-a}{n} = \frac{1}{n}$ - length of each subinterval $[x_{i-1}, x_i]$ in I

$S_i = \sum_{i=1}^{n} \Delta x f(x_{i-1})$ - lower sum (sum of "inscribed" rectangles to the graph of f in I).

$S_s = \sum_{i=1}^{n} \Delta x f(x_i)$ - upper sum (sum of "circumscribed" rectangles to the graph of f in I).

$\omega_i = S_s - S_i$ – oscillation of the function f on interval I.

As an example, we fill for n = 1, n = 2, and n = 32. Continue filling the table for other values of n.



n	1	2				32						
$\Delta x = \frac{1}{n}$	1	0,5				0,0313						
$S_i = \sum_{i=1}^{n} \Delta x f(x_{i-1})$	0	0,125				0,3179						
$S_s = \sum_{i=1}^{n} \Delta x f(x_i)$	1	0,625				0,3491						
$\omega_i = S_s - S_i$	1	0,5				0,0312						

Reflections

a) What happens to the monotony of the following elements?

n ____

Δx ____

S_i ____

S_s____

b) The answers to the question a) are related. Try to find the meaning of these responses when compared to the number and sizes of rectangles involved in different iterations.

c) What must be the exact area of region under analysis? Why?

d) What must mean this exact area for this situation?

e) Assuming that we do not have Geogebra, develop an algebraic way that allows us to determine the same result.

Figure 1. Form of tasks for the students.

The second session was aimed to build the notions of Riemann lower and upper sums, the notion of oscillation of f in I and the corresponding Riemann integral. Bellow we present some extracts of this session.

At beginning, the researcher writes on the whiteboard the programming steps of the task in geogebra the students could follow:

Function: *f(x) = if [0 <= x <= 1, x ^ 2];*

Ends of the range: *a = 0, b = 1*;

Slider: *k = [0, 32], step 1;*

Number of rectangles: *n = 2 ^ k*;

Base of rectangle = *(b - a) / n*;

Lower sum *(si) = lowersum (f, a, b, n)*;

Upper sum *(Ss) = uppersum (f, a, b, n*).

After this script, the researcher distributes the form with the tasks for students to work in pairs (Figure 1).

After receiving this form (Figure 1), the students worked on it in pairs, with help of geogebra software.

5. Results

Next, we include some passages of the discussion, highlighting some sequences of occurred dialogues.

Tutor: Now is to fill that. Then we draw conclusions. For example, is just to see all the elements which are there. We have lower sum, upper sum. We have delta, we have n. We have oscillation. Well, I'll leave this, at this side here (the researcher refers to the script that he leaves on whiteboard corner) to be all clear. We have here (refer to the Figure1, for n = 1) means that the area (in the chart) is divided into one part only. As we were saying, to determine the upper sum, as our partition is on this interval ([0, 1]), it means that, if I want the lower sum, I take the lower end.

Students: We have one (width of rectangle) times f of zero. Then it will give this area, this line here (one student points the line [0, 1]). One student performs: $s(f, P1) = 1 \cdot f(0) = 1 \cdot 0 = 0$.

Tutor: In fact, it is that lower sum that we have over there. And the upper sum?

Students: It will be one times *f* of one. (They write $S_1 (f, P_1) = 1 \cdot f(1) = 1 \cdot 1 = 1$).

Tutor: Then it will be that area (writes down as the students did: $S_1(f, P1) = 1 \cdot f(1) = 1 \cdot 1 = 1$). So this is how you will fill this sheet here. The first part is already fulfilled, for example. For n = 1, the delta, which is it?

Students: One.

Tutor: The delta is the length of this division ($\Delta x = \frac{b-a}{n}$). So we have s_1, on the first column, already filled on.

You will continue, for n equals to 2, which delta, which s_i, lower sum, which is the upper sum, which is the oscillation.

Student: The oscillation!

Tutor: Yes. The oscillation is the difference between S_s and S_i, […]

The students work in computers, in their respective groups.

Group 2: Delta becomes constant ... Only n changes.

At this stage the students have the numerical results and corresponding graph, as shown at the Figure 2.

Tutor: […] The size of rectangles, what happens? And, as consequence, how is it S_i and S_s? How is it about lower sum? Upper sum, in those conditions? There are consequences. There are some consequences. Then which are the consequences? You have to talk about n, about delta, talk about lower sum, about upper sum […].

Tutor: What will be the area of this part here, under the graph (of $f(x) = x^2$), on interval I = [0, 1]? You are already seeing, is not it? Each one must try to show what must be the exact area.

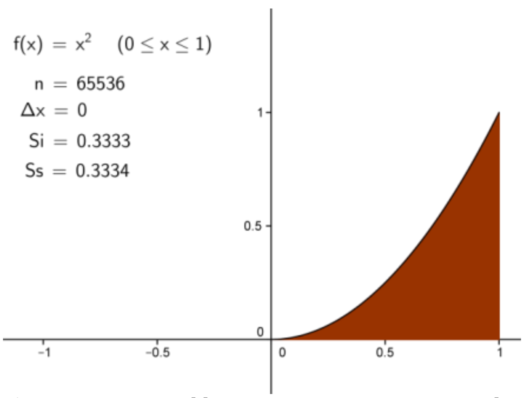


Figure 2. Upper and lower approximating areas under graph of function f: [0, 1] → IR, $f(x) = x^2$.

Up to this moment the students have the tables filled, as it is shown at a small part at Figure 3, accompanied with the corresponding graphic image, at each iteration, as in Figure 2. We note that this table comes from group 2, as the students worked in pairs, and all four pairs had similar responses.

n	4096	8192	16384	32762 ~~(26214)~~	65536	131072
$\Delta x = \frac{1}{n}$	.0,0002	0,0001	0,0001	.0	0	0
$S_i = \sum_{i=1}^{n} \Delta x f(x_{i-1})$	0,3332	0,3333	0,3333	0,3333	0,3333	0,3333
$S_s = \sum_{i=1}^{n} \Delta x f(x_i)$	0,3335	0,3334	0,3334	0,3334	0,3334	0,3334
$W_i = S_s - S_i$	0,0002	0,0001	0,0001	0,0001	0,0001	0,0001

Figure 3. Leading processes to the Riemann sums and integral, using geogebra software.

Tutor: Yes. What is the exact area, it is what I'm asking for, what you are seeing from there (in Figure 3). [...] How is it Si, and Ss in this case? Don't you have any idea what should be the exact area … or is there a group that has an idea about what should be the exact area?

Student (Micas): Yes. I have some ideas: (the students respond to the questions a), b) and c) on reflections section of the form):

a) What happens to the monotony for each of the following elements: n, Δx, S_i, S_s?

Student: The values of n are growing up, increasing; the values of Δx are diminishing, decreasing; the values of Si are growing up, increasing; the values of S_s are diminishing, decreasing

b) The answers to the question a) are related. Try to find the meaning of these responses when compared to the number and sizes of rectangles involved in different iterations.

Student: In regard to the size of rectangles involved in different iterations, in case of number 1 we have big rectangle, when value of n increases, the rectangles diminish their size, and the values of upper sum diminish as well, while the values of lower sum increase.

c) What must be the exact area of region under analysis? Why?

Student: With the increase of n, the exact area of the region in analysis will be the lower sum (0,3333). Because each time we increase the value of n, the part of upper sum tends to vanish.

Tutor: Well, this is the Mica's response, I don't know what others groups say?

The extract above gives some clues about how the discussion was developed and how the key ideas of convergence emerged in the processes of instrumentation (constructing of schemes of knowing) and instrumentalization (institution of function to the artefact), in the use of a computer as an instrument. We leave bellow some comments and conclusions about this teaching experiment.

6. Comments

We believe that the strategy used to study the construction procedure of upper sum and lower sum approximations using computer resource has been effective, it is achieved very good approximate results for the exact area. At this level the software was used as an ostensible object (Chevallard, 2002), the "materializer" of the mathematical processes to build the mathematical techniques and technologies (ways of solving mathematical questions and their explanations), according to Chevallard (1999, 2014) and as an instrument, mediator of actions to the target mathematical object (Rabardel, 1995, 2002). The graphical, numerical and algebraic visual images actually show what is wanted to be constructed: the ideas of approximation (the refinement of the partition) and its implications to the quality of approximation, inducing to the inferential reasoning for limit processes. We have the perception that the computer aided enough to characterize the target mathematical object by providing graphical visual information and numerical approximations.

We can formulate our perception from the perspective of Rabardel (1995), noting that the computational resource has expanded the possibilities of representation of the actions of students and the object of activity. To better understand this statement, just imagining how it would be possible, in traditional modes to obtain 32,768 rectangles in a range from 0 to 1, with about 0.00003 cm wide? The graphical result is effectively suggesting that the area under the graph of the function must be 0.3333. So, for us, this approach is dramatically different from traditional handlings on Riemann integral. The limit process arises naturally and meaningfully. Therefore, according to Rabardel (1995), the computational tool played, in this phase of activity, a real role of cognitive tool. We consider as cognitive resource at the disposal to the students, the feedback the machine gives as the student works on her. So, we stress that the geogebra software structured the behavior of students and the environment of learning: the milieu of learning.

On the other hand, we noticed some disturbing influences to the correct learning process, arising from information provided by the computer in relation to its limited capability to present the correct decimal places of the result and the overlap of the upper and lower sums that led the student Mica, as we see in his speech, in which he concluded that the upper decreasing area was vanishing. The student seems to have confused the upper area with the oscillation, as this is what really disappeared in the approximation process of the upper to the lower sum.

In general we believe that the strategy was effective in the way that it provided a construction of some elements of meaning (approximate sums) leading to the formal definition of the definite integral as generalization of process that was graphically well produced.

We still believe the articulation between the results (tables and figures) obtained through the computer and the theoretical analysis was consistent, because according to Rabardel (1995), the students show they have adapted themselves to the epistemic, pragmatic and heuristic functions of the instrument in the sense that it has provided an understanding of the task, its transformation to obtain the result, guidance and control of their actions. We interpret these processes as meaning a learning, i.e., the construction of knowledge by students when it is considered in the perspective of Chevallard (1999), as a work with a particular question to produce a satisfactory answer. So the learning corresponds to put in place the target notions in the exercises or in the problems. We base our claims on the students' behavior, both in computer use, in articulation between the result obtained by computer and on filling the table and in their interventions on the results obtained. These actions match with our expectances presented at a priori analysis (although they are not presented in this article).

It is our feeling that the strategy for the experiment was fully interpreted: articulate the computational resource with the usual practices in introduction to the Riemann integral, although we didn't use the idea of the norm of a partition, but it helped us to produce the generalization.

7. Conclusions

The notion of Riemann integral was effectively built with the didactical mediation with computational resources. The intervention stressed the central concepts of the definite integral, which are the concepts of partition refinement and numerical approximation (Labarre, 2008). We emphasize that for the students who participated to the study, it was the first moment for them to attend on the subject, as this content is not studied in high school. However, their performance was good, at least in relation to the type of interventions and arguments used to justify their ideas, as we have seen above.

References

Araújo, J. L. (2002). *Cálculo, Tecnologias e Modelagem Matemática: As discussões dos alunos* (Unpublished doctoral dissertation) – Universidade Estadual Paulista – UNESP. Instituto de Geociências e Ciências Exatas. Campus de Rio Claro.

Artigue, M. (2010). *Engénierie didactique: Notas do Seminário da EAE- Escola de Altos Estudos da CAPES*. Unpublished Manuscript, Pós-graduação em Educação Matemática, Universidade Bandeirante de São Paulo, São Paulo, Brasil.

Chevallard, Y. (2014). Théorie Anthropologique du Didactique & Ingénierie Didactique du Développement. *Journal du Seminaire tad/idd. UMR ADEF*.

Chevallard, Y. (2002). Organiser l'etude 3. Ecologie & Regulation. Retrieved from http://yves.chevallard.free.fr/spip/spip/.

Chevallard, Y. (1999). El análisis de las prácticas docentes en la teoría antropológica de lo didáctico. *Recherches en didactique des mathématiques*, 19(2), 221-266.

Labarre, A. E. Jr. (2008) Intermediate Mathematical Analysis. Mineola, New York: Dover Publications, Inc.

Leng, N. W. (2011). Using an Advanced Graphing Calculator in the Teaching and Learning of Calculus. National Institute of Education, Nanyang Technological University, Singapore. International Journal of Mathematical Education in Science and Technology, 42(7), 925-938.

Rabardel, P. (1995). Les hommes et les technologies, une approche cognitive des instruments contemporains. Paris: Armand Colin.

Vianna, C. C. S. (1998). *Students' Understanding of the Fundamental Theorem of Calculus: An Exploration of Definitions, Theorems and Visual Imagery* (Unpublished Doctoral Dissertation). Institute of Education, University of London.