

A Reissner-Nordström+ Λ Black Hole in the Friedman-Robertson-Walker Universe

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Abstract

A charged, non-rotating, spherically symmetric black hole which has cosmological constant Λ (Reissner-Nordström+ Λ or RN+ Λ), active gravitational mass M and electric charge Q is studied in exterior Friedman-Robertson-Walker (FRW) universe in (2+1) dimensional spacetime. We find new class of exact solutions of the charged black hole. It is found that the cosmological constant is negative inside the black hole. We confirm it from the geodesic equations too. The cosmological constant is found to be dependent on R , Q and $a(v)$ which correspond to the areal radius, charge, of the black hole and the scale factor of the universe respectively. We note that the expansion of the universe affects the size and the mass of the black hole. An important observation is that, for an observer at infinity, both the mass and charge of black hole increase with the contraction of the universe and decrease with the expansion of the universe.

Keywords

Black holes, Expanding Universe, Cosmological constant, Darms-Israel formalism

1. Introduction

Ever since their advent black holes have been studied in a great detail. However, almost all previous studies have focused either on isolated or binary black holes. But in reality, black holes are neither isolated nor only in binaries. They are actually embedded in the background of expanding universe. Therefore, we must study black holes in non-flat backgrounds in order to understand the black holes in real universe.

The main motivation to study (2 + 1)-dimensional spacetimes admitting black hole solutions is that the $d = 3$ cases have now attracted more attention and interest as compared to other (4) space -times with special mass and charge dependence. The solution of the Einstein field equations in (2 + 1)-d exhibit many characteristics of the (3 + 1)-d black hole. Moreover, the structure of (2 + 1)-d black hole is simple enough to derive a number of exact computations, particularly in the quantum realm and string theory, which are not possible in (3 + 1) dimensions. Such study may also give us a way to unify gravity and quantum theory. We know that the entropy of a black hole is proportional to its surface area, which is also consistent with a (2+1)-d black hole. We are further interested to derive the equation of motion for geodesics in vicinity of space-time of a (2 + 1)-dimensional charged black hole.

It is well known also that at the core or center of a black hole, according to general relativity, is a gravitational singularity, which is indeed a one-dimensional point. Its huge mass is located in an infinitesimal small space, where density as well as gravity is infinite and space-time curves infinitely. Hence, the notion of an actual physical singularity appears

quite unlikely, and possibly points to general relativity being rather incomplete.

The work on $(2 + 1)$ -d gravity theories has seen a great increase after the discovery that $(2 + 1)$ -d general relativity possesses a black hole solution [1]. It is the first example in this regard. The authors have observed that the fascinating properties of classical and especially quantum black hole, have long made it desirable to work on a lower-dimensional analog which could exhibit the key features and avert the unnecessary complications. It has been observed that such analog does exist in standard $(2 + 1)$ -d Einstein-Maxwell theory with a negative cosmological constant. This has further motivated us to work on $(2 + 1)$ -d black hole including the cosmological constant inside it.

Later on, Einstein-Maxwell [2] and Einstein-Maxwell-dilaton [3] extensions were also found. The authors studied black hole solutions which include all special characteristics that are observed in $(3+1)$ or higher dimensional black holes like horizon(s), black hole thermodynamics as well as Hawking radiation.

The dimensional reduction of black hole solutions in 4D general relativity is done and new 3D black hole solutions with an isotropic event horizon are obtained by Zanchin et al. [4]. Such event horizon is a typical characteristic of black hole and is an important study in our research too. The authors considered a 4D space-time with one spacelike Killing vector and observed that it is possible to split the Einstein-Hilbert-Maxwell action with a cosmological term in terms of 3D quantities. The authors in [5] have further formulated the three-dimensional Einstein-Maxwell-dilaton theory from the usual four-dimensional Einstein-Maxwell-Hilbert action for general relativity and observed that the 3D static spherically symmetric solution is analogous to the 4D Reissner-Nordström-AdS black hole. A particular case of the 3D action which presents Maxwell field, dilaton field and an extra scalar field, besides gravity field and a negative cosmological constant is chosen by them, and new 3D static black hole solutions are obtained.

In [6], the authors have constructed a large class of black hole solutions by the power Maxwell field. Here the Maxwell scalar has the form $(F^{\mu\nu} F_{\mu\nu})^k$. The particular choice of $k = d/4$, yields in general a traceless Maxwell's energy-momentum tensor. They however observed that $k = 3/4$ yields a general solution in $(2 + 1)$ -dimensional Einstein-power-Maxwell (EPM) spacetime which is devoid of the traceless condition.

The references are much more, but limited here, which have aroused the interest to study the Reissner-Nordström+ Λ black hole in the Friedman-Robertson-Walker universe.

The study on black holes is not completely new. It started long back in 1933 when McVittie [7] obtained his celebrated metric for a mass-particle in an expanding universe. This metric is nothing but the Schwarzschild black hole which is embedded in the Friedman-Robertson-Walker universe. In 1993, Kastor and Traschen found the multi-black holes solution in the background of de Sitter universe [8, 9]. The Kastor-Traschen solution describes the dynamical system of arbitrary number of extreme Reissner-Nordstrom black holes in the background of de Sitter universe. In 1999, Shiromizu and Gen studied charged rotating black hole in de Sitter background [10]. In 2000, Nayak et al. studied the solutions for the Schwarzschild and Kerr black holes in the background of the Einstein universe [11, 12]. In 2004, Gao et al. studied Reissner-Nordström black hole in the expanding universe [13].

In this paper, we extend the above studies from charged black holes into charged black holes which have cosmological constant inside them. It has been found in the literature that there are three possible black hole solutions depending on the cosmological constant being (1) positive (2) negative and (3) zero [14]. We first deduce the metric for a Reissner-Nordström+ Λ black hole in the expanding universe. We show that several special cases of our solution are exactly the same as some solutions discovered previously. We then study the effects of the evolution of the universe on the size, mass and charge of the black hole. We know that black holes exert a strong gravitational influence due to their mass, just like every other massive object in the Universe. This is how we actually discover and measure the mass of black holes, by watching their effect through gravitational lensing, accretion, X-ray emissions etc. For instance, the supermassive black hole at the center of the Milky Way galaxy is so strong gravitationally that the stars very near it orbit at a very, very high rate. Using this and the equations that describe the orbits of these stars, we can actually estimate the mass of the black hole.

N. Kaloper et al. [15] has analyzed the McVittie solutions of Einsteins field equations for describing the gravitational fields of spherically symmetric mass distributions in expanding FRW universes. They focused on spatially flat McVittie geometries and showed that the McVittie solutions which asymptote to FRW universes and dominated by a positive cosmological constant at late times are black holes with regular event horizons. Near the hole, the charge contributions correct the effective potential for the scalar and give it a large mass, as the supersymmetric attractor mechanism in asymptotically flat black holes.

T. Maki et al. [16] have studied $(N+1)$ -dimensional cosmological solutions describing the multi-black hole configuration in the same system with a cosmological constant. They investigated that the cosmological evolution of the scale factor depends on the coupling of the dilaton to the cosmological constant. The outline of our paper is envisaged as follows:

In section II we have solved the Einstein-Maxwell field equations for the static spherically symmetric line element for interior spacetime of a RN+ Λ black hole. The event horizons have been studied. The pressure, matter density and proper

charge density of the black hole have been expressed in terms of the mass, charge and the cosmological constant (Λ). The geodesics have been further verified. In section III we transform the RN+ Λ metric to the McVittie form [7] under suitable transformation conditions for compatible study with respect to the FRW universe. The various subconditions are specified. In Section IV, the boundary conditions are discussed and we further confirm the negative value of cosmological constant inside the black hole. The value of the curvature parameter k in the FRW metric is discussed. That the transformed RN+ Λ metric is an exact solution of the EM field equations and the metric is physically relevant has been studied in section V. We discuss the Darmois-Israel matching conditions in Section VI. In section VII, we further study the surface continuity. The study ends with a concluding remark in Section VIII.

2. Interior Reissner-Nordström with Λ metric

We know that if an electrically charged particle falls into the Schwarzschild black hole it becomes charged. To describe such a black hole one has to solve the Einstein-Maxwell equations considering the stress-energy tensor of the electromagnetic field. RN+ Λ metric is a static solution to the Einstein-Maxwell field equations, which corresponds to the electrovacuum gravitational field of a charged, non-rotating, spherically symmetric black hole of mass M . Hence, we follow the analogue of the RN+ Λ solution with exterior FRW metric for a spacetime with a cosmological constant. Under such conditions, the metric of the line element for the interior space-time of a static spherically symmetric charged distribution of matter in $(2 + 1)$ dimensions is of the form,

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda r^2}{3}\right)dt^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda r^2}{3}\right)^{-1}dr^2 + r^2 d\theta^2, \quad (1)$$

where M and Q are the mass and charge of the black hole, respectively and Λ , the cosmological constant.

We have included the charge Q inside the metric. Unlike our $2 + 1$ -d metric, we find that the $3 + 1$ -d metric in [17], is devoid of any charge. The coordinate speed of light signal [Null geodesic] is obtained with $ds^2 = 0$, hence we obtain from eqn.(1),

$$0 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda r^2}{3}\right)dt^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda r^2}{3}\right)^{-1}dr^2 + r^2 d\theta^2, \quad (2)$$

$$\left(\frac{dr}{dt}\right)^2 = \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda r^2}{3}\right) \cdot \left[\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda r^2}{3}\right) - r^2\left(\frac{d\theta}{dt}\right)^2\right], \quad (3)$$

This implies,

At the surface $r = R$ on which $dr/dt = 0$ (i.e. on the RN+ Λ blackhole surface), light cannot escape from this black hole surface, thus,

$$1 - \frac{2M}{R} + \frac{Q^2}{R^2} - \frac{\Lambda R^2}{3} = 0, \quad (4)$$

2.1 Horizons in the RN+ Λ spacetime:

On solving the above eqn.(4), we get four values of the radial parameter, r , given by,

$$r_1 = \frac{1}{2} \cdot \left[\frac{2}{\Lambda} - \frac{a}{\Lambda b} - \frac{b}{c}\right]^{\frac{1}{2}} - \frac{1}{2} \cdot \left[\frac{4}{\Lambda} + \frac{a}{\Lambda b} + \frac{b}{c} - \frac{12M}{\Lambda\left(\frac{2}{\Lambda} - \frac{a}{\Lambda b} - \frac{b}{c}\right)^{\frac{1}{2}}}\right]^{\frac{1}{2}}, \quad (5)$$

$$r_2 = \frac{1}{2} \cdot \left[\frac{2}{\Lambda} - \frac{a}{\Lambda b} - \frac{b}{c} \right]^{\frac{1}{2}} + \frac{1}{2} \cdot \left[\frac{4}{\Lambda} + \frac{a}{\Lambda b} + \frac{b}{c} - \frac{12M}{\Lambda \left(\frac{2}{\Lambda} - \frac{a}{\Lambda b} - \frac{b}{c} \right)^{\frac{1}{2}}} \right]^{\frac{1}{2}}, \quad (6)$$

$$r_3 = -\frac{1}{2} \cdot \left[\frac{2}{\Lambda} - \frac{a}{\Lambda b} - \frac{b}{c} \right]^{\frac{1}{2}} - \frac{1}{2} \cdot \left[\frac{4}{\Lambda} + \frac{a}{\Lambda b} + \frac{b}{c} - \frac{12M}{\Lambda \left(\frac{2}{\Lambda} - \frac{a}{\Lambda b} - \frac{b}{c} \right)^{\frac{1}{2}}} \right]^{\frac{1}{2}}, \quad (7)$$

and

$$r_4 = -\frac{1}{2} \cdot \left[\frac{2}{\Lambda} - \frac{a}{\Lambda b} - \frac{b}{c} \right]^{\frac{1}{2}} + \frac{1}{2} \cdot \left[\frac{4}{\Lambda} + \frac{a}{\Lambda b} + \frac{b}{c} - \frac{12M}{\Lambda \left(\frac{2}{\Lambda} - \frac{a}{\Lambda b} - \frac{b}{c} \right)^{\frac{1}{2}}} \right]^{\frac{1}{2}}, \quad (8)$$

where

$$a = 3.2^{\frac{1}{3}} \cdot (1 - 4Q^2\Lambda), \quad (9)$$

$$b = [54 - 972M^2\Lambda + 648Q^2\Lambda + [(54 - 972M^2\Lambda + 648Q^2\Lambda)^2 - 4(9 - 36Q^2\Lambda)^3]^{\frac{1}{2}}]^{\frac{1}{3}}, \quad (10)$$

$$c = 3.2^{\frac{1}{3}}\Lambda, \quad (11)$$

From the above equations, we observe that a depends on the charge of the black hole and the cosmological constant, b depends on the mass, the charge of the black hole and the cosmological constant whereas c depends only on the cosmological constant.

We find from above that r₃ is negative and hence unphysical. r₁, r₂, r₄ are real and positive, depending upon suitable choice of a, b, c, M, Q, Λ. Hence, there are three possible horizons, from the innermost (depending on the values of r), they are Cauchy horizon, event horizon and the cosmological horizon.

We get the values of the horizons for Reissner-Nordström metric, if we put Λ = 0 in eq.(4) above. Hence, the values of the radius of the horizon of the charged black hole in case of RN metric is,

$$r_{\pm} = M \pm \sqrt{M^2 - Q^2}, \quad (12)$$

The larger one r₊, is the event horizon, while the smaller one, r₋, is the inner or Cauchy horizon located inside the black hole. The event horizon corresponds to,

$$r_+ = M + \sqrt{M^2 - Q^2}, \quad (13)$$

This is analog of the Schwarzschild radius, and for Q = 0, r₊ = r_s = 2M.

2.2 Solutions of Einstein-Maxwell equations in RN+Λ spacetime:

The metric (1) [considering spherical and planar (2+1)- dimensional black holes as [4], [5]] can be written in the form,

$$ds^2 = -e^{\nu(r)} dt^2 + e^{\lambda(r)} dr^2 + r^2 d\theta^2, \quad (14)$$

where we take,

$$e^{\nu(r)} = e^{-\lambda(r)} = \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda r^2}{3} \right), \quad (15)$$

The Hilbert action coupled given by,

$$I = \int dx^3 \sqrt{-g} \left(\frac{R - 2\Lambda}{16\pi} - \frac{1}{4} F_a^c F_{bc} + L_m \right), \quad (16)$$

where L_m is the Lagrangian for matter. The variation with respect to the metric gives the following self consistent Einstein-Maxwell equations with cosmological constant Λ for a charged cosmological constant effective fluid distribution,

$$G_{ab} = R_{ab} - \frac{1}{2} R g_{ab} + \Lambda g_{ab} = -8\pi (T_{ab}^{PF} + T_{ab}^{EM}), \quad (17)$$

The explicit forms of the energy momentum tensor (EMT) components for the matter source (we assumed that the matter distribution at the interior of the black hole is cosmological constant effective fluid type) and electromagnetic fields are given by,

$$T_{ab}^{PF} = (\rho + p) u_a u_b + p g_{ab}, \quad (18)$$

$$T_{ab}^{EM} = -\frac{1}{4\pi} \left(F_a^c F_{bc} - \frac{1}{4} g_{ab} F_{cd} F^{cd} \right), \quad (19)$$

where ρ , p , u_i and F_{ab} are, respectively, matter density, fluid pressure and velocity three vector of a fluid element and electromagnetic field. Here, the electromagnetic field is related to current three vector

$$J^c = \sigma(r) u^c, \quad (20)$$

as

$$F_{;b}^{ab} = -4\pi J^a, \quad (21)$$

where, $\sigma(r)$ is the proper charge density of the distribution. In our consideration, the three velocity is assumed as $u_a = \delta_a^t$ and consequently the electromagnetic field tensor can be given as,

$$F_{ab} = E(r) (\delta_a^t \delta_b^r - \delta_a^r \delta_b^t), \quad (22)$$

where $E(r)$ is the electric field.

The Einstein-Maxwell equations with the assumption, cosmological constant ($\Lambda < 0$), for the black hole metric in eqn.(14) together with the energy-momentum tensor given in eqns. (18),(19) along with eqns. (20), (21) and (22) yield (rendering $G = c = 1$)

$$\frac{\lambda' e^{-\lambda}}{2r} = 8\pi \rho(r) + E^2(r) + \Lambda \quad (23)$$

$$\frac{\nu' e^{-\lambda}}{2r} = 8\pi p(r) - E^2(r) - \Lambda \quad (24)$$

$$\frac{e^{-\lambda}}{2} \left(\frac{1}{2} \nu'^2 + \nu'' - \frac{1}{2} \nu' \lambda' \right) = 8\pi p(r) + E^2(r) - \Lambda \quad (25)$$

$$\sigma(r) = \frac{e^{-\frac{\lambda}{2}}}{4\pi r^2} (r^2 E(r))' \quad (26)$$

where a ' ' denotes differentiation with respect to the radial parameter r . When $E=0$, the Einstein-Maxwell system given above reduces to the uncharged Einstein system.

Here the term $\sigma(r) e^{\lambda(r)/2}$ [19] is equivalent to the volume charge density in (2 + 1)-d. We consider the proper charge density $\sigma(r)$ as a polynomial function of r .

The E-M equations in (23) (24) and (25), using eqn.(15) reduce to,

$$8\pi \rho(r) + E^2(r) + \Lambda = -\frac{1}{r} \left(\frac{M}{r^2} - \frac{Q^2}{r^3} - \frac{\Lambda r}{3} \right) \quad (27)$$

$$8\pi p(r) - E^2(r) - \Lambda = \frac{1}{r} \left(\frac{M}{r^2} - \frac{Q^2}{r^3} - \frac{\Lambda r}{3} \right) \quad (28)$$

$$8\pi p(r) + E^2(r) - \Lambda = \frac{3Q^2}{r^4} - \frac{2M}{r^3} - \frac{\Lambda}{3} \tag{29}$$

On adding eqns. (27) and (28) we obtain,

$$\begin{aligned} 16\pi p(r) &= -16\pi\rho(r) \\ &= \frac{2Q^2}{r^4} - \frac{M}{r^3} + \frac{4\Lambda}{3} \end{aligned} \tag{30}$$

The equations for pressure and matter density are evident from eqn.(30). Both should be dependent on the radial parameter r . It is observed from the above eqn.(30), that the particular case of $(\Lambda = 0, r^2 < 2Q^2 / M)$ is not valid, which makes the energy density negative. Hence, the cosmological constant should be negative, which is further verified in the following sections. The electric field $E(r)$ which is also dependent on r but is independent of the cosmological constant is given by

$$E(r) = \left(\frac{2Q^2}{r^4} - \frac{3M}{2r^3}\right)^{\frac{1}{2}} \tag{31}$$

It is observed from the above eqn.(31), that the electric field is non-vanishing and imaginary, if $Q = 0$, and hence the field is limited to our RN + Λ black hole, and cannot be reduced to the Schwarzschild form, as a particular case.

The proper charge density is also dependent on r , and from eqn.(26) is evaluated as

$$\sigma(r) = \frac{\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda r^2}{3}\right)^{\frac{1}{2}} \left(\frac{3M}{2r^3} - \frac{3Q^2}{r^4}\right)}{2\pi r \left(\frac{2Q^2}{r^4} - \frac{3M}{2r^3}\right)^{\frac{1}{2}}} \tag{32}$$

2.3 Physical significance of pressure and matter density:

Thus for interior solutions we have deduced that $p = -\rho$. It is equivalent to $p = \omega\rho$, where we take $\omega = -1$. This type of equation of state is available in the literature and is known as a false vacuum, degenerate vacuum, or ρ -vacuum and represents a repulsive pressure.

We choose the following values of the parameters,

$$\Lambda = -10^{-46} \text{km}^{-2}, \quad M = 3.8M_{\odot}, \quad Q = 0.00089 \text{km}, \tag{33}$$

The figures below show the variation of $p(r)$ and $\rho(r)$ against r .

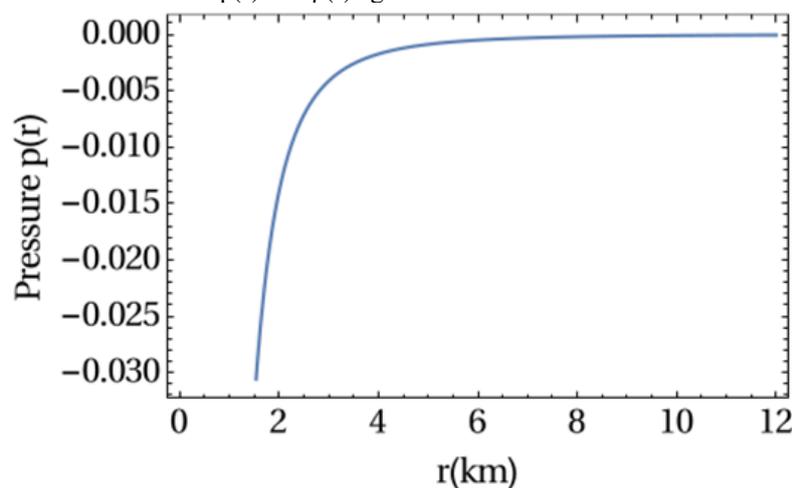


Figure 1. Pressure $p(r)$ has been depicted against r . The geometric unit of pressure here is in km^{-2} .

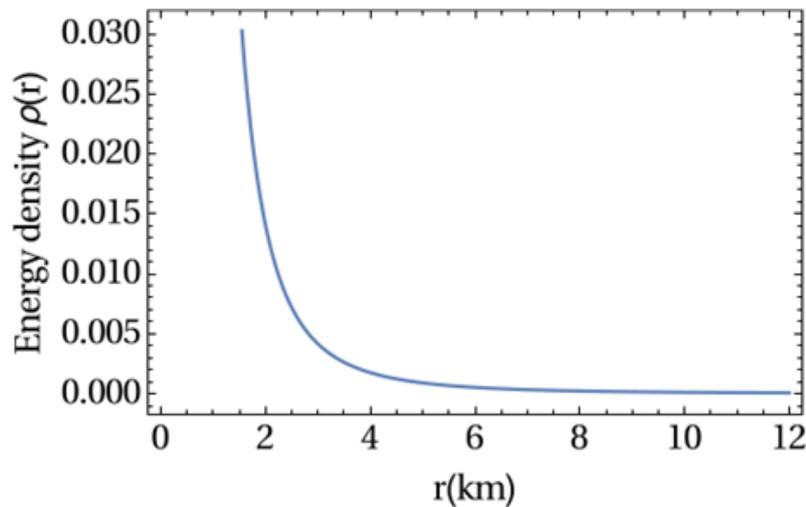


Figure 2. Density $p(r)$ has been depicted against r . The geometric unit of density here is in km^{-2} .

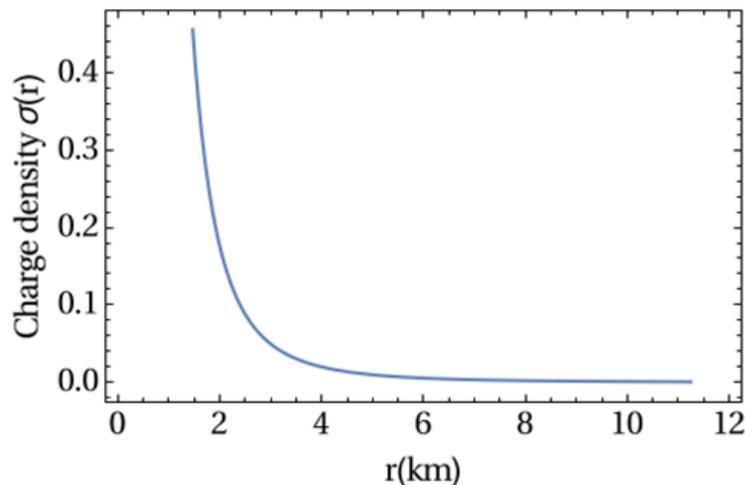


Figure 3. Proper charge density $\sigma(r)$ has been depicted against r . The geometric unit of proper charge density here is in km^{-2} .

Hence, we observe from the figure that the black hole has a negative pressure and positive matter density inside which is due to the presence of exotic matter. The pressure $p(r)$ increases with the increase in radius and the matter density $\rho(r)$ decreases with the increase in radius.

We have assumed that the effective mass of the black hole is $M = 3.8M_{\odot}$.

It can also be observed via eqn.(30) that the physical parameters, viz. Density and pressure are dependent on the charge. Also, if $\sigma(r) = 0$, then from eqn.(32), we get $M = 2Q^2 / r$, and both the parameters $p(r)$ and $\rho(r)$ in eqn.(30), become constant, being dependent only on Λ . Therefore, our solutions provide electromagnetic mass model, such that for vanishing charge density $\sigma(r)$, the physical parameters (pressure and density) becomes constant.

Figure 3 shows the variation of the proper charge density against r . We observe that the proper charge density is maximum at the centre and decreases with the increase in radius.

2.4 Geodesic equations in RN+ Λ spacetime:

We write the geodesic equations as follows:

$$\frac{d^2 t}{ds^2} + \nu' \frac{dt}{ds} \frac{dr}{ds} = 0, \quad (34)$$

$$\frac{d^2 r}{ds^2} + \frac{1}{2} \nu' e^{(\nu-\lambda)} \left(\frac{dt}{ds}\right)^2 + \frac{1}{2} \lambda' \left(\frac{dr}{ds}\right)^2 - r e^{-\lambda} \left(\frac{d\theta}{ds}\right)^2 = 0, \quad (35)$$

$$\frac{d^2 \theta}{ds^2} + \frac{2}{r} \frac{d\theta}{ds} \frac{dr}{ds} = 0, \quad (36)$$

Using eqn.(15), since $\nu = -\lambda$, $\nu' = -\lambda'$ and $\nu - \lambda = 2\nu$, we find,

$$\nu' = \frac{2}{\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda r^2}{3}\right)} \cdot \left(\frac{M}{r^2} - \frac{Q^2}{r^3} - \frac{\Lambda r}{3}\right), \quad (37)$$

On integrating eqns.(34) and (36), we obtain

$$\frac{d\theta}{ds} = \frac{k_1^2}{r^2}, \quad (38)$$

and

$$\frac{dt}{ds} = \frac{k_2^2}{\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda r^2}{3}\right)}, \quad (39)$$

Putting the above values of $d\theta / ds$ and dt / ds in eqn. (35) we get,

$$\begin{aligned} \frac{d^2 r}{ds^2} + \frac{k_2^2 \left(\frac{M}{r^2} - \frac{Q^2}{r^3} - \frac{\Lambda r}{3}\right)}{\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda r^2}{3}\right)} \\ - \frac{\left(\frac{M}{r^2} - \frac{Q^2}{r^3} - \frac{\Lambda r}{3}\right)}{\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda r^2}{3}\right)} \cdot \left(\frac{dr}{ds}\right)^2 \\ - \frac{k_1^2}{r^3} \cdot \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda r^2}{3}\right) = 0, \end{aligned} \quad (40)$$

Also from the metric eqn.(14), on dividing each side by ds^2 , we find that,

$$1 = -e^\nu \left(\frac{dt}{ds}\right)^2 + e^\lambda \left(\frac{dr}{ds}\right)^2 + r^2 \left(\frac{d\theta}{ds}\right)^2, \quad (41)$$

Using eqns.(38) and (39), eqn.(41) reduces to,

$$\left(\frac{dr}{ds}\right)^2 = k_2^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda r^2}{3}\right) \left(1 - \frac{k_1^2}{r^2}\right), \quad (42)$$

Using eqn.(42) in eqn.(40), we obtain

$$\begin{aligned} \frac{d^2 r}{ds^2} = \left(\frac{M}{r^2} - \frac{Q^2}{r^3} - \frac{\Lambda r}{3}\right) \\ + k_1^2 \left(\frac{1}{r^3} - \frac{3M}{r^4} + \frac{2Q^2}{r^5}\right), \end{aligned} \quad (43)$$

Multiplying the above equation by $2 dr / ds$ and integrating both sides w.r.t ds we get,

$$\left(\frac{dr}{ds}\right)^2 = 2 \left[-\frac{M}{r} + \frac{Q^2}{2r^2} - \frac{\Lambda r^2}{6} + k_1^2 \left(-\frac{1}{2r^2} + \frac{M}{r^3} - \frac{Q^2}{2r^4}\right)\right], \quad (44)$$

On equating eqns.(42) and (44), we observe that,

$$k_2^2 = -\left(\frac{\Lambda k_1^2}{3} + 1\right), \quad (45)$$

Hence, we observe that the cosmological constant can have a negative value as is confirmed from the above eqn.(45) [20]. Hence our assumption is found to be true.

3. Metric for interior RN+ Λ and exterior FRW spacetimes

The metric for RN+ Λ black hole in (2 + 1) dimensions is given by eqn.(1). For the sake of convenience, we transform the metric under isotropic conditions with the following transformations, using $x^0 = v$ and $x^1 = x$, as

$$2t = v, 2s = l, 2r = x\left[\left(1 + \frac{M}{x}\right)^2 - \frac{Q^2}{x^2} + \Lambda e^{-2x}\right], \quad (46)$$

Hence eqn.(1) is transformed as,

$$\begin{aligned} dl^2 = & -\frac{\left(1 - \frac{M^2}{x^2} + \frac{Q^2}{x^2} - \Lambda e^{-2x}\right)^2}{\left[\left(1 + \frac{M}{x}\right)^2 - \frac{Q^2}{x^2} + \Lambda e^{-2x}\right]^2} dv^2 \\ & + \left[\left(1 + \frac{M}{x}\right)^2 - \frac{Q^2}{x^2} + \Lambda e^{-2x}\right]^2 \\ & \cdot (dx^2 + x^2 d\theta^2), \end{aligned} \quad (47)$$

We consider the line element for the exterior spacetime in FRW metric in the form,

$$dl^2 = -dv^2 + \frac{a^2(v)}{\left(1 + \frac{kx^2}{4}\right)^2} (dx^2 + x^2 d\theta^2), \quad (48)$$

Here $a(v)$ is the scale factor of the universe and k denotes the space-time curvature. The above RN+ Λ metric embedded in FRW universe is represented as follows:

$$dl^2 = -P^2(v, x)dv^2 + T^2(v, x)(dx^2 + x^2 d\theta^2), \quad (49)$$

where,

$$\begin{aligned} T(v, x) = & \left[g(v, x) + \frac{z(v)}{x}\right]^2 - \frac{h(v)}{x^2} \\ & + \Lambda g^2 e^{-2bgx}, \end{aligned} \quad (50)$$

$$\begin{aligned} P(v, x) = & f(v) \frac{\dot{T}}{2T} \\ = & \left[\frac{\dot{g}f}{g} + (gz + \dot{g}z)\frac{f}{g^2x} + \frac{z\dot{z}f}{g^2x^2} - \frac{\dot{h}f}{2g^2x^2}\right. \\ & \left. - \Lambda e^{-2bgx}\right] \cdot \left[\left(1 + \frac{z}{gx}\right)^2 - \frac{h}{g^2x^2}\right] \\ & + \Lambda e^{-2bgx}]^{-1}, \end{aligned} \quad (51)$$

where “.” denotes differentiation with respect to v and $b(v)$ is the scale factor under the transformed conditions. We consider the limit when $f \dot{g}(bx - 1/g) \rightarrow 1$.

We consider asymptotic flat conditions where $P(v = \text{const.}, x)$ is reduced to $\sqrt{g_{00}}$ term in eqn.(47). Hence, on comparing the $\sqrt{g_{00}}$ term of eqn.(51) with that of (47), we find that the following identities

$$\begin{aligned} \text{hold: (i) } \frac{\dot{g}f}{g} = 1, \text{ (ii) } (gz + \dot{g}z)\frac{f}{g^2x} = 0, \text{ (iii) } \frac{z\dot{z}f}{g^2x^2} = -\left(\frac{z}{gx}\right)^2 \\ \text{and (iv) } -\frac{\dot{h}f}{2g^2x^2} = \frac{h}{g^2x^2}. \end{aligned}$$

Now, (i)-(iv) reduce to (v) $\dot{g}f = g$, $\dot{z}f = -z$, $\dot{h}f = -2h$; On suitable transformations assuming, $f = b / \dot{b}$ and

$$g = \frac{b(v)}{\sqrt{1 + \frac{kx^2}{4}}}, \text{ we obtain, (vi) } z = \frac{M}{b}, h = \frac{Q^2}{b^2}.$$

Here the integration constants M and Q are related to the mass and charge of the black hole respectively. On substituting the above eqns.(49), (50) and (51) in (47) with the above transformations under suitable conditions, the final RN+Λ metric in (2 + 1) dimensions in the FRW background is observed as follows:

$$\begin{aligned}
 dl^2 = & - \frac{(1 - \frac{M^2(1+\frac{kx^2}{4})}{a^2x^2} + \frac{Q^2(1+\frac{kx^2}{4})}{a^2x^2} - \Lambda e^{-\frac{2ax}{\sqrt{1+\frac{kx^2}{4}}})^2}{[(1 + \frac{M\sqrt{1+\frac{kx^2}{4}}}{ax})^2 - \frac{Q^2(1+\frac{kx^2}{4})}{a^2x^2} + \Lambda e^{-\frac{2ax}{\sqrt{1+\frac{kx^2}{4}}}]^2} \\
 & \cdot dv^2 + \frac{a^2}{(1 + \frac{kx^2}{4})^2} \cdot [(1 + \frac{M\sqrt{1+\frac{kx^2}{4}}}{ax})^2 \\
 & - \frac{Q^2(1+\frac{kx^2}{4})}{a^2x^2} + \Lambda e^{-\frac{2ax}{\sqrt{1+\frac{kx^2}{4}}}]^2 \\
 & \cdot (dx^2 + x^2d\theta^2), \tag{52}
 \end{aligned}$$

Here a = a(v) → b²(v). If k=0, the above eqn.(52) reduces to,

$$\begin{aligned}
 dl^2 = & - \frac{(1 - \frac{M^2}{a^2x^2} + \frac{Q^2}{a^2x^2} - \Lambda e^{-2ax})^2}{[(1 + \frac{M}{ax})^2 - \frac{Q^2}{a^2x^2} + \Lambda e^{-2ax}]^2} dv^2 \\
 & + a^2[(1 + \frac{M}{ax})^2 - \frac{Q^2}{a^2x^2} + \Lambda e^{-2ax}]^2 \\
 & \cdot (dx^2 + x^2d\theta^2), \tag{53}
 \end{aligned}$$

We know that a(v) = e^{Hv} where H is the Hubble constant. If further H = 0, then a(v) = 1 and the eqn.(47) is restored from eqn.(53). However, Q = 0 reduces the above eqn.(52) to,

$$\begin{aligned}
 dl^2 = & - \frac{(1 - \frac{M^2(1+\frac{kx^2}{4})}{a^2x^2} - \Lambda e^{-\frac{2ax}{\sqrt{1+\frac{kx^2}{4}}})^2}{[(1 + \frac{M\sqrt{1+\frac{kx^2}{4}}}{ax})^2 + \Lambda e^{-\frac{2ax}{\sqrt{1+\frac{kx^2}{4}}}]^2} dv^2 \\
 & + \frac{a^2}{(1 + \frac{kx^2}{4})^2} \cdot [(1 + \frac{M\sqrt{1+\frac{kx^2}{4}}}{ax})^2 \\
 & + \Lambda e^{-\frac{2ax}{\sqrt{1+\frac{kx^2}{4}}}]^2 \cdot (dx^2 + x^2d\theta^2), \tag{54}
 \end{aligned}$$

which is just the McVittie solution. For the extreme RN black hole case M = Q and k = 0 in presence of cosmological constant, the eqn.(52) is reduced to

$$\begin{aligned}
 dl^2 = & - \frac{(1 - \Lambda e^{-2ax})^2}{(1 + \frac{2M}{ax} + \Lambda e^{-2ax})^2} dv^2 \\
 & + a^2(1 + \frac{2M}{ax} + \Lambda e^{-2ax})^2 \\
 & \cdot (dx^2 + x^2d\theta^2), \tag{55}
 \end{aligned}$$

If Λ = 0 the above eqn.(55) further takes the form,

$$\begin{aligned}
 dl^2 = & - \frac{1}{(1 + \frac{2M}{ax})^2} dv^2 + a^2(1 + \frac{2M}{ax})^2 \\
 & \cdot (dx^2 + x^2d\theta^2), \tag{56}
 \end{aligned}$$

If the scale factor a(v) = 1, when H = 0, the above eqn.(56) reduces to the Schwarzschild metric in an FRW present day accelerating universe as,

$$dl^2 = -\frac{1}{\left(1 + \frac{2M}{x}\right)^2} dv^2 + \left(1 + \frac{2M}{x}\right)^2 (dx^2 + x^2 d\theta^2), \quad (57)$$

4. Boundary and matching conditions with the exterior FRW universe:

We use matching conditions of g_{vv} , g_{xx} and $\partial g_{vv} / \partial x$ at $x = R$ we find from eqns. (48) and (52) three results which are enunciated below,

A. Continuity of g_{vv} :

$$\frac{\left(1 - \frac{M^2(1 + \frac{kx^2}{4})}{a^2x^2} + \frac{Q^2(1 + \frac{kx^2}{4})}{a^2x^2} - \Lambda e^{-\frac{2ax}{\sqrt{1 + \frac{kx^2}{4}}}}\right)^2}{\left[\left(1 + \frac{M\sqrt{1 + \frac{kx^2}{4}}}{ax}\right)^2 - \frac{Q^2(1 + \frac{kx^2}{4})}{a^2x^2} + \Lambda e^{-\frac{2ax}{\sqrt{1 + \frac{kx^2}{4}}}}\right]^2} = 1 \quad (58)$$

Thus, the scale factor $a(v)$ is expressed by the following equation,

$$\frac{M^2(1 + \frac{kx^2}{4})}{a^2x^2} + \frac{M\sqrt{1 + \frac{kx^2}{4}}}{ax} - \frac{Q^2(1 + \frac{kx^2}{4})}{a^2x^2} + \Lambda e^{-\frac{2ax}{\sqrt{1 + \frac{kx^2}{4}}}} = 0, \quad (59)$$

Hence, Λ is negative for a positive mass. For the extreme R-N case when $Q = M$,

$$\begin{aligned} & \frac{M\sqrt{1 + \frac{kx^2}{4}}}{ax} + \Lambda e^{-\frac{2ax}{\sqrt{1 + \frac{kx^2}{4}}}} \\ &= \frac{Q\sqrt{1 + \frac{kx^2}{4}}}{ax} + \Lambda e^{-\frac{2ax}{\sqrt{1 + \frac{kx^2}{4}}}} = 0, \end{aligned} \quad (60)$$

As Λ is constant the above eqn.(60) indicates that, for an observer at infinity, the mass and charge of the black hole decreases with the expansion of the universe whereas both increases with the contraction of the universe.

If $Q = 0$ we get,

$$\left[\frac{M\sqrt{1 + \frac{kx^2}{4}}}{ax}\right] \cdot \left[1 + \frac{M\sqrt{1 + \frac{kx^2}{4}}}{ax}\right] + \Lambda e^{-\frac{2ax}{\sqrt{1 + \frac{kx^2}{4}}}} = 0, \quad (61)$$

B. Continuity of g_{xx} :

$$\begin{aligned} \frac{a^2(v)}{\left(1 + \frac{kx^2}{4}\right)^2} &= \frac{a^2(v)}{\left(1 + \frac{kx^2}{4}\right)^2} \cdot \left[\left(1 + \frac{M\sqrt{1 + \frac{kx^2}{4}}}{ax}\right)^2\right. \\ &\quad \left. - \frac{Q^2(1 + \frac{kx^2}{4})}{a^2x^2} + \Lambda e^{-\frac{2ax}{\sqrt{1 + \frac{kx^2}{4}}}}\right]^2 \end{aligned} \quad (62)$$

The above eqn.(62) gives another expression for $a(v)$ as,

$$\frac{M^2(1 + \frac{kx^2}{4})}{a^2x^2} + \frac{2M\sqrt{1 + \frac{kx^2}{4}}}{ax} - \frac{Q^2(1 + \frac{kx^2}{4})}{a^2x^2} + \Lambda e^{-\frac{2ax}{\sqrt{1 + \frac{kx^2}{4}}}} = 0, \tag{63}$$

We observe via eqns.(59) and (63) that as Λ has a negative value of the order 10^{-46} , the term containing Λ can be eliminated to obtain the value of the constant $k = -0.0278$ in both the equations, considering the scale factor, $a(v) = 1$ for the present day accelerating universe.

C. Continuity of $\partial g_{vv} / \partial x$ at $x = R$:

$$2(2 - \frac{\sqrt{1 + \frac{kx^2}{4}}}{ax}) [(\frac{M^2}{(\frac{ax}{\sqrt{1 + \frac{kx^2}{4}})})^3} - \frac{Q^2}{(\frac{ax}{\sqrt{1 + \frac{kx^2}{4}})})^3} + \Lambda e^{-\frac{2ax}{\sqrt{1 + \frac{kx^2}{4}}}}] + \frac{\sqrt{1 + \frac{kx^2}{4}}}{ax} = 0, \tag{64}$$

Hence at $x = R$ we get,

$$2(2 - \frac{\sqrt{1 + \frac{kR^2}{4}}}{aR}) [(\frac{M^2}{(\frac{aR}{\sqrt{1 + \frac{kR^2}{4}})})^3} - \frac{Q^2}{(\frac{aR}{\sqrt{1 + \frac{kR^2}{4}})})^3} + \Lambda e^{-\frac{2aR}{\sqrt{1 + \frac{kR^2}{4}}}}] + \frac{\sqrt{1 + \frac{kR^2}{4}}}{aR} = 0, \tag{65}$$

In the extreme R-N case when $Q = M$ we find,

$$\Lambda = -e^{-\frac{2aR}{\sqrt{1 + \frac{kR^2}{4}}}} \cdot \frac{1}{2(\frac{2aR}{\sqrt{1 + \frac{kR^2}{4}}} - 1)} \tag{66}$$

As Λ is negative, $2aR / \sqrt{1 + kR^2 / 4} \geq 1$, Hence $R > 2 / \sqrt{16a^2 - k}$, where k is the curvature of space-time. The curvature parameter k may take values of 0, +1 or -1, depending on whether 3-D spacetime is assumed to be Euclidean, spherical, or hyperbolic, respectively. Here, we observe that R is always positive for the accelerating universe, if we take $a(v) = 1$ and $k = 1$.

For $Q = 0$ in eqn.(64),

$$\Lambda = -e^{-\frac{2aR}{\sqrt{1 + \frac{kR^2}{4}}}} \cdot [(\frac{M^2}{(\frac{aR}{\sqrt{1 + \frac{kR^2}{4}})})^3} + \frac{1}{2(\frac{2aR}{\sqrt{1 + \frac{kR^2}{4}}} - 1)}] \tag{67}$$

which represents the cosmological constant inside the Schwarzschild black hole and also has a negative value.

We have considered $Q = 0.00089$ km and $c = 1$, as geometric units. However when converted to SI units we get $Q = 1.03419 \times 10^{17}$ coulomb.

5. Physical relevance of RN+ Λ metric:

It is found that the final RN+ Λ metric in eqn.(52) satisfies the field equations (23)-(26). The Einstein tensor $G_{\mu\nu}$ and the energy momentum tensor T_{ab}^{PF} and T_{ab}^{EM} for cosmological constant effective fluid and electromagnetic fields w.r.t the metric are easily obtained. Using eqns. (18)-(22) we deduce the non-vanishing components of the electromagnetic tensor

F_{ab} as,

$$\begin{aligned}
F^{01} = E^1 = & [2(4+kx^2)(e^{\frac{4ax}{\sqrt{4+kx^2}}}(-4a^2x^2+M^2 \\
& (4+kx^2)-Q^2(4+kx^2))+4a^2x^2\Lambda)(e^{\frac{4ax}{\sqrt{4+kx^2}}} \\
& (4a^2x^2+4aMx\sqrt{4+kx^2}+M^2(4+kx^2) \\
& -Q^2(4+kx^2))+4a^2x^2\Lambda)^4]^{-\frac{1}{2}} \times [32a^3e^{\frac{12ax}{\sqrt{4+kx^2}}}x^3 \\
& ((4+kx^2)(e^{\frac{8ax}{\sqrt{4+kx^2}}}M(M^2-Q^2)^2(-2+kx^2) \\
& (4+kx^2)^{\frac{5}{2}}+4ae^{\frac{8ax}{\sqrt{4+kx^2}}}(M^2-Q^2)x(4+kx^2)^2 \\
& (2kM^2x^2+Q^2(4-kx^2))+8a^2e^{\frac{4ax}{\sqrt{4+kx^2}}} \\
& M(M^2-Q^2)x^2(4+kx^2)^{\frac{3}{2}}(3e^{\frac{4ax}{\sqrt{4+kx^2}}} \\
& (2+kx^2)+8\Lambda)+16a^3e^{\frac{4ax}{\sqrt{4+kx^2}}}x^3(4+kx^2) \\
& (e^{\frac{4ax}{\sqrt{4+kx^2}}}(2M^2(4+kx^2)-Q^2(8+kx^2)) \\
& +(8M^2-Q^2(8+kx^2))\Lambda)+16a^4Mx^4\sqrt{4+kx^2} \\
& (6+kx^2)(e^{\frac{8ax}{\sqrt{4+kx^2}}}-\Lambda^2))+16a^3x^3 \\
& \Lambda(e^{\frac{4ax}{\sqrt{4+kx^2}}}(M^3(-16+kx^2)(4+kx^2)^2 \\
& +6aM^2x\sqrt{4+kx^2}(-40-6kx^2+k^2x^4)+ \\
& 2ax\sqrt{4+kx^2}(4a^2x^2(2+kx^2)+Q^2(88+18kx^2 \\
& -k^2x^4))-M(4+kx^2)(-12a^2x^2(-4+kx^2) \\
& +Q^2(-64-12kx^2+k^2x^4)))+4a^2x^2 \\
& (2ax(2+kx^2)\sqrt{4+kx^2}+M(32+12kx^2 \\
& +k^2x^4))\Lambda)+128a^4x^4(2ae^{\frac{4ax}{\sqrt{4+kx^2}}}(3M^2 \\
& -Q^2)x(4+kx^2)+e^{\frac{4ax}{\sqrt{4+kx^2}}}M(M^2-Q^2) \\
& (4+kx^2)^{\frac{3}{2}}+8a^3x^3(e^{\frac{4ax}{\sqrt{4+kx^2}}}-2\Lambda)+4a^2Mx^2 \\
& \sqrt{4+kx^2}(3e^{\frac{4ax}{\sqrt{4+kx^2}}}-2\Lambda))\Lambda]^{\frac{1}{2}}(68)
\end{aligned}$$

which reduces significantly as follows when $Q = M$,

$$\begin{aligned}
F^{01} = E^1 = & [2(4+kx^2)(e^{\frac{4ax}{\sqrt{4+kx^2}}}(-4a^2x^2)+4a^2x^2\Lambda) \\
& (e^{\frac{4ax}{\sqrt{4+kx^2}}}(4a^2x^2+4aMx\sqrt{4+kx^2}+4a^2x^2\Lambda)^4]^{-\frac{1}{2}} \\
& \times [32a^3e^{\frac{12ax}{\sqrt{4+kx^2}}}x^3((4+kx^2)(16a^3e^{\frac{4ax}{\sqrt{4+kx^2}}}x^3(4+kx^2) \\
& (e^{\frac{4ax}{\sqrt{4+kx^2}}}(kM^2x^2)-kM^2x^2\Lambda)+16a^4Mx^4\sqrt{4+kx^2} \\
& (6+kx^2)(e^{\frac{8ax}{\sqrt{4+kx^2}}}-\Lambda^2))+16a^3x^3\Lambda(e^{\frac{4ax}{\sqrt{4+kx^2}}} \\
& (M^3(-16+kx^2)(4+kx^2)^2+6aM^2x\sqrt{4+kx^2} \\
& (-40-6kx^2+k^2x^4)+2ax\sqrt{4+kx^2}(4a^2x^2 \\
& (2+kx^2)+M^2(88+18kx^2-k^2x^4))-M(4+kx^2) \\
& (-12a^2x^2(-4+kx^2)+M^2(-64-12kx^2+k^2x^4))) \\
& +4a^2x^2(2ax(2+kx^2)\sqrt{4+kx^2}+M(32+12kx^2 \\
& +k^2x^4))\Lambda)+128a^4x^4(4aM^2e^{\frac{4ax}{\sqrt{4+kx^2}}} \\
& x(4+kx^2)+8a^3x^3(e^{\frac{4ax}{\sqrt{4+kx^2}}}-2\Lambda)+4a^2Mx^2 \\
& \sqrt{4+kx^2}(3e^{\frac{4ax}{\sqrt{4+kx^2}}}-2\Lambda))\Lambda]^{\frac{1}{2}}(69)
\end{aligned}$$

Also since $F_{ab} = A_{a;b} - A_{b;a}$, using eqn. (68), we get the non-vanishing components of the potential A_a as,

$$A_0 = \int F^{01} g_{00} g_{11} dx \tag{70}$$

Furthermore, the eqn. (68) satisfies $F^{ab}{}_{;b} = 0$. From eqn. (17) $G^{ab}{}_{;b} = 0$ always holds, hence we get, $T^{PF}{}_{ab;b} + T^{EM}{}_{ab;b} = 0$. We also find that the above relation is satisfied using equations (18) and (19) as

$$\begin{aligned} u_a T^{PF}{}_{ab;b} = T^{PF}{}_{0b;b} &= p_{,b} g_{0b} + p g_{0b;b} + (\rho + p)_{,b} u_b u_0 + (\rho + p) [u_{b;b} u_0 + u_{0;b} u_b] = 0 \\ \text{and } 4\pi T^{ab(EM)}{}_{;b} &= F^{a\alpha}{}_{;b} F^b{}_{\alpha} + F^{a\alpha} F^b{}_{\alpha;b} - \frac{1}{2} g^{ab} F_{\alpha\beta} F^{\alpha\beta} = \\ F^{a\alpha} F^b{}_{\alpha;b} + \frac{1}{2} g^{a\zeta} F^{b\kappa} (F_{\zeta\kappa;b} - F_{\zeta b;\kappa} - F_{\kappa b;\zeta}) &= -F^{a\alpha} J_{\alpha} = 0. \end{aligned}$$

So both $T^{PF}{}_{ab}$ and $T^{EM}{}_{ab}$ satisfy Bianchi identity. The proof is indicative of the fact that eqn.(52) is an exact solution of the EM field equations and the metric is physically relevant.

6. Darmois-Israel matching conditions:

The Darmois-Israel matching conditions have been studied [21, 22]. The junction conditions to match the inner and exterior metrics across the boundary surface $x = x_0$, are the continuity of first and second fundamental forms across that surface. We define a surface Σ , where $x = x_0$, the junction surface being an one dimensional ring of matter, by the metric [23],

$$dl^2_{\Sigma} = -d\tau^2 + x_0^2 d\theta^2, \tag{71}$$

with the intrinsic coordinates of Σ being $\xi^m = (\tau, \theta)$. The inner and outer metrics from eqns. (52) and (48) are given as,

$$\begin{aligned} dl^2_{-} &= - \frac{(1 - \frac{M^2(1+\frac{kx^2}{4})}{a^2x^2} + \frac{Q^2(1+\frac{kx^2}{4})}{a^2x^2} - \Lambda e^{-\frac{2ax}{\sqrt{1+\frac{kx^2}{4}}})^2}{[(1 + \frac{M\sqrt{1+\frac{kx^2}{4}}}{ax})^2 - \frac{Q^2(1+\frac{kx^2}{4})}{a^2x^2} + \Lambda e^{-\frac{2ax}{\sqrt{1+\frac{kx^2}{4}}})^2]} \\ &\cdot dv^2 + \frac{a^2}{(1 + \frac{kx^2}{4})^2} \cdot [(1 + \frac{M\sqrt{1+\frac{kx^2}{4}}}{ax})^2 \\ &- \frac{Q^2(1 + \frac{kx^2}{4})}{a^2x^2} + \Lambda e^{-\frac{2ax}{\sqrt{1+\frac{kx^2}{4}}})^2] \\ &\cdot (dx^2 + x^2 d\theta^2), \end{aligned} \tag{72}$$

and

$$dl^2_{+} = -dv^2 + \frac{a^2(v)}{(1 + \frac{kx^2}{4})^2} (dx^2 + x^2 d\theta^2), \tag{73}$$

Here the coordinates (v, x, θ) are recognised in both the regions of the spacetime.

Now we consider the boundary surface Σ which would imply as timelike which would imply,

$$\begin{aligned} &\frac{(1 - \frac{M^2(1+\frac{kx^2}{4})}{a^2x^2} + \frac{Q^2(1+\frac{kx^2}{4})}{a^2x^2} - \Lambda e^{-\frac{2ax}{\sqrt{1+\frac{kx^2}{4}}})^2}{[(1 + \frac{M\sqrt{1+\frac{kx^2}{4}}}{ax})^2 - \frac{Q^2(1+\frac{kx^2}{4})}{a^2x^2} + \Lambda e^{-\frac{2ax}{\sqrt{1+\frac{kx^2}{4}}})^2]} > 0 \end{aligned} \tag{74}$$

The radial coordinate x is P used as the matching parameter along the generators on Σ , the normal η_m to the surface has only the radial component $\eta_x = \sqrt{g_{xx}}$. We thus obtain the extrinsic curvature in the form [where, $x^0 = v, x^1 = x, x^2 = \theta$],

$$K_{mn}^{\pm} = -\eta_x^{\pm} \Gamma_{ab}^{x(\pm)} \frac{\partial x^a}{\partial \xi^m} \frac{\partial x^b}{\partial \xi^n}, \quad (75)$$

Now, the line elements in eqns.(72) and (73) are continuous at $x = x_0$. The continuity of the first fundamental form at the boundary indicates that $g_{vv}^+ = g_{vv}^-$, and $g_{xx}^+ = g_{xx}^-$, i.e.,

$$\frac{(1 - \frac{M^2(1 + \frac{kx_0^2}{4})}{a^2x_0^2}) + \frac{Q^2(1 + \frac{kx_0^2}{4})}{a^2x_0^2} - \Lambda e^{-\frac{2ax_0}{\sqrt{1 + \frac{kx_0^2}{4}}}}}{[(1 + \frac{M\sqrt{1 + \frac{kx_0^2}{4}}}{ax_0})^2 - \frac{Q^2(1 + \frac{kx_0^2}{4})}{a^2x_0^2} + \Lambda e^{-\frac{2ax_0}{\sqrt{1 + \frac{kx_0^2}{4}}}}]} = 1 \quad (76)$$

and

$$\frac{a^2(v)}{(1 + \frac{kx_0^2}{4})^2} = \frac{a^2(v)}{(1 + \frac{kx_0^2}{4})^2} \cdot [(1 + \frac{M\sqrt{1 + \frac{kx_0^2}{4}}}{ax_0})^2 - \frac{Q^2(1 + \frac{kx_0^2}{4})}{a^2x_0^2} + \Lambda e^{-\frac{2ax_0}{\sqrt{1 + \frac{kx_0^2}{4}}}}] \quad (77)$$

Hence, we retrieve eqns.(58) and (62) on replacing x_0 by x in the above eqns.(76) and (77) respectively. It is also evident that,

$$K_{\tau\tau}^- = -\eta_x^- \Gamma_{vv}^{(-)x} \frac{dv}{d\tau} \frac{dv}{d\tau}, \quad (78)$$

But (by construction),

$$g_{vv}^{\pm} \frac{dv^2}{d\tau^2} = -1$$

$$\eta_x^- = \frac{a}{(1 + \frac{kx_0^2}{4})} [(1 + \frac{M\sqrt{1 + \frac{kx_0^2}{4}}}{ax_0})^2 - \frac{Q^2(1 + \frac{kx_0^2}{4})}{a^2x_0^2} + \Lambda e^{-\frac{2ax_0}{\sqrt{1 + \frac{kx_0^2}{4}}}}] \quad (79)$$

$$\begin{aligned} \frac{\partial g_{vv}^-}{\partial x} &= [(4 + kx_0^2)^{-3/2} (e^{\frac{4ax_0}{\sqrt{4+kx_0^2}}} (4a^2x_0^2 \\ &+ 4aMx_0\sqrt{4 + kx_0^2} + M^2(4 + kx_0^2) - Q^2 \\ &(4 + kx_0^2)) + 4a^2x_0^2\Lambda)^{-3}] [32ae^{\frac{4ax_0}{\sqrt{4+kx_0^2}}} \\ &(e^{\frac{4ax_0}{\sqrt{4+kx_0^2}}} (4ax_0^2 - M^2(4 + kx_0^2) + Q^2(4 + kx_0^2)) \\ &- 4a^2x_0^2\Lambda)((4 + kx_0^2)(e^{\frac{4ax_0}{\sqrt{4+kx_0^2}}} (4aM^2x_0\sqrt{4 + kx_0^2} \\ &- 4aQ^2x_0\sqrt{4 + kx_0^2} + M^3(4 + kx_0^2) \\ &- M(-4a^2x_0^2 + Q^2(4 + kx_0^2))) - 4a^2Mx_0^2\Lambda) \\ &+ 16a^3x_0^3(2ax_0 + M\sqrt{4 + kx_0^2})\Lambda)], \quad (80) \end{aligned}$$

It is found that

$$\begin{aligned} \Gamma_{vv}^{x(-)} = & [e^{\frac{4ax_0}{\sqrt{4+kx_0^2}}} (4a^2x_0^2 + 4aMx_0\sqrt{4+kx_0^2} + M^2 \\ & (4+kx_0^2) - Q^2(4+kx_0^2)) + 4a^2x_0^2\Lambda]^{-5} \\ & \times [16a^3e^{\frac{12ax_0}{\sqrt{4+kx_0^2}}} x_0^4\sqrt{4+kx_0^2} (e^{\frac{4ax_0}{\sqrt{4+kx_0^2}}} (-4a^2x_0^2 \\ & + M^2(4+kx_0^2) - Q^2(4+kx_0^2)) + 4a^2x_0^2\Lambda) \\ & ((4+kx_0^2)(e^{\frac{4ax_0}{\sqrt{4+kx_0^2}}} (4aM^2x_0\sqrt{4+kx_0^2} \\ & - 4aQ^2x_0\sqrt{4+kx_0^2} + M^3(4+kx_0^2) \\ & - M(-4a^2x_0^2 + Q^2(4+kx_0^2))) - 4a^2Mx_0^2\Lambda) \\ & + 16a^3x_0^3(2ax_0 + M\sqrt{4+kx_0^2})\Lambda)], \quad (81) \end{aligned}$$

Thus

$$\begin{aligned} K_{\tau\tau}^- = & -[16a^2e^{\frac{8ax_0}{\sqrt{4+kx_0^2}}} x_0^2((4+kx_0^2)(e^{\frac{4ax_0}{\sqrt{4+kx_0^2}}} \\ & (4aM^2x_0\sqrt{4+kx_0^2} - 4aQ^2x_0\sqrt{4+kx_0^2} \\ & + M^3(4+kx_0^2) - M(-4a^2x_0^2 + Q^2(4+kx_0^2))) \\ & - 4a^2Mx_0^2\Lambda) + 16a^3x_0^3(2ax_0 + M\sqrt{4+kx_0^2}) \\ & \Lambda)] \times [\sqrt{4+kx_0^2} (e^{\frac{4ax_0}{\sqrt{4+kx_0^2}}} (-4a^2x_0^2 \\ & + M^2(4+kx_0^2) - Q^2(4+kx_0^2)) \\ & + 4a^2x_0^2\Lambda) (e^{\frac{4ax_0}{\sqrt{4+kx_0^2}}} (4a^2x_0^2 + 4aMx_0\sqrt{4+kx_0^2} \\ & + M^2(4+kx_0^2) - Q^2(4+kx_0^2)) + 4a^2x_0^2\Lambda)^2]^{-1}, \quad (82) \end{aligned}$$

Similarly, the extrinsic curvature arising from the exterior FRW region is calculated and we find,

$$K_{\tau\tau}^+ = 0, \quad (83)$$

In order to match $K_{\tau\tau}^-$, and $K_{\tau\tau}^+$, this would simply imply

$$\begin{aligned} (4+kx_0^2)(e^{\frac{4ax_0}{\sqrt{4+kx_0^2}}} (4aM^2x_0\sqrt{4+kx_0^2} - 4aQ^2x_0 \\ \sqrt{4+kx_0^2} + M^3(4+kx_0^2) - M(-4a^2x_0^2 \\ + Q^2(4+kx_0^2))) - 4a^2Mx_0^2\Lambda) + 16a^3x_0^3 \\ (2ax_0 + M\sqrt{4+kx_0^2})\Lambda = 0, \quad (84) \end{aligned}$$

Hence, the metric as well as the extrinsic curvature are continuous at the boundary surface.

7. Further discussion on surface continuity:

We now prove the surface continuity alternatively. Let the surface be discontinuous. Then on the contrary, the discontinuity in the extrinsic curvature determine the surface stress energy and surface tension of the junction surface at $x = x_0$ where the surface stress-energy tensor components are determined [24-27].

Let,

$$f(x) = \frac{\left(1 - \frac{M^2(1 + \frac{kx^2}{4})}{a^2x^2} + \frac{Q^2(1 + \frac{kx^2}{4})}{a^2x^2} - \Lambda e^{-\frac{2ax}{\sqrt{1 + \frac{kx^2}{4}}}\right)^2}{\left[\left(1 + \frac{M\sqrt{1 + \frac{kx^2}{4}}}{ax}\right)^2 - \frac{Q^2(1 + \frac{kx^2}{4})}{a^2x^2} + \Lambda e^{-\frac{2ax}{\sqrt{1 + \frac{kx^2}{4}}}\right]^2}, \quad (85)$$

$$g(x) = \frac{a^2}{\left(1 + \frac{kx^2}{4}\right)^2} \left[\left(1 + \frac{M\sqrt{1 + \frac{kx^2}{4}}}{ax}\right)^2 - \frac{Q^2(1 + \frac{kx^2}{4})}{a^2x^2} + \Lambda e^{-\frac{2ax}{\sqrt{1 + \frac{kx^2}{4}}}\right]^2, \quad (86)$$

$$h(x) = \frac{a^2x^2}{\left(1 + \frac{kx^2}{4}\right)^2} \left[\left(1 + \frac{M\sqrt{1 + \frac{kx^2}{4}}}{ax}\right)^2 - \frac{Q^2(1 + \frac{kx^2}{4})}{a^2x^2} + \Lambda e^{-\frac{2ax}{\sqrt{1 + \frac{kx^2}{4}}}\right]^2, \quad (87)$$

Hence

$$\begin{aligned} f'(x) = & [(4 + kx^2)^{-3/2} (e^{\frac{4ax}{\sqrt{4+kx^2}}} (4a^2x^2 \\ & + 4aMx\sqrt{4+kx^2} + M^2(4+kx^2) - Q^2 \\ & (4+kx^2)) + 4a^2x^2\Lambda)^{-3}] [32ae^{\frac{4ax}{\sqrt{4+kx^2}}} \\ & (e^{\frac{4ax}{\sqrt{4+kx^2}}} (4ax^2 - M^2(4+kx^2) + Q^2(4+kx^2)) \\ & - 4a^2x^2\Lambda) ((4+kx^2)(e^{\frac{4ax}{\sqrt{4+kx^2}}} (4aM^2x\sqrt{4+kx^2} \\ & - 4aQ^2x\sqrt{4+kx^2} + M^3(4+kx^2) \\ & - M(-4a^2x^2 + Q^2(4+kx^2))) - 4a^2Mx^2\Lambda) \\ & + 16a^3x^3(2ax + M\sqrt{4+kx^2})\Lambda)], \quad (88) \end{aligned}$$

The jump of the extrinsic curvature components at the surface $x = x_0$, is associated with the surface energy density [28] as,

$$\lambda(x_0) = -\frac{1}{8\pi} \frac{h'(x_0)}{h(x_0)} \sqrt{\frac{1 + \dot{x}_0^2 g(x_0)}{g(x_0)}} \quad (89)$$

and the surface pressure as,

$$P(x_0) = \frac{1}{8\pi} \sqrt{\frac{1 + \dot{x}_0^2 g(x_0)}{g(x_0)}} [2\ddot{x}_0 + \dot{x}_0^2 \left(\frac{f'(x_0)}{f(x_0)} + \frac{g'(x_0)}{g(x_0)}\right) + \frac{f'(x_0)}{f(x_0)g(x_0)}], \quad (90)$$

For a static configuration of radius x_0 , we obtain (assuming $\dot{x}_0 = 0$ and $\ddot{x}_0 = 0$)

$$\begin{aligned} \lambda(x_0) &= -\frac{1}{8\pi} \frac{h'(x_0)}{h(x_0)} \sqrt{\frac{1}{g(x_0)}} \\ &= [4\pi x_0(4+kx_0^2)^{\frac{3}{2}}(e^{\frac{4ax_0}{\sqrt{4+kx_0^2}}}(4a^2x_0^2+4aMx_0\sqrt{4+kx_0^2}+M^2(4+kx_0^2)-Q^2(4+kx_0^2)+4a^2x_0^2\Lambda)]^{-1} [4ae^{\frac{4ax_0}{\sqrt{4+kx_0^2}}}kMx_0^3(4+kx_0^2) \\ &\quad +e^{\frac{4ax_0}{\sqrt{4+kx_0^2}}}(M^2-Q^2)(4+kx_0^2)^{\frac{5}{2}}+4a^2x_0^2(-4+kx_0^2)\sqrt{4+kx_0^2}(e^{\frac{4ax_0}{\sqrt{4+kx_0^2}}}+\Lambda)+64a^3x_0^3\Lambda] \\ &\times [\frac{(4+kx_0^2)}{4a(1+\frac{M\sqrt{4+kx_0^2}}{ax_0}+\frac{(M^2-Q^2)(4+kx_0^2)}{4a^2x_0^2}+\Lambda e^{-\frac{4ax_0}{\sqrt{4+kx_0^2}}})}], \end{aligned} \tag{91}$$

and

$$\begin{aligned} P(x_0) &= \frac{1}{8\pi} \sqrt{\frac{1}{g(x_0)}} [\frac{f'(x_0)}{f(x_0)g(x_0)}] \\ &= -[\pi(e^{\frac{4ax_0}{\sqrt{4+kx_0^2}}}(-4a^2x_0^2+M^2(4+kx_0^2)-Q^2(4+kx_0^2)+4a^2x_0^2\Lambda)(e^{\frac{4ax_0}{\sqrt{4+kx_0^2}}}(4a^2x_0^2+4aMx_0\sqrt{4+kx_0^2}+M^2(4+kx_0^2)-Q^2(4+kx_0^2))+4a^2x_0^2\Lambda)^3)^{-1} \\ &\quad \times [a^2e^{\frac{12ax_0}{\sqrt{4+kx_0^2}}}x_0^4(4+kx_0^2)^{\frac{3}{2}}(1+\frac{M\sqrt{4+kx_0^2}}{ax_0} \\ &\quad +\frac{(M^2-Q^2)(4+kx_0^2)}{4a^2x_0^2}+\Lambda e^{-\frac{4ax_0}{\sqrt{4+kx_0^2}}})^{-1}((4+kx_0^2) \\ &\quad (e^{\frac{4ax_0}{\sqrt{4+kx_0^2}}}(4aM^2x_0\sqrt{4+kx_0^2}-4aQ^2x_0\sqrt{4+kx_0^2}+M^3(4+kx_0^2)-M(-4a^2x_0^2+Q^2(4+kx_0^2)))-4a^2Mx_0^2\Lambda) \\ &\quad +16a^3x_0^3(2ax_0+M\sqrt{4+kx_0^2})\Lambda)], \end{aligned} \tag{92}$$

which significantly reduces to $P(x_0) = 0$ using eqn.(84). The vanishing surface pressure thus proves the metric continuity at the boundary surface, i.e. on the horizon $x = x_0$, as envisaged.

8. Conclusions

We thus study a charged, non-rotating, spherically symmetric black hole which has cosmological constant Λ (Reissner-Nordström+ Λ), active gravitational mass M and electric charge Q in exterior Friedman-Robertson-Walker (FRW) universe. The Einstein-Maxwell equations of the RN+ Λ black hole embedded in the FRW background are solved. As a procedure, we have started with a (2+1)-d RN+ Λ black hole and then performed a simple transformation only under suitable conditions to obtain a metric which matches with the exterior Friedman-Robertson-Walker universe and also derived a negative cosmological constant inside the black hole. New classes of exact solutions of the charged black hole are found. Literature reveals that there are three possible black hole solutions where the cosmological constant is (1) positive (2) negative and (3) zero. The cosmological constant found negative inside the black hole is also confirmed by the geodesic equations. Here, the cosmological constant is dependent on R , Q and $a(v)$ which correspond to the areal ra-

dius, charge, of the black hole and the scale factor of the universe respectively.

The century-old problem of describing a gravitationally bound system in an expanding universe in the frame-set of general relativity has seen many attempts to find a solution. Assuming that scale factor does not alter with the metric transformation, we find a maximum limit of the universal expansion. Despite its apparent simplicity, a full understanding of the mechanisms involved when general and realistic systems are considered has yet to be found. We also observe that the size, mass and charge of the black hole is affected by the expansion of the universe. An important observation is that for an observer at infinity, both the mass and charge of black hole increase with the contraction of the universe and decrease with the expansion of the universe. The cosmological constant has been found to be negative in a previous work too [29]. The AdS/CFT correspondence tells us that the case $\Lambda < 0$ is still worthy of consideration. In future, we plan to study the stability of such black hole with cosmological constant in an expanding universe.

We justify the use of two different methods for matching spacetimes. Boundary and matching conditions with the exterior FRW universe are studied in section IV., to arrive at a conclusion that the cosmological constant can have a negative value inside the black hole. However, the Darmois-Israel matching conditions have been studied in section VI., to deduce that the metric as well as the extrinsic curvature are continuous at the boundary surface. Alternatively, the vanishing surface pressure also proves the metric continuity at the boundary surface, i.e. on the horizon ($x = x_0$).

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