Journal of Applied Mathematics and Computation, 2022, 6(1), 30-40
https://www.hillpublisher.com/journals/JAMC/
ISSN Online: 2576-0653
ISSN Print: 2576-0645

Estimated Parameters of Rain Flow Distribution Using L-Moment Method in South Sulawesi, Indonesia

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How to cite this paper: Wahidah Sanusi, Srie Chaerunnisa, Suwardi Annas, Syafruddin Side, Muhammad Abdy. (2022) Estimated Parameters of Rain Flow Distribution Using L-Moment Method in South Sulawesi, Indonesia. *Journal of Applied Mathematics and Computation*, 6(1), 30-40.
DOI: 10.26855/jamc.2022.03.006

Received: December 30, 2021
Accepted: January 25, 2022
Published: February 23, 2022

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Abstract

This research is an applied research which aims to determine the type of distribution of each district/city in South Sulawesi Province. The data used are secondary data belonging to the Water Resources Management Office (PSDA) from 58 rainfall stations with a span of 31 years (1988-2018). The L-Moment method is a method that defines the probability density function of a distribution with its first 4 characteristics. The four characteristics are L-location λ_1, L-variability λ_2, L-skewness τ_3 dan L-curtosis τ_4. Several types of distribution in this study are the distribution of Generalized Extreme Value (GEV), Generalized Logistics (GLo), Generalized Pareto (GPa), Lognormal III (LN3), Pareto type III (Pe3) and Gumbel. Determination of the best distribution based on 3 test indicators, namely the minimum value of Root Mean Square Error (RMSE) and Mean Absolute Error (MAE) and the maximum value of the Correlation Coefficient (CC). The results obtained are Soppeng district have the GPa distribution; The results of this research can be utilized by the PSDA Service, Public Works Service, civil engineering sector, and researchers in the fields of hydrology, climatology and geography to plan irrigation development, prevention of future disasters or the need for science.

Keywords

Parameter estimation, rainfall distribution, L-moment method

1. Introduction

Indonesia is a country that is in a disaster-prone zone. Disasters often cause a lot of loss of life and property. One of the efforts to avoid the consequences of floods and landslides is to build a good irrigation system. The appropriate method used in analyzing rain data is the method of frequency analysis because it is due to the absence of certainty about the occurrence of rain both in space and time. Analysis of rainfall data is the initial stage in a series of planning in the field of water resources such as irrigation, flood control and so on, so that accurate analysis of rainfall data is needed [1].

Frequency analysis methods commonly used by researchers for forecasting include the Bayessian Nonparametric Regression & Adaptive Neuro-Fuzzy Inferences Systems (ANFIS) method to predict rainfall in Makassar City [2, 3, 4]; Automatic Clustering-Fuzzy Logical method for predicting population in Makassar City [5] apart from forecasting, Neuro-Fuzzy method is also used in modeling [6, 7]; Wavelet method to build a rainfall prediction system to support flood disaster early detection systems [8]; and the L-Moment method has been used as an alternative to the conventional moment estimation method and the maximum likelihood method, especially in research in the fields of hydrology, climatology, and meteorology [9, 10], but also in the economic and socio-economic fields [11].

The application of the L-Moment method approach in determining the type of frequency distribution of extreme rainfall data has been carried out by several researchers [12-16] which have been carried out in various regions around the world including Indonesia including Selangor, Malaysia; Sicily, Italy; Makassar City, South Sulawesi Province; West Sumatra and Medina, Saudi Arabia. The L-Moment method is used because it is easier to use, has the ability to characterize a wider range of probability distributions and is more prone to biased estimation. Based on the amount of rainfall, natural conditions in South Sulawesi Province and the analytical method used to forecast rainfall, therefore the L-Moment method is better used to estimate the parameters of the rainfall distribution in South Sulawesi.

2. Method

The data used in this study are the maximum annual rainfall data from the PSDA Office for 31 years (1988-2018) at 58 selected rainfall stations spread across South Sulawesi Province. With an area of 45,764.53 square kilometers covering 21 districts and 3 cities with Makassar City as its capital, the province of South Sulawesi is located between $0^o12'$-8^o latitude and $116^o48'$-122^o36 'east longitude [17]. The type of research used is theoretical and applied research. This study is divided into two major parts, namely (1) estimating distribution parameters using the L-Moment method and (2) determining the appropriate distribution for each district using the L-Moment method. Determination of the best distribution is obtained based on 3 test indicators, namely the minimum value of RMSE, MAE, and the maximum value of CC.

3. Result and Discussion

The estimation of the maximum rainfall distribution parameters in this study was carried out by listing all the bulk stations belonging to the Water Resources Management Agency (PSDA) which have the most complete data in each district/city and are considered to be representative of the South Sulawesi Province. As many as 154 rainfall stations were recorded and 58 stations were selected that had complete data from 1988 to 2018 (31 years), can be seen in Figure 1.

The data obtained were then performed data screening to see and find out the incomplete data. If there is incomplete data, the data estimate will be carried out using the algebraic mean method. The Average Algebra Method is presented in equation (1).

$$p = \frac{p_1 + p_1 + \cdots + p_n}{n} \tag{1}$$

The next step is to estimate the distribution parameters of the cumulative function of each distribution. Prior to that, the quantile functions of each were distributed. There are three parameters to be calculated, namely the parameters of location, shape and scale. The following is the parameter estimate for the candidate distribution used:

1) Generalized Extreme Value (GEV) Distribution

Comulative function

$$F = \exp[-exp(-y)] \ , y = \begin{cases} -\kappa^{-1} \ln\left\{1 - \frac{\kappa(x-\xi)}{\alpha}\right\}; \kappa \neq 0 \\ \frac{(x-\xi)}{\alpha} \qquad\qquad\qquad ; \kappa = 0 \end{cases} \tag{2}$$

for $\kappa \neq 0$

$$F = \exp\left[-\exp\left(-\left(-\kappa^{-1}\ln\left\{1 - \frac{\kappa(x-\xi)}{\alpha}\right\}\right)\right)\right]$$

$$(\ln F^{-1})^{-1} = \left\{1 - \frac{\kappa(x-\xi)}{\alpha}\right\}^{-\frac{1}{\kappa}}$$

$$x = \xi + \frac{\alpha(1 - (-\ln F)^{\kappa})}{\kappa}$$

for $\kappa = 0$

$$F = \exp\left[-exp\left(-\frac{(x-\xi)}{\alpha}\right)\right]$$

$$\ln(-\ln F) = -\frac{(x-\xi)}{\alpha}$$

$$x = \xi - \alpha \ln(-\ln F)$$

Thus, the quantile function of the cumulative function (F) of the Generalized Extreme Value (GEV) distribution is

$$x(F) = \begin{cases} \xi + \frac{\alpha\{1-(-\ln F)^{\kappa}\}}{\kappa} & ; \quad \kappa \neq 0 \\ \xi - \alpha \ln(-\ln F) & ; \quad \kappa = 0 \end{cases} \tag{3}$$

Figure 1. Coordinate Points for Rainfall Stations in South Sulawesi Province.

According to the equation (3), it is used to estimate the distribution parameters

$$\beta_0 = \frac{1}{(0+1)}\left(\xi + \frac{\alpha}{\kappa} - \frac{\alpha}{\kappa(r+1)^{\kappa}}\Gamma(1+\kappa)\right) = \xi + \frac{\alpha\left(1-\Gamma(1+\kappa)\right)}{\kappa}$$

$$\beta_1 = \frac{1}{1+1}\left(\xi + \frac{\alpha}{\kappa} - \frac{\alpha}{\kappa(r+1)^{\kappa}}\Gamma(1+\kappa)\right) = \frac{1}{2}\left(\xi + \frac{\alpha}{\kappa} - \frac{\alpha}{\kappa 2^{\kappa}}\Gamma(1+\kappa)\right)$$

$$\beta_2 = \frac{1}{2+1}\left(\xi + \frac{\alpha}{\kappa} - \frac{\alpha}{\kappa(r+1)^{\kappa}}\Gamma(1+\kappa)\right) = \frac{1}{3}\left(\xi + \frac{\alpha}{\kappa} - \frac{\alpha}{\kappa 3^{\kappa}}\Gamma(1+\kappa)\right)$$

$$\beta_3 = \frac{1}{3+1}\left(\xi + \frac{\alpha}{\kappa} - \frac{\alpha}{\kappa(r+1)^{\kappa}}\Gamma(1+\kappa)\right) = \frac{1}{4}\left(\xi + \frac{\alpha}{\kappa} - \frac{\alpha}{\kappa 4^{\kappa}}\Gamma(1+\kappa)\right)$$

So that the 4 characteristics first of the Generalized Extreme Value (GEV) distribution are obtained

$$\boldsymbol{\lambda_1} = \beta_0 = \xi + \frac{\alpha\left(1-\Gamma(1+\kappa)\right)}{\kappa} \tag{4}$$

$$\boldsymbol{\lambda_2} = 2\beta_1 - \beta_0 = 2\left(\frac{1}{2}\left(\xi + \frac{\alpha}{\kappa} - \frac{\alpha}{\kappa 2^{\kappa}}\Gamma(1+\kappa)\right)\right) - \left(\xi + \frac{\alpha\left(1-\Gamma(1+\kappa)\right)}{\kappa}\right) = \frac{\alpha(1-2^{-\kappa})\Gamma(1+\kappa)}{\kappa} \tag{5}$$

$$\boldsymbol{\lambda_3} = 6\beta_2 - 6\beta_1 + \beta_0 = \frac{\alpha\left(\Gamma(1+\kappa)\right)}{\kappa}\left(-2\frac{1}{3^{\kappa}} + 3\frac{1}{2^{\kappa}} - 1\right) \tag{6}$$

$$\boldsymbol{\lambda_4} = 20\beta_3 - 30\beta_2 + 12\beta_1 - \beta_0 = \frac{\alpha\left(\Gamma(1+\kappa)\right)}{\kappa}\left(-5\frac{1}{4^{\kappa}} + 10\frac{1}{3^{\kappa}} - 6\frac{1}{2^{\kappa}} + 1\right) \tag{7}$$

$$\boldsymbol{\tau_3} = \frac{\lambda_3}{\lambda_2} = \frac{\frac{\alpha\left(\Gamma(1+\kappa)\right)}{\kappa}\left(-2\frac{1}{3^{\kappa}} + 3\frac{1}{2^{\kappa}} - 1\right)}{\frac{\alpha\Gamma(1+\kappa)}{\kappa}(1-2^{-\kappa})} = \frac{2(1-3^{-\kappa})}{1-2^{-\kappa}} - 3 \tag{8}$$

$$\boldsymbol{\tau_4} = \frac{\lambda_4}{\lambda_2} = \frac{5(1-4^{\kappa})-10(1-3^{\kappa})+6(1-2^{\kappa})}{1-2^{-\kappa}} \tag{9}$$

From the L-Moment value, the Generalized Extreme Value (GEV) distribution parameter is obtained

$$\kappa \approx 7.8590c + 2.9554c^2\,, \quad c = \frac{2}{3+\tau_3} - \frac{\ln 2}{\ln 3} \tag{10}$$

$$\alpha = \frac{\lambda_2 \kappa}{(1-2^{-\kappa})\Gamma(1+\kappa)} \tag{11}$$

$$\xi = \lambda_1 - \frac{\alpha\{1-\Gamma(1+\kappa)\}}{\kappa} \tag{12}$$

2) Generelized Logistic (GLo) Distribution
Comulative function

$$F = \frac{1}{1-\ \exp(y)}, \qquad y = \begin{cases} -\kappa^{-1}\ln\left\{1-\dfrac{\kappa(x-\xi)}{\alpha}\right\}, \kappa \neq 0 \\ \dfrac{(x-\xi)}{\alpha}\,, \qquad\qquad \kappa = 0 \end{cases}$$

for $\kappa \neq 0$

$$F = \frac{1}{\left(1+e^{-\left(-\kappa^{-1}\ln\left\{1-\frac{\kappa(x-\xi)}{\alpha}\right\}\right)}\right)}$$

$$F + F\left(e^{-\left(-\kappa^{-1}\ln\left\{1-\frac{\kappa(x-\xi)}{\alpha}\right\}\right)}\right) = 1$$

$$\kappa^{-1}\ln\left\{1-\frac{\kappa(x-\xi)}{\alpha}\right\} = \ln\left(\frac{1-F}{F}\right)$$

$$x = \xi + \frac{\alpha\left(1-\left(\frac{1-F}{F}\right)^{\kappa}\right)}{\kappa}$$

for $\kappa = 0$

$$F = \frac{1}{\left(1+e^{-\left(\frac{x-\xi}{\alpha}\right)}\right)}$$

$$F + F\left(e^{-\left(\frac{x-\xi}{\alpha}\right)}\right) = 1$$

$$x = \xi - \alpha\ln\left(\frac{1-F}{F}\right)$$

Thus, the quantile function of the cumulative function (F) of the Generalized Logistic distribution is

$$x(F) = \begin{cases} \xi + \dfrac{\alpha\left[1-\left\{\frac{(1-F)}{F}\right\}^{\kappa}\right]}{\kappa}\,; & \kappa \neq 0 \\ \xi - \alpha\ln\left\{\frac{(1-F)}{F}\right\}; & \kappa = 0 \end{cases} \tag{13}$$

Equation (13) is used to estimate the distribution parameters

$$\beta_0 = \xi + \frac{\alpha}{\kappa}[1-\Gamma(1+\kappa)\Gamma(1-\kappa)]$$

$$\beta_1 = \frac{\xi}{2} + \frac{\alpha}{2\kappa}[1-\Gamma(1+\kappa)\Gamma(2-\kappa)]$$

$$\beta_2 = \frac{\xi}{3} + \frac{\alpha}{3\kappa}\left[1-\frac{\Gamma(1+\kappa)\Gamma(3-\kappa)}{2}\right]$$

$$\beta_3 = \frac{\xi}{4} + \frac{\alpha}{4\kappa}\left[1-\frac{\Gamma(1+\kappa)\Gamma(4-\kappa)}{6}\right]$$

So that the 4 characteristics first of the Generalized Logistic distribution are obtained

$$\boldsymbol{\lambda_1} = \beta_0 = \xi + \frac{\alpha}{\kappa}[1 - \Gamma(1+\kappa)\Gamma(1-\kappa)] \tag{14}$$

$$\boldsymbol{\lambda_2} = 2\beta_1 - \beta_0 = \alpha\Gamma(1+\kappa)\Gamma(1-\kappa) \tag{15}$$

$$\boldsymbol{\lambda_3} = 6\beta_2 - 6\beta_1 + \beta_0 = -\alpha\kappa\Gamma(1+\kappa)\Gamma(1-\kappa) \tag{16}$$

$$\boldsymbol{\lambda_4} = 20\beta_3 - 30\beta_2 + 12\beta_1 - \beta_0 = \frac{\alpha}{6}(1+5\kappa^2)\Gamma(1+\kappa)\Gamma(1-\kappa) \tag{17}$$

$$\boldsymbol{\tau_3} = \frac{\lambda_3}{\lambda_2} = \frac{-\alpha\kappa\Gamma(1+\kappa)\Gamma(1-\kappa)}{\alpha\Gamma(1+\kappa)\Gamma(1-\kappa)} = -\kappa \tag{18}$$

$$\boldsymbol{\tau_4} = \frac{\lambda_4}{\lambda_2} = \frac{\frac{\alpha}{6}(1+5\kappa^2)\Gamma(1+\kappa)\Gamma(1-\kappa)}{\alpha\Gamma(1+\kappa)\Gamma(1-\kappa)} = \frac{(1+5\kappa^2)}{6} \tag{19}$$

From the L-Moment value, the Generalized Logistic (GLo) distribution parameter is obtained

$$\kappa = -\tau_3 \tag{20}$$

$$\alpha = \frac{\lambda_2 \sin \kappa\pi}{\kappa\, \pi} \tag{21}$$

$$\xi = \lambda_1 - \alpha\left(\frac{1}{\kappa} - \frac{\pi}{\sin \kappa\pi}\right) \tag{22}$$

3) *Generalized Pareto* (GPa) Distribution

Comulative function

$$F = 1 - \exp(-y) \ , y = \begin{cases} -\kappa^{-1}\ln\left\{1 - \dfrac{\kappa(x-\xi)}{\alpha}\right\} , & \kappa \neq 0 \\ \dfrac{(x-\xi)}{\alpha} , & \kappa = 0 \end{cases}$$

for $\kappa \neq 0$

$$F = 1 - e^{-\left(-\kappa^{-1}\ln\left\{1-\frac{\kappa(x-\xi)}{\alpha}\right\}\right)}$$

$$\ln(1-F)^{-1} = \ln\left\{1 - \frac{\kappa(x-\xi)}{\alpha}\right\}^{-\frac{1}{\kappa}}$$

$$x = \xi + \frac{\alpha(1-(1-F)^{\kappa})}{\kappa}$$

for $\kappa = 0$

$$F = 1 - e^{-\left(\frac{x-\xi}{\alpha}\right)}$$

$$-\left(\frac{x-\xi}{\alpha}\right) = \ln(1-F)$$

$$x = \xi - \alpha\ln(1-F)$$

Thus, the quantile function of the cumulative function (F) of the Generalized Pareto distribution is

$$x(F) = \begin{cases} \xi + \frac{\alpha(1-(1-F)^{\kappa})}{\kappa} , & \kappa \neq 0 \\ \xi - \alpha\ln(1-F) \ , & \kappa = 0 \end{cases} \tag{23}$$

Equation (23) is used to estimate the distribution parameters

$$\beta_0 = \xi + \frac{\alpha}{(1+\kappa)}$$

$$\beta_1 = \frac{1}{2}\xi F^2\Big|_0^1 + \frac{\alpha F^2}{2\kappa}\Big|_0^1 - \frac{\alpha}{\kappa}\left(\frac{1}{(\kappa+1)(\kappa+2)}\right) = \frac{1}{2}\xi + \frac{\alpha}{2\kappa} - \frac{\alpha}{\kappa}\left(\frac{1}{(\kappa+1)(\kappa+2)}\right)$$

$$\beta_2 = \frac{1}{3}\xi F^3\Big|_0^1 + \frac{\alpha F^3}{3\kappa}\Big|_0^1 - \frac{\alpha}{\kappa}\left(\frac{2}{(1+\kappa)(2+\kappa)(3+\kappa)}\right) = \frac{1}{3}\xi + \frac{\alpha}{3\kappa} - \frac{\alpha}{\kappa}\left(\frac{2}{(1+\kappa)(2+\kappa)(3+\kappa)}\right)$$

$$\beta_3 = \frac{1}{4}\xi F^4\Big|_0^1 + \frac{\alpha F^4}{4\kappa}\Big|_0^1 - \frac{\alpha}{\kappa}\left(\frac{1}{1+\kappa} - \frac{3}{2+\kappa} + \frac{3}{3+\kappa} - \frac{1}{4+\kappa}\right) = \frac{1}{4}\xi + \frac{\alpha}{4\kappa} - \frac{\alpha}{\kappa}\left(\frac{6}{(1+\kappa)(2+\kappa)(3+\kappa)(4+\kappa)}\right)$$

So that the 4 characteristics first of the Generalized Pareto distribution are obtained

$$\boldsymbol{\lambda_1} = \beta_0 = \xi + \frac{\alpha}{(1+\kappa)} \tag{24}$$

$$\boldsymbol{\lambda_2} = 2\beta_1 - \beta_0 = \frac{\alpha}{(1+\kappa)(2+\kappa)} \tag{25}$$

$$\boldsymbol{\lambda_3} = 6\beta_2 - 6\beta_1 + \beta_0 = \frac{\alpha(1-\kappa)}{(1+\kappa)(2+\kappa)(3+\kappa)} \tag{26}$$

$$\boldsymbol{\lambda_4} = 20\beta_3 - 30\beta_2 + 12\beta_1 - \beta_0 = \frac{\alpha(1+\kappa)(2+\kappa)(3+\kappa) - \alpha(1+\kappa)(96+42\kappa)(4+\kappa) + \kappa(2+\kappa)(3+\kappa)\alpha\left(\kappa^2 - 16\alpha - 20\right)}{\kappa(1+\kappa)(2+\kappa)(3+\kappa)(4+\kappa)} \tag{27}$$

$$\boldsymbol{\tau_3} = \frac{\lambda_3}{\lambda_2} = \frac{\frac{\alpha(1-\kappa)}{(1+\kappa)(2+\kappa)(3+\kappa)}}{\frac{\alpha}{(1+\kappa)(2+\kappa)}} = \frac{1-\kappa}{3+\kappa} \tag{28}$$

$$\boldsymbol{\tau_4} = \frac{\boldsymbol{\tau_3}(\mathbf{1} + \mathbf{5}\boldsymbol{\tau_3})}{5 + \boldsymbol{\tau_3}} = \frac{\left(\frac{1-\kappa}{3+\kappa}\right)\left(1+5\left(\frac{1-\kappa}{3+\kappa}\right)\right)}{5+\left(\frac{1-\kappa}{3+\kappa}\right)} = \frac{(1-\kappa)(2-\kappa)}{(3+\kappa)(4+\kappa)} \tag{29}$$

From the L-Moment value, the Generalized Pareto (GPa) distribution parameter is obtained

$$\kappa = \frac{(1-3\tau_3)}{(1+\tau_3)} \tag{30}$$

$$\alpha = (1+\kappa)(2+\kappa)\lambda_2 \tag{31}$$

$$\xi = \lambda_1 - (2+\kappa)\lambda_2 \tag{32}$$

4) Lognormal III (LN3) Distribution

Comulative function

$$F = \int_{-\infty}^{x} 2\pi^{-\frac{1}{2}} \exp\left(-\frac{1}{2}x^2\right) dx$$

According to Hosking (1997) the cumulative function of the LN3 distribution does not have an explicit quantile function solution [13]. So that the reparameterization of the lognormal distribution is carried out in this case the parameters ξ, α, and κ are small modifications of the reparameterization [14], so that the parameters for the lognormal distribution III (LN3) is

$$\kappa \approx -\tau_3 \frac{E_0 + E_1\tau_3^2 + E_2\tau_3^4 + E_3\tau_3^6}{1 + F_1\tau_3^2 + F_2\tau_3^4 + F_3\tau_3^6} \tag{33}$$

with Estimated Parameter Coefficients

$$E_0 = 2{,}0466534$$

$$E_1 = -3{,}6544371$$

$$E_2 = 1{,}8396733$$

$$E_3 = -0{,}20360244$$

$$F_1 = -2{,}0182173$$

$$F_2 = 1{,}2420401$$

$$F_3 = -0{,}21741801$$

$$\alpha = \frac{\lambda_2 \kappa e^{-\frac{\kappa^2}{2}}}{1 - 2\Phi\left(\frac{-\kappa}{\sqrt{2}}\right)} \tag{34}$$

$$\xi = \lambda_1 - \frac{\alpha}{\kappa}\left(1 - e^{\frac{\kappa^2}{2}}\right) \tag{35}$$

5) Pearson Type III (Pe3) Distribution

Comulative function

If $\gamma > 0$ then the range of x is $\xi \leq x < \infty$ and

$$F = \frac{G\left(\alpha, \frac{x-\xi}{\beta}\right)}{\Gamma(\alpha)}, \quad G(\alpha, x) = \int_0^x t^{\alpha-1} e^{-t}\, dt$$

If $\gamma < 0$ then the range of x is $-\infty < x \leq \xi$ and

$$F = 1 - \frac{G\left(\alpha, \frac{x-\xi}{\beta}\right)}{\Gamma(\alpha)}, \quad G(\alpha, x) = \int_0^x t^{\alpha-1} e^{-t}\, dt$$

The cumulative function of the Pearson type III distribution does not have an explicit solution for the quantile function. The type III pearson distribution is usually considered to consist only of cases γ> 0 and is parameterized by the parameters α, β, and ξ. The parameterization was extended to include the usual type III pearson distribution, with positive slope and lower bound ξ, inverse type III pearson distribution, with negative slope and upper limit ξ, and the normal distribution which is included as a special case of the type III pearson distribution. It is possible that the type III Pearson distribution is used when the slope of the observed data may have a negative value [13].

The parameters of the Pearson type III distribution are as follows:

If $0 < |\tau_3| < \frac{1}{3}$ and $z = 3\pi\tau_3^2$ then, it is used

$$c \approx \frac{1+0{,}2906z}{z+0{,}1882z^2+0{,}0442z^3} \tag{36}$$

If $\frac{1}{3} \leq |\tau_3| < 1$ then, it is used

$$c \approx \frac{0{,}36067z-0{,}59567z^2+0{,}25361z^3}{1-2{,}78861z+2.56096z^2-0{,}77045z^3} \tag{37}$$

We found

$$\kappa = 2c^{-\frac{1}{2}} \operatorname{sign}(\tau_3) \tag{38}$$

$$\alpha = \frac{\lambda_2 \pi^{\frac{1}{2}} c^{\frac{1}{2}} \Gamma(c)}{\Gamma\left(c+\frac{1}{2}\right)} \tag{39}$$

$$\xi = \lambda_1 \tag{40}$$

6) Gumbel Distribution

Comulative function

$$F = \exp\left[-\exp\left(-\frac{x-\xi}{\alpha}\right)\right], -\infty < x < \infty$$

$$F = \exp\left[-\exp\left(-\frac{x-\xi}{\alpha}\right)\right]$$

$\ln(-\ln F) = -\frac{x-\xi}{\alpha}$

$$x(F) = \xi - \alpha \ln(-\ln F)$$

Thus, the quantile function of the cumulative function *F(x)* of the Gumbel distribution is

$$x(F) = \xi - \alpha \ln(-\ln F) \tag{41}$$

From equation (40), the estimation of the distribution parameters is obtained

$$\beta_0 = \frac{1}{0+1}\big(\xi + \alpha(\gamma + \ln(0+1))\big) = \xi + \alpha\gamma$$

$$\beta_1 = \frac{1}{1+1}\big(\xi + \alpha(\gamma + \ln(1+1))\big) = \frac{1}{2}(\xi + \alpha\gamma + \alpha \ln 2)$$

$$\beta_2 = \frac{1}{2+1}\big(\xi + \alpha(\gamma + \ln(2+1))\big) = \frac{1}{3}(\xi + \alpha\gamma + \alpha \ln 3)$$

$$\beta_3 = \frac{1}{3+1}\left(\xi + \alpha(\gamma + \ln(3+1))\right) = \frac{1}{4}(\xi + \alpha\gamma + \alpha \ln 4)$$

So that the 4 characteristics first of the Gumbel distribution are obtained

$$\boldsymbol{\lambda_1} = \beta_0 = \xi + \alpha\gamma \tag{42}$$

$$\boldsymbol{\lambda_2} = 2\beta_1 - \beta_0 = 2\left(\frac{1}{2}(\xi + \alpha\gamma + \alpha \ln 2)\right) - (\xi + \alpha\gamma) = \alpha \ln 2 \tag{43}$$

$$\boldsymbol{\lambda_3} = 6\beta_2 - 6\beta_1 + \beta_0 = 2\alpha \ln 3 - 3\alpha \ln 2 \tag{44}$$

$$\boldsymbol{\lambda_4} = 20\beta_3 - 30\beta_2 + 12\beta_1 - \beta_0 = 5\alpha \ln 4 - 10\alpha \ln 3 + 6\alpha \ln 2 \tag{45}$$

$$\boldsymbol{\tau_3} = \frac{\lambda_3}{\lambda_2} = \frac{2\alpha \ln 3 - 3\alpha \ln 2}{\alpha \ln 2} = \frac{2{,}19722 - 2{,}07944}{0{,}69315} = 0{,}1699 \tag{46}$$

$$\boldsymbol{\tau_4} = \frac{\lambda_4}{\lambda_2} = \frac{5\alpha \ln 4 - 10\alpha \ln 3 + 6\alpha \ln 2}{\alpha \ln 2} = \frac{6{,}93147 - 10{,}98612 + 4{,}15888}{0{,}69315} = 0{,}15037 \tag{47}$$

From the L-Moment value, the Gumbel distribution parameter is obtained

$$\alpha = \frac{\lambda_2}{\ln 2} \tag{48}$$

$$\xi = \lambda_1 - \gamma\alpha\ ;\ where\ \gamma\ (Euler's\ Constant) = 0{,}5772\ ... \tag{49}$$

Furthermore, the parameter value of each distribution using the parameters and data that has been obtained can be seen in Table 1.

Table 1. Distribution Parameter Values

Distribution	Parameter		
	Xi (ξ)	**Alpha (α)**	**K (κ)**
GEV	51,037	20,030	0.143
GLo	58,498	14,011	-0.081
GPa	22,159	64,168	0.699
LN3	-27,168	4,442	0.273
PE3	60,387	17,405	0.496
Gumbel	48,592	20,435	–

The parameter values in Table 1 are used to determine the best distribution in a district using 3 indicator tests, namely the minimum value of RMSE, MAE and the maximum value of CC so that the results in Table 2 and Figure 2 are obtained.

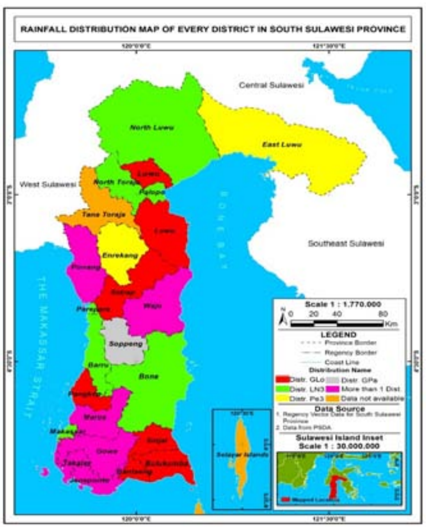

Figure 2. Distribution of Rainfall in Each Regency in South Sulawesi Province.

Based on Table 2, districts that follow one, two or three distribution models happen to have geographic and climatic locations. For example, the districts of East Luwu and Enrekang follow the Pearson Type III distribution model where their remote areas (yellow) have a soil type, namely podsolic with sandy and loamy soils, easy to wet, relatively low soil pH, which can be used as plantations or rice fields. Districts that follow 2 or 3 distribution models (in white) also have diversity of soil types, such as Pinrang district which follows the GEV, LN3, Pe3 distribution models with alluvial, regosol and podsolic soil types. Wajo Regency which follows the distribution model of LN3, GPa, Gumbel with alluvial, clay, podzolic, mediterranean, and grumosol soil types. Meanwhile, Maros district which follows the Pe3, LN3, GLo distribution model has regosol, mediterranean, lithosol and alluvial soil types; Takalar district follows the Pe3, GEV, GPa distribution model having alluvial soil types. This lies in the geographic location and climate that affects the rainfall of an area so that the distribution models vary.

Table 2. Districts Distribution

District	RMSE	MAE	CC	Distribution
Bantaeng	0.15246	0.11689	0.97264	*Generalized Logistic* (GLo)
Bulukumba	0.15929	0.12431	0.91208	*Generalized Logistic* (GLo)
Sinjai	0.11971	0.08350	0.93605	*Generalized Logistic* (GLo)
Bone	0.08878	0.05484	0.95759	*Lognormal III* (LN3)
Wajo	0.06879	0.05023	0.99520	*Lognormal III* (LN3)
	0.08840	0.05477	0.99146	*Generalized Pareto* (GPa)
	0.11805	0.06872	0.99387	Gumbel
Sidrap	0.17364	0.09656	0.94084	*Generalized Logistic* (GLo)
Luwu	0.15326	0.08017	0.97912	*Generalized Logistic* (GLo)
Enrekang	0.03694	0.02507	0.98888	*Pareto type III* (Pe3)
Palopo	0.08384	0.07201	0.98986	*Lognormal III* (LN3)
North Toraja	0.06757	0.04666	0.98561	*Lognormal III* (LN3)
North Luwu	0.03253	0.02524	0.99354	*Lognormal III* (LN3)
East Luwu	0.19199	0.13331	0,92182	*Pareto type III* (Pe3)
Pinrang	0.14384	0.10272	0.86214	*Generalized Extreme Value* (GEV)
	0.04591	0.03440	0.99281	*Lognormal III* (LN3)
	0.05873	0.04768	0.97502	*Pareto type III* (Pe3)
Parepare	0.07322	0.04095	0.92800	*Lognormal III* (LN3)
Barru	0.14885	0.07946	0.98691	*Lognormal III* (LN3)
Soppeng	0.11706	0.09526	0.98772	*Generalized Pareto* (GPa)
Pangkep	0.22018	0.13631	0.92262	*Generalized Logistic* (GLo)
Maros	0.06623	0.05116	0.98096	*Pareto type III* (Pe3)
	0.06124	0.04287	0.96376	*Lognormal III* (LN3)
	0.07648	0.05868	0.98436	*Generalized Logistic* (GLo)
Makassar	0.04699	0.03398	0.97573	*Lognormal III* (LN3)
Gowa	0.06660	0.04450	0.96148	*Lognormal III* (LN3)
	0.04968	0.03450	0.98926	*Pareto type III* (Pe3)
Takalar	0.09265	0.04426	0.99285	*Pareto type III* (Pe3)
	0.10649	0.07496	0.97903	*Generalized Extreme Value* (GEV)
	0.09099	0.07288	0.99225	*Generalized Pareto* (GPa)
Jeneponto	0.18990	0.08215	0.97016	*Generalized Logistic* (GLo)
	0.03453	0.02736	0.99630	*Lognormal III* (LN3)
	0.02790	0.02257	0.99501	*Pareto type III* (Pe3)

4. Discussion

Research on the distribution of rainfall has been carried out by [14] using 2 criteria tests namely Chi Square Test and Kolmogorov Smirnov Test to test 10 candidate probability distribution functions namely Uniform, Exponential, Gumbel, Logistics, Log Normal II, GLo, GEV, Gamma and Pearson , GPa in Aceh; [1] 3 test indicators namely Chi Square Test, Standard Error and Kolmogorov Smirnov Test to test 4 candidate probability distribution functions namely Normal, Gumbel, Log III Person and Iway Kadoya in West Sumatra.

Meanwhile, research abroad was carried out by [15] using 2 test indicators, namely the Z test and the L-Moment Ratios Diagram (LMRD) to test five candidate probability distribution functions that are considered to be able to model rainfall in Medina, namely GLo, GEV, GNo, GPa, Pe3; [12] RMSE test indicator to test three candidate probability distribution functions namely LN3, GEV, TCEV in Sicily, Italy; [16] 2 test indicators namely Mean Absolute Deviation Index (MADI) and Mean Square Deviation Index (MSDI) to test 8 candidate probability distribution functions namely NOM, LN2, LN3, LOG, GLO, EV1, GEV, GPA in Selangor and Kuala Lumpur . This study determines the rainfall distribution model for each district in South Sulawesi Province based on 3 test indicators, namely RMSE, MAE and CC.

5. Conclusion

Based on the results obtained, it can be concluded that the regions in South Sulawesi Province follow the distribution model of Generalized Extreme Value (GEV), Generalized Logistic (GLo), Generalized Pareto (GPa), Lognormal III (LN3), Pareto type III (Pe3) and Gumbel so that The government can use the results of this research in planning irrigation development and early prevention of natural disasters, especially those related to rainfall in South Sulawesi Province. Henceforth, the type of distribution used must be more diverse and the determination of the distribution model for each district can be used as data sources from the BMKG or divide the area based on other natural factors so that the distribution model is more specific.

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