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Global Existence and Stability of Time-Periodic Solution to 1-D Isentropic Compressible Euler Equations with Source Term

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Abstract

In this paper, we study the global existence and stability of time-periodic solution to 1-D isentropic compressible Euler equations with a source term $\alpha(t)\rho u$ near the subsonic background solution $(\underline{\rho},\ e^{\int_0^t \alpha(s)ds}\underline{u})^T$. We first introduce the Riemann invariants to transform equations to a symmetrical system, and then rewrite the dominating equations which indicate the difference between Riemann invariant and background subsonic solution. For the proof of the existence and stability of time-periodic solutions, we all use the iterative method. Since we consider the subsonic flow, we need to the left and right boundary conditions. When proving the existence, we transform the boundary value problem into two initial value problems and the linearized system is now decoupled in each iteration. For the isothermal compressible Euler equations, we can also prove the global existence and stability of time-periodic solution of initial-boundary problem in the same way as this paper. The proof process will not be described in detail.

Keywords

Isentropic compressible Euler equations, source term, subsonic flow, time-periodic solutions

1. Introduction

In practical applications, people use various ducts to transmit gas and control the flow of gas. However, since the wall of a duct is generally not sufficiently smooth, there is a certain resistance in the process of gas flow. In this paper, we consider the isentropic compressible Euler equations with a nonlinear term:

$$\begin{cases} \partial_t\rho + \partial_x(\rho u) = 0, \\ \partial_t(\rho u) + \partial_x(\rho u^2 + p) = \alpha(t)\rho u, \end{cases} (t,x) \in \mathbb{R} \times [0,L], \tag{1.1}$$

where ρ, u represent density and velocity of gas. $p = p(\rho)$ is the pressure satisfying the equation of state

$$p = A\rho^\gamma,$$

where $A = 1$ is considered and $\gamma > 1$ is the adiabatic gas exponent. The sonic speed c is given by

$$c = \sqrt{\frac{\partial p}{\partial \rho}} = \sqrt{\gamma}\rho^{\frac{\gamma-1}{2}}.$$

The term $\alpha\rho u$ in the second equation of (1.1) is so-called damping (acceleration) effect on the fluid for $\alpha < 0$ ($\alpha > 0$). Here $\alpha = \alpha(t) \in C^1(0, +\infty)$ denotes the external force coefficient satisfying

$$\int_0^t \alpha(s)ds \leq 0, \tag{1.2}$$

$$\int_t^{t+p} \alpha(s)ds = 0 \tag{1.3}$$

for any $t > 0$.

Obviously, by (1.3), $\alpha = \alpha(t)$ is a time-periodic function with a period $P > 0$, namely

$$\alpha(t+P) = \alpha(t). \tag{1.4}$$

Moreover, there exists a small enough constant $\varepsilon_0 > 0$, such that for any given constant $\varepsilon \in (0, \varepsilon_0)$,

$$|\alpha| < \varepsilon, \tag{1.5}$$

$$|\alpha^{'}| < \varepsilon, \tag{1.6}$$

where $'$ represents the time derivative.

In [1], $\alpha \equiv 0$ is considered and Yuan showed the existence of the time-periodic supersonic solution drived by the periodic boundary condition around $(\underline{\rho}, \underline{u})$. In this paper, we consider the global existence and stability problem of time-periodic subsonic solution in one dimensional isentropic compressible Euler system. We also consider the case including time-dependent damping.

We would like to show global existence and exponential stability of time-periodic solutions of initial-boundary value problem. There are many works on the studies of time-periodic solutions of the partial differential equations, for example, the vicous fluids equations [2-6] and the hyperbolic conservation laws [7-11]. All of the studies mentioned above discuss the time-periodic solutions which are derived by the time-periodic external forces or the piston motion. But there are few works on the time-periodic solutions of the hyperbolic conservations laws derived by the time-periodic boundary condition. In [1], Yuan studied time-periodic solutions for the isentropic compressible Euler equation (i.e. $\alpha = 0$) triggered by periodic supersonic boundary condition. For the quasilinear hyperbolic system with a more general time-periodic boundary conditions, Qu showed the existence and stability of the time-periodic solutions around a small neighborhood of $u \equiv 0$ in [12]. Recently, Wei et al. [13] studied the global stability problem for supersonic flows in one dimensional compressible Euler equations with a friction term $-\mu\rho|u|u, \mu > 0$.

In fact, when $\gamma = 1$, equations (1.1) becomes the isothermal Euler equationswith a nonlinear term:

$$\begin{cases} \partial_t\rho + \partial_x(\rho u) = 0, \\ \partial_t(\rho u) + \partial_x(\rho u^2 + c^2\rho) = \alpha(t)\rho u, \end{cases} (t,x) \in \mathbb{R} \times [0, L], \tag{1.7}$$

where sound speed $c > 0$ is a constant.

In this case, we can also prove the global existence and stability of time-periodic solution of initial-boundary problem in the same way as this paper. The proof process will not be described in detail here.

We organize the article as follows. In Section 2, we first introduce the Riemann invariants to transform equations (1.1) to a symmetrical system (2.3), and then rewrite the dominating equations which indicate the difference between Riemann invariant and background subsonic solution. Finally, we give the main results of this paper. In Section 3 and Section 4, we give the proof of our main result.

2. Preliminaries and main result

In this section, we provide some preliminaries and state the main results.

By simple computation, the eigenvalues of the homogenous Euler system are

$$\lambda_1 = u - c, \lambda_2 = u + c,$$

using (1.2) and $\underline{u} < \underline{c} = \sqrt{\gamma}\underline{\rho}^{\frac{\gamma-1}{2}}$, we have

$$\lambda_1(\underline{\rho}, e^{\int_0^t \alpha(s)ds}\underline{u}) < 0 < \lambda_2(\underline{\rho}, e^{\int_0^t \alpha(s)ds}\underline{u})$$

or

$$\lambda_1(\rho, u) < 0 < \lambda_2(\rho, u), \qquad (\rho, u) \in \mho, \tag{2.1}$$

where $\mho$ is a small neighborhood of $(\underline{\rho},\ e^{\int_0^t \alpha(s)ds}\underline{u})$.

Introducing the Riemann invariants r and s:

$$r = u - \frac{2}{\gamma - 1}c, \quad s = u + \frac{2}{\gamma - 1}c, \tag{2.2}$$

then system (1.1) become

$$\begin{cases} r_t + \lambda_1(r,s)r_x = \frac{\alpha(t)(r+s)}{2}, \\ s_t + \lambda_2(r,s)s_x = \frac{\alpha(t)(r+s)}{2}, \end{cases} \tag{2.3}$$

where $\lambda_1(r,s) = \frac{\gamma+1}{4}r + \frac{3-\gamma}{4}s$, $\lambda_2(r,s) = \frac{3-\gamma}{4}r + \frac{\gamma+1}{4}s$.

We assume the boundary conditions are

$$x = 0:\ s(t,0) = s_l(t), \tag{2.4}$$

$$x = L:\ r(t,0) = r_l(t), \tag{2.5}$$

where $r_l(t), s_l(t)$ are periodic functions with a period $P > 0$.

Let

$$m(t,x) = \left(m_1(t,x), m_2(t,x)\right)^T = \left(r(t,x) - r_\alpha(t), s(t,x) - s_\alpha(t)\right)^T,$$

$$m_\alpha(t) = \left(r_\alpha(t),\ s_\alpha(t)\right)^T,$$

where

$$r_\alpha(t) = e^{\int_0^t \alpha(s)ds}\underline{u} - \frac{2}{\gamma - 1}\underline{c}, \quad s_\alpha(t) = e^{\int_0^t \alpha(s)ds}\underline{u} + \frac{2}{\gamma - 1}\underline{c}, \tag{2.6}$$

is a background solution of system (2.3) with $\underline{c} = \sqrt{\gamma}\underline{\rho}^{\frac{\gamma-1}{2}}$.

Then system (2.3) can be converted into following form:

$$\begin{cases} \partial_t m_1 + \lambda_1(m + m_\alpha)\partial_x m_1 = \dfrac{\alpha(t)}{2}(m_1 + m_2), \\ \partial_t m_2 + \lambda_2(m + m_\alpha)\partial_x m_2 = \dfrac{\alpha(t)}{2}(m_1 + m_2). \end{cases} \tag{2.7}$$

The initial data are

$$t = 0:\ m(0,x) = m_0(x) = \left(m_{1_0}(x), m_{2_0}(x)\right)^T. \tag{2.8}$$

It is easy to see that boundary conditions (2.4)-(2.5) can be rewritten as

$$x = 0:\ m_2(t,0) = m_{2_l}(t) = s_l(t) - s_\alpha(t),\ t \ge 0, \tag{2.9}$$

$$x = L:\ m_1(t,l) = m_{1_l}(t) = r_l(t) - r_\alpha(t),\ t \ge 0, \tag{2.10}$$

where further by (1.3), we know that $m_{i_l}(t)$ $(i = 1,2)$ are also periodic functions with a period $P > 0$, i.e. $m_{i_l}(t + P) = m_{i_l}(t)$. And we assume the following compatibility conditions hold:

$$\begin{cases} m_{1_l}(0) = m_{1_0}(L),\ m_{2_l}(0) = m_{2_0}(0), \\ m'_{1_l}(0) + \lambda_1\left(m_0(L) + m_\alpha(0)\right)m'_{1_0}(L) \\ = \dfrac{\alpha(0)}{2}\left(\left(m_{1_l}(0) + m_{2_l}(0)\right) + \left(m_{1_0}(L) + m_{2_0}(L)\right)\right), \\ m'_{2_l}(0) + \lambda_2\left(m_0(0) + m_\alpha(0)\right)m'_{2_0}(0) \\ = \dfrac{\alpha(0)}{2}\left(\left(m_{1_l}(0) + m_{2_l}(0)\right) + \left(m_{1_0}(0) + m_{2_0}(0)\right)\right). \end{cases} \tag{2.11}$$

We treat $(\underline{\rho}, e^{\int_0^t \alpha(s)ds}\underline{u})^T$ as a background solution of system (1.1) with $\underline{u} < \underline{c}$ and by (1.2), it is a subsonic flow. Then we can take $O = (0,0)^T$ as a background solution of system (2.7). M is a small neighborhood of O corresponding to Ʊ.

Noting (2.1), one has

$$\lambda_1(m + m_\alpha) < 0 < \lambda_2(m + m_\alpha), \quad \forall m \in M. \tag{2.12}$$

Then, we can further set

$$\mu_i(m + m_\alpha) = \lambda_i^{-1}(m + m_\alpha), \quad i = 1,2$$

and denote

$$\mu_{max} = \max_{i=1,2} \sup_{m\in M} |\mu_i(m + m_\alpha)|.$$

By rescaling the time variable if needed, we have

$$\mu_{max} \leq 1. \tag{2.13}$$

Next, we shall prove the following theorems.

Theorem 2.1. (Existence of time-periodic solutions) There exist a small enough constant $\varepsilon_1 \in (0, \varepsilon_0)$, a constant $0 < \beta < 1$ and two constants $C_P > C_E > 0$, such that for any given $\varepsilon \in (0, \varepsilon_1)$ and any given $P > 0$, there exist C^2 smooth functions $m_{i_l}(t)(i = 1,2)$ satisfying

$$m_{i_l}(t + P) = m_{i_l}(t), \tag{2.14}$$

$$\| m_{i_l}(t) \|_{C^1(\mathbb{R}_+)} \leq \beta C_P \varepsilon, \tag{2.15}$$

$$\| m_{i_l}(t) \|_{C^2(\mathbb{R}_+)} \leq C_E, \tag{2.16}$$

there exists a C^1 smooth function $m_0 = m_0(x)$ satisfying

$$\| m_0 \|_{C^1(D)} \leq C_P \varepsilon \tag{2.17}$$

such that the initial-boundary value problem (2.7)-(2.10) admits a C^1 classical solution $m = m^{(P)}(t,x)$ on $D = \{(t,x) | t \in \mathbb{R}, x \in [0, L]\}$ satisfying

$$m^{(P)}(t + P, x) = m^{(P)}(t,x), \quad \forall (t,x) \in D, \tag{2.18}$$

$$\| m^{(P)} \|_{C^1(D)} \leq C_P \varepsilon. \tag{2.19}$$

Theorem 2.2. (Stability of time-periodic solutions) There exist a small constant $\varepsilon_2 \in (0, \varepsilon_1)$ and a constant $C_S > 0$, such that for any given $\varepsilon \in (0, \varepsilon_2)$ and for any given $P > 0$, there exist C^1 smooth functions $m_{i_l}(t)(i = 1,2)$ satisfying (2.14)-(2.15) and given C^1 smooth function $m_0 = m_0(x)$ satisfying (2.17) with compatibility conditions (2.11), such that the initial-boundary value problem (2.7)-(2.10) admits a unique global C^1 classical solution $m = m(t,x)$ on $E = \{(t,x) | t \geq 0, x \in [0, L]\}$, satisfying

$$\| m(t,\cdot) - m^{(P)}(t,\cdot) \|_{C^0} \leq C_S \varepsilon \delta^{[t/T_0]}, \quad \forall t \geq 0, \tag{2.20}$$

where $m^{(P)}$, depending on $m_{i_l}(t)(i = 1,2)$, is the time-periodic solution given by Theorem 2.1 and $\delta \in (0,1)$ is a constant,

$$T_0 = \max_{1\leq i\leq 2} \sup_{m\in M} \frac{L}{|\lambda_i(m + m_\alpha)|} = L\mu_{max}. \tag{2.21}$$

By taking $t \to +\infty$, this result directly shows the uniqueness.

Corollary 2.1. (Uniqueness of the time-periodic solution) There exists a constant $\varepsilon_3 \in (0, \varepsilon_2)$, such that for any given $\varepsilon \in (0, \varepsilon_3)$, any given $P > 0$, there exist C^1 smooth functions $m_{i_l}(t)(i = 1,2)$ satisfying (2.14)-(2.15) and given C^1 smooth function $m_0 = m_0(x)$ satisfying (2.17) with compatibility conditions (2.11), such that the corresponding time-periodic solution $m = m^{(P)}(t,x)$ is unique.

3. Existence of Time-periodic Solutions

In this section, we will use a linearized iteration method to construct the time-periodic solution of Theorem 2.1.

Using (2.7), we can establish our linearized system as

$$\partial_t m_i^{(k)} + \lambda_i\big(m^{(k-1)} + m_\alpha\big)\partial_x m_i^{(k)} = \frac{\alpha(t)}{2}\big(m_1^{(k-1)} + m_2^{(k-1)}\big), \tag{3.1}$$

$$x = 0: \qquad m_2^{(k)}(t,0) = \begin{cases} m_{2_l}(t), & t \geq 0, \\ 0 \quad , & t < 0, \end{cases} \tag{3.2}$$

$$x = L: \qquad m_1^{(k)}(t,L) = \begin{cases} m_{1_l}(t), & t \geq 0, \\ 0 \quad , & t < 0. \end{cases} \tag{3.3}$$

This means that we can iterate from

$$m^{(0)}(t,x) = O. \tag{3.4}$$

The proof of Theorem 2.1 is based on the uniform a priori estimates. Similar to the method in [14], once we show Proposition 3.1, we can directly prove Theorem 2.1.

Proposition 3.1. There are an enough small constant $\varepsilon_1 > 0$ and enough large constant $C_P > 0$ such that the sequence of C^1 solutions $m_i^{(k)}(t,x)(i = 1,2)$ to system (3.1)-(3.3) satisfies

$$m^{(k)}(t + P, x) = m^{(k)}(t,x), \qquad \forall (t,x) \in D, \forall k \in \mathbb{N}_+, \tag{3.5}$$

$$\| m^{(k)} \|_{C^1(D)} \leq C_P\varepsilon, \qquad \forall k \in \mathbb{N}_+, \tag{3.6}$$

$$\| m^{(k)} - m^{(k-1)} \|_{C^0(D)} \leq C_P\varepsilon, \qquad \forall k \in \mathbb{N}_+, \tag{3.7}$$

and

$$\max_{i=1,2}\{w(\xi|\partial_t m_i^{(k)}) + w(\xi|\partial_x m_i^{(k)})\} \leq G_P(\xi), \forall k \in \mathbb{N}_+, \tag{3.8}$$

where

$$\| m^{(k)} \|_{C^1(D)} \triangleq \max_{i=1,2}\{\| m_i^{(k)} \|_{C^0(D)}, \| \partial_t m_i^{(k)} \|_{C^0(D)}, \| \partial_x m_i^{(k)} \|_{C^0(D)}\},$$

$w(\xi|\cdot)$ is modulus of continuity:

$$w(\xi|g) = \sup_{\substack{|t_1-t_2|\leq\xi \\ |x_1-x_2|\leq\xi}} |g(t_1, x_1) - g(t_2, x_2)|,$$

and $G_P(\xi)$ is a continuous function of $\xi \in (0,1)$, independent of k, to be determined later with

$$\lim_{\xi\to 0^+} G_P(\xi) = 0.$$

Proof. Firstly, we set the uniform a priori estimates (3.5)-(3.8) inductively, namely, for each $k \in \mathbb{N}_+$, we prove

$$m_i^{(k)}(t + P, x) = m_i^{(k)}(t,x), \qquad \forall (t,x) \in D, \forall i = 1,2, \tag{3.9}$$

$$\max_{i=1,2} \| m_i^{(k)} \|_{C^1(D)} \leq C_P\varepsilon, \tag{3.10}$$

$$\max_{i=1,2} \| m_i^{(k)} - m_i^{(k-1)} \|_{C^0(D)} \leq C_P\varepsilon, \tag{3.11}$$

$$\max_{i=1,2} w(\xi|\partial_t m_i^{(k)}(\cdot,x)) \leq \frac{1}{8}G_P(\xi), \qquad \forall x \in [0,L] \tag{3.12}$$

and

$$\max_{i=1,2}\{w(\xi|\partial_t m_i^{(k)}) + w(\xi|\partial_x m_i^{(k)})\} \leq G_P(\xi) \tag{3.13}$$

under the assumptions

$$m_i^{(k-1)}(t + P, x) = m_i^{(k-1)}(t,x), \qquad \forall (t,x) \in D, \forall i = 1,2, \tag{3.14}$$

$$\max_{i=1,2} \| m_i^{(k-1)} \|_{C^1(D)} \leq C_P\varepsilon, \tag{3.15}$$

$$\max_{i=1,2} \| m_i^{(k-1)} - m_i^{(k-2)} \|_{C^0(D)} \leq C_P\varepsilon, \quad \forall k \geq 2, \tag{3.16}$$

$$\max_{i=1,2} w(\xi|\partial_t m_i^{(k-1)}(\cdot,x)) \leq \frac{1}{8}G_P(\xi), \quad \forall x \in [0,L] \tag{3.17}$$

and

$$\max_{i=1,2}\{w(\xi|\partial_t m_i^{(k-1)}) + w(\xi|\partial_x m_i^{(k-1)})\} \le G_P(\xi). \tag{3.18}$$

Especially, By (3.15), we know $m^{(k-1)} \in M$ for small ε_1, which makes our assumption (2.12) hold for (3.1)-(3.3).

Next, we multiply the i-th equation of (3.1) for $i = 1,2$ by $\mu_i(m^{(k-1)} + m_\alpha) = \lambda_i^{-1}(m^{(k-1)} + m_\alpha)$, and then exchange the roles of t and x to get

$$\partial_x m_1^{(k)} + \mu_1\big(m^{(k-1)} + m_\alpha\big)\partial_t m_1^{(k)} = \frac{\alpha(t)}{2}\mu_1\big(m^{(k-1)} + m_\alpha\big)\big(m_1^{(k-1)} + m_2^{(k-1)}\big), \tag{3.19}$$

$$x = L: \qquad m_1^{(k)}(t,L) = \begin{cases} m_{1_l}(t), & t \ge 0, \\ 0 \quad , & t < 0. \end{cases} \tag{3.20}$$

$$\partial_x m_2^{(k)} + \mu_2\big(m^{(k-1)} + m_\alpha\big)\partial_t m_2^{(k)} = \frac{\alpha(t)}{2}\mu_2\big(m^{(k-1)} + m_\alpha\big)\big(m_1^{(k-1)} + m_2^{(k-1)}\big), \tag{3.21}$$

$$x = 0: \qquad m_2^{(k)}(t,0) = \begin{cases} m_{2_l}(t), & t \ge 0, \\ 0 \quad , & t < 0. \end{cases} \tag{3.22}$$

In this way, we transform the boundary value problem (3.1)-(3.3) into two initial value problems (3.19)-(3.22) and the linearized system is now decoupled in each iteration.

Furthermore, let the characteristic curves $t = t_i^{(k)}(x; t_*, x_*)$ of μ_i for $i = 1,2$ and $k \in \mathbb{N}_+$:

$$\begin{cases} \dfrac{dt_i^{(k)}}{dx}(x; t_*, x_*) = \mu_i\big(m^{(k-1)} + m_\alpha\big)\big(t_i^{(k)}(x; t_*, x_*), x\big), \\ \qquad\qquad t_i^{(k)}(x_*; t_*, x_*) = t_*. \end{cases} \tag{3.23}$$

Since we start our iteration from (3.4), then $m_i^{(0)}(i = 1,2)$ satisfy (3.14)-(3.15) and (3.17)-(3.18) naturally.

Let's start by proving estimates (3.9)-(3.13). At first, by (1.3), (1.4) and (3.14), we know that if $m_i^{(k)}(t,x)(i = 1,2)$ is the solution of problem (3.19)-(3.22), so does $m_i^{(k)}(t+P,x)(i = 1,2)$, and then by the uniqueness to this linear system, we get (3.9).

Due to (2.15), we can directly get that the boundary satisfy (3.10). Thus, we only need to prove that $m_i^{(k)}$ for $i = 1,2$ satisfy (3.10) in the interior of region D.

Next, we first show the C^0 estimate of $m_i^{(k)}$for $i = 1,2$. Integrating (3.19) along the characteristic curve $t = t_1^{(k)}(x; t_*, L)$, we get

$$\begin{aligned} m_1^{(k)}\big(t_1^{(k)}(x; t_*, L), x\big) = m_1^{(k)}(t_*, L) + \int_L^x \frac{\alpha}{2}\mu_1\big(m^{(k-1)} + m_\alpha\big) \\ \big(m_1^{(k-1)} + m_2^{(k-1)}\big)\big(t_1^{(k)}(y; t_*, L), y\big)dy. \end{aligned}$$

Then by (1.5), (2.13), (2.15) and (3.15), we get

$$\| m_1^{(k)} \|_{C^0(D)} \le \beta C_P\varepsilon + LC_P\varepsilon^2 \le C_P\varepsilon. \tag{3.24}$$

Similarly, integrating (3.21) along $t = t_2^{(k)}(x; t_*, 0)$, we get

$$\| m_2^{(k)} \|_{C^0(D)} \le C_P\varepsilon. \tag{3.25}$$

By (3.24) and (3.25), we get the C^0 norm estimates in (3.10).

In order to get the estimates for the temporal derivative of $m_i^{(k)}(t,x)(i = 1,2)$, let

$$n_i^{(k)} = \partial_t m_i^{(k)}, \qquad i = 1,2, \qquad k \in \mathbb{N}_+. \tag{3.26}$$

Differentiating the equations (3.19) and (3.21) with respect to t and get the following formulas of wave decomposition for $n_i^{(k)}(i = 1,2)$:

$$\begin{aligned}
&\partial_x n_1^{(k)} + \mu_1(m^{(k-1)} + m_\alpha)\partial_t n_1^{(k)} \\
&= -\left(\nabla\mu_1(m^{(k-1)} + m_\alpha)\cdot(\partial_t m^{(k-1)} + m'_\alpha)\right) n_1^{(k)} \\
&+\frac{\alpha}{2}\mu_1(m^{(k-1)} + m_\alpha)\left(\partial_t m_1^{(k-1)} + \partial_t m_2^{(k-1)}\right) \\
&+\frac{\alpha}{2}\left(\nabla\mu_1(m^{(k-1)} + m_\alpha)\cdot(\partial_t m^{(k-1)} + m'_\alpha)\right)\left(m_1^{(k-1)} + m_2^{(k-1)}\right) \\
&+\frac{\alpha'}{2}\mu_1(m^{(k-1)} + m_\alpha)\left(m_1^{(k-1)} + m_2^{(k-1)}\right),
\end{aligned} \tag{3.27}$$

$$\begin{aligned}
&\partial_x n_2^{(k)} + \mu_2(m^{(k-1)} + m_\alpha)\partial_t n_2^{(k)} \\
&= -\left(\nabla\mu_2(m^{(k-1)} + m_\alpha)\cdot(\partial_t m^{(k-1)} + m'_\alpha)\right) n_2^{(k)} \\
&+\frac{\alpha}{2}\mu_2(m^{(k-1)} + m_\alpha)\left(\partial_t m_1^{(k-1)} + \partial_t m_2^{(k-1)}\right) \\
&+\frac{\alpha}{2}\left(\nabla\mu_2(m^{(k-1)} + m_\alpha)\cdot(\partial_t m^{(k-1)} + m'_\alpha)\right)\left(m_1^{(k-1)} + m_2^{(k-1)}\right) \\
&+\frac{\alpha'}{2}\mu_2(m^{(k-1)} + m_\alpha)\left(m_1^{(k-1)} + m_2^{(k-1)}\right),
\end{aligned} \tag{3.28}$$

where $'$ represents the time derivative.

Integrating (3.27) along the 1-characteristic curve $t = t_1^{(k)}(x; t_*, L)$, we get

$$\begin{aligned}
&n_1^{(k)}\left(t_1^{(k)}(x;t_*,L),x\right) \\
&= n_1^{(k)}(t_*,L) - \int_L^x \left(\nabla\mu_1(m^{(k-1)} + m_\alpha)\cdot(\partial_t m^{(k-1)} + m'_\alpha)\right) n_1^{(k)} \\
&\left(t_1^{(k)}(y;t_*,L),y\right)dy + \int_L^x \frac{\alpha}{2}\mu_1(m^{(k-1)} + m_\alpha)\left(\partial_t m_1^{(k-1)} + \partial_t m_2^{(k-1)}\right) \\
&+\frac{\alpha}{2}\left(\nabla\mu_1(m^{(k-1)} + m_\alpha)\cdot(\partial_t m^{(k-1)} + m'_\alpha)\right)\left(m_1^{(k-1)} + m_2^{(k-1)}\right) \\
&+\frac{\alpha'}{2}\mu_1(m^{(k-1)} + m_\alpha)\left(m_1^{(k-1)} + m_2^{(k-1)}\right)\left(t_1^{(k)}(y;t_*,L),y\right)dy.
\end{aligned}$$

Due to $\mu_i(m^{(k-1)} + m_\alpha) = \lambda_i^{-1}(m^{(k-1)} + m_\alpha)$ and (2.13), we can assume

$$\sup_{m\in M}|\nabla\mu_i(m + m_\alpha)| \le \mu_{max}^2 \sup_{m\in M}|\nabla\lambda_i(m + m_\alpha)| \le \sup_{m\in M}|\nabla\lambda_i(m + m_\alpha)| \le C_1, \tag{3.29}$$

where constant $C_1 > 0$ is independent of k.

Moreover, by (1.2), (1.5) and (2.6), we can get

$$\| m'_\alpha \|_{C^0(\mathbb{R})} \le \underline{u}\varepsilon, \tag{3.30}$$

where $\underline{u} > 0$ is a constant.

Then by (1.5), (1.6), (2.13), (2.15), (3.15), (3.29) and (3.30), we get

$$\begin{aligned}
\| n_1^{(k)} \|_{C^0(D)} &\le (\beta C_P\varepsilon + 2LC_P\varepsilon^2 + LC_1\varepsilon(C_P\varepsilon)^2 \\
&\quad + \underline{u}LC_1\varepsilon^2 C_P\varepsilon)e^{LC_1(C_P\varepsilon + \underline{u}\varepsilon)} \\
&\le \beta_1 C_P\varepsilon,
\end{aligned} \tag{3.31}$$

where constant $0 < \beta < \beta_1 < 1$ is independent of k.

Similarly, integrating (3.28), we get

$$\| n_2^{(k)} \|_{C^0(D)} \le \beta_1 C_P\varepsilon. \tag{3.32}$$

Then by (3.19) and (3.21) and using (1.5), (2.13), (3.15) and (3.31)-(3.32), we get

$$\| \partial_x m_i^{(k)} \|_{C^0(D)} \leq \beta_1 C_P \varepsilon + C_P \varepsilon^2 \leq C_P \varepsilon. \tag{3.33}$$

Finally, by the estimates (3.24)-(3.25) and (3.31)-(3.33), we get the C^1 estimates (3.10).

Due to the boundary are given functions, we have that (3.11) clearly holds at the boundary. Next, we try to get the Cauchy sequence property (3.11) in the interior of region D. For $k = 1$, noting (3.4) and using (3.24)-(3.25), we can directly get (3.11). For $k \geq 2$, by equation (3.19), we get

$$\begin{aligned}
&\left(\partial_x + \mu_1(m^{(k-1)} + m_\alpha)\partial_t\right)\left(m_1^{(k)} - m_1^{(k-1)}\right) \\
&= -\left(\mu_1(m^{(k-1)} + m_\alpha) - \mu_1(m^{(k-2)} + m_\alpha)\right)\partial_t m_1^{(k-1)} \\
&+\frac{\alpha}{2}\mu_1(m^{(k-1)} + m_\alpha)\left(m_1^{(k-1)} - m_1^{(k-2)} + m_2^{(k-1)} - m_2^{(k-2)}\right) \\
&+\frac{\alpha}{2}\left(\mu_1(m^{(k-1)} + m_\alpha) - \mu_1(m^{(k-2)} + m_\alpha)\right)\left(m_1^{(k-1)} + m_2^{(k-1)}\right).
\end{aligned} \tag{3.34}$$

Integrating (3.34) along the 1-characteristic curve $t = t_1^{(k)}(x; t_*, L)$, one has

$$\begin{aligned}
&\left|m_1^{(k)}\left(t_1^{(k)}(x; t_*, L), x\right) - m_1^{(k-1)}\left(t_1^{(k)}(x; t_*, L), x\right)\right| \\
&\leq \left|m_1^{(k)}(t_*, L) - m_1^{(k-1)}(t_*, L)\right| \\
&+L \sup_{m\in M} |\nabla\mu_1(m + m_\alpha)| \, \| m^{(k-1)} - m^{(k-2)} \|_{C^0(D)} \| \partial_t m^{(k-1)} \|_{C^0(D)} \\
&+L|\alpha|\mu_{max} \| m^{(k-1)} - m^{(k-2)} \|_{C^0(D)} \\
&+L|\alpha| \sup_{m\in M} |\nabla\mu_1(m + m_\alpha)| \, \| m^{(k-1)} - m^{(k-2)} \|_{C^0(D)} \| m^{(k-2)} \|_{C^0(D)}.
\end{aligned}$$

Then by (1.5), (2.13), (3.15)-(3.16), (3.20) and (3.29), we get

$$\begin{aligned}
\| m_1^{(k)} - m_1^{(k-1)} \|_{C^0(D)} &\leq LC_1(C_P\varepsilon)^2 + LC_P\varepsilon^2 + LC_1\varepsilon(C_P\varepsilon)^2 \\
&\leq C_P\varepsilon.
\end{aligned} \tag{3.35}$$

Similarly, we have

$$\| m_2^{(k)} - m_2^{(k-1)} \|_{C^0(D)} \leq C_P\varepsilon. \tag{3.36}$$

Finally, by (3.35)-(3.36), we get (3.11).

Next, we prove the estimates (3.12). First, we choose

$$G_P(\xi) = 40LC_1C_P\left(C_P + \underline{u}\right)\varepsilon\xi + 20C_P\xi,$$

where $\xi \in (0,1)$ and obviously, $\lim_{\xi\to 0^+} G_P(\xi) = 0$.

For given points (t_1, L) and (t_2, L) with $|t_1 - t_2| \leq \xi$ at the boundary $x = L$, by (2.16) and $C_P > C_E$, we get

$$\left|n_1^{(k)}(t_1, L) - n_1^{(k)}(t_2, L)\right| \leq C_E\varepsilon \leq \frac{1}{10}G_P(\xi). \tag{3.37}$$

Similarly, we can get

$$\left|n_1^{(k)}(t_1, 0) - n_1^{(k)}(t_2, 0)\right| \leq C_E\varepsilon \leq \frac{1}{10}G_P(\xi).$$

Then, we give any two points (t_1, x_*) and (t_2, x_*) in the domain D with $|t_1 - t_2| \leq \xi$ and $x_* \in [0, L]$, by definition (3.23), we have

$$\begin{aligned}
&\left|t_1^{(k)}(x; t_1, x_*) - t_1^{(k)}(x; t_2, x_*)\right| \\
&\leq |t_1 - t_2| + \left|\int_{x_*}^{x} \mu_1\left(m^{(k-1)} + m_\alpha\right)\left(t_1^{(k)}(y; t_1, x_*), y\right)\right.
\end{aligned}$$

$$-\mu_1\big(m^{(k-1)}+m_\alpha\big)\big(t_1^{(k)}(y;t_2,x_*),y\big)dy|$$
$$\le |t_1-t_2|+|\int_{x_*}^{x}\big|t_1^{(k)}(y;t_1,x_*)-t_1^{(k)}(y;t_2,x_*)\big|$$
$$\int_0^1\Big(\nabla\mu_1\big(m^{(k-1)}+m_\alpha\big)\cdot\big(\partial_t m^{(k-1)}+m'_\alpha\big)\Big)$$
$$\big(\gamma t_1^{(k)}(y;t_1,x_*)+(1-\gamma)t_1^{(k)}(y;t_2,x_*),y\big)d\gamma dy|.$$

By Gronwall's inequality, (3.15), (3.29) and (3.30), we get

$$\begin{aligned}&\left|t_1^{(k)}(x;t_1,x_*)-t_1^{(k)}(x;t_2,x_*)\right|\\ &\le |t_1-t_2|e^{LC_1C_P\varepsilon+\underline{u}LC_1\varepsilon}\\ &\le \xi\big(1+LC_1C_P\varepsilon+\underline{u}LC_1\varepsilon\big),\quad \forall x\in[0,L].\end{aligned}\tag{3.38}$$

Then we integrate (3.27) along $t=t_1^{(k)}(x;t_1,x_*)$ and $t=t_1^{(k)}(x;t_2,x_*)$ to get

$$\begin{aligned}&n_1^{(k)}(t_2,x_*)-n_1^{(k)}(t_1,x_*)\\ &\le n_1^{(k)}\Big(t_1^{(k)}(L;t_2,x_*),L\Big)-n_1^{(k)}\Big(t_1^{(k)}(L;t_1,x_*),L\Big)\\ &+\int_L^{x_*}-\Big(\nabla\mu_1\big(m^{(k-1)}+m_\alpha\big)\cdot\big(\partial_t m^{(k-1)}+m'_\alpha\big)\Big)n_1^{(k)}\Big|_{\left(t_1^{(k)}(x;t_1,x_*),x\right)}^{\left(t_1^{(k)}(x;t_2,x_*),x\right)}dx\\ &+\int_L^{x_*}\frac{\alpha}{2}\mu_1\big(m^{(k-1)}+m_\alpha\big)\Big(\partial_t m_1^{(k-1)}+\partial_t m_2^{(k-1)}\Big)\\ &+\frac{\alpha}{2}\Big(\nabla\mu_1\big(m^{(k-1)}+m_\alpha\big)\cdot\big(\partial_t m^{(k-1)}+m'_\alpha\big)\Big)\Big(m_1^{(k-1)}+m_2^{(k-1)}\Big)\\ &+\frac{\alpha'}{2}\mu_1\big(m^{(k-1)}+m_\alpha\big)\Big(m_1^{(k-1)}+m_2^{(k-1)}\Big)\Big|_{\left(t_1^{(k)}(x;t_1,x_*),x\right)}^{\left(t_1^{(k)}(x;t_2,x_*),x\right)}dx.\end{aligned}$$

Using Gronwall's inequality, (1.5)-(1.6), (2.13), (3.15), (3.29), (3.30), (3.37) and (3.38), we get

$$w\left(\xi\middle|n_1^{(k)}(\cdot,x)\right)\le\frac{1}{8}G_P(\xi),\qquad \forall x\in[0,L].\tag{3.39}$$

Similarly, we can get

$$w\left(\xi\middle|n_2^{(k)}(\cdot,x)\right)\le\frac{1}{8}G_P(\xi),\qquad \forall x\in[0,L].\tag{3.40}$$

Thus, we get (3.12).

Next, in order to show (3.13), we first discuss the special case that two given points (t_1,x_1) and (t_2,x_2) with $|t_1-t_2|\le\xi,|x_1-x_2|\le\xi$ locate on the same characteristic curve $t=t_1^{(k)}(x;t_*,x_*)$, namely, $t_2=t_1^{(k)}(x_2;t_1,x_1)$. Integrating the corresponding formulas of wave decomposition (3.27) along $t=t_1^{(k)}(x;t_1,x_1)$, we get

$$\begin{aligned}&n_1^{(k)}(t_2,x_2)-n_1^{(k)}(t_1,x_1)\\ &=\int_{x_1}^{x_2}-\Big(\nabla\mu_1\big(m^{(k-1)}+m_\alpha\big)\cdot\big(\partial_t m^{(k-1)}+m'_\alpha\big)\Big)n_1^{(k)}\\ &+\frac{\alpha}{2}\mu_1\big(m^{(k-1)}+m_\alpha\big)\Big(\partial_t m_1^{(k-1)}+\partial_t m_2^{(k-1)}\Big)\\ &+\frac{\alpha}{2}\Big(\nabla\mu_1\big(m^{(k-1)}+m_\alpha\big)\cdot\big(\partial_t m^{(k-1)}+m'_\alpha\big)\Big)\Big(m_1^{(k-1)}+m_2^{(k-1)}\Big)\\ &+\frac{\alpha'}{2}\mu_1\big(m^{(k-1)}+m_\alpha\big)\Big(m_1^{(k-1)}+m_2^{(k-1)}\Big)\Big(t_1^{(k)}(x;t_1,x_1),x\Big)dx.\end{aligned}$$

Then using (1.5)-(1.6), (2.13), (3.15), (3.29) and (3.30), one has

$$\left|n_1^{(k)}(t_2,x_2)-n_1^{(k)}(t_1,x_1)\right|\le\frac{1}{12}G_P(\xi).\tag{3.41}$$

For general two points (t_1, x_1) and (t_2, x_2) with $|t_1 - t_2| \le \xi, |x_1 - x_2| \le \xi$, we can choose a point (t_3, x_2) locating on the 1-th characteristic curve passing through (t_1, x_1), namely,

$$t_3 = t_1^{(k)}(x_2; t_1, x_1).$$

Using (2.13) and definition (3.23), we get

$$|t_3 - t_1| \le |\mu_1||x_2 - x_1| \le |x_2 - x_1| \le \xi,$$

then

$$|t_3 - t_2| \le 2\xi.$$

With the aid of the estimates (3.39) and (3.41), we have

$$\begin{aligned}
&\left|n_1^{(k)}(t_2, x_2) - n_1^{(k)}(t_1, x_1)\right| \\
&\le \left|n_1^{(k)}(t_2, x_2) - n_1^{(k)}\left(\frac{t_2 + t_3}{2}, x_2\right)\right| + \left|n_1^{(k)}\left(\frac{t_2 + t_3}{2}, x_2\right) - n_1^{(k)}(t_3, x_2)\right| \\
&\quad + \left|n_1^{(k)}(t_3, x_2) - n_1^{(k)}(t_1, x_1)\right| \\
&\le \frac{1}{4} G_P(\xi) + \frac{1}{12} G_P \\
&= \frac{1}{3} G_P(\xi),
\end{aligned}$$

then

$$w\left(\xi \middle| n_1^{(k)}\right) \le \frac{1}{3} G_P(\xi). \tag{3.42}$$

Similarly, we have

$$w\left(\xi \middle| n_2^{(k)}\right) \le \frac{1}{3} G_P(\xi). \tag{3.43}$$

Finally, using equations (3.19), (3.21) and by (1.5), (2.13), (3.15), (3.29), (3.30) and (3.42)-(3.43), we have

$$w\left(\xi \middle| \partial_x m_i^{(k)}\right) \le \frac{1}{2} G_P(\xi), \quad i = 1,2.$$

Therefore, we complete the proof of (3.13). Meanwhile, we also finish the proof of Proposition 3.1.

Proof of Theorem 2.1. First, by (3.7), we get that the sequence $\{m^{(k)}\}$ is a Cauchy sequence in C^0 space. Since C^0 space is complete, $\{m^{(k)}\}$ converges to some C^0 function $m^{(P)}$. Then by (3.5), we deduce that $m^{(P)}$ is time-periodic.

Furthermore, by (3.6) and (3.8) and applying Arzelà-Ascoli theorem, we know that $\{m^{(k)}\}$ is a sequential compact set, which indicates $\{m^{(k)}\}$ has a convergent subsequence in C^1 space. Then applying the uniqueness of limit, we know that whole original sequence $\{m^{(k)}\}$ converges to $m^{(P)}$in C^1 space.

Thus $m^{(P)}$ is C^1 smooth and is a classical solution to problem (2.7)-(2.10), satisfying (2.18). Meanwhile, using (3.6), we can get (2.19). Then we regard $m_0(x) = m^{(P)}(0, x)$ as the initial data of system (2.7) and obviously, $m_0(x)$ satisfies (2.17).

4. Stability of the Time-periodic Solution

In this section, we will prove Theorem 2.2. According to the existence and uniqueness of local C^1 solution to the mixed initial-boundary value problem (cf. Chapter 4 in Li Ta-tsien and Yu Wen-ci [14]), in order to prove the existence of the classical solutions $m = m(t, x)$, it suffices to show the following Lemma 4.1. Similar to the method in [15], we can show Lemma 4.1.

Lemma 4.1. There exists a small constant $\varepsilon' \in (0, \varepsilon_2)$ such that for any fixed ε with $0 < \varepsilon < \varepsilon'$, on the whole existence domain of the C^1 solution $m(t, x)$, the following inequality holds:

$$\| m(t, x) \|_{C^1(E)} \le C\varepsilon, \tag{4.1}$$

with constant $C > C_P$.

By Lemma 4.1 and Theorem 2.1, we can get the global existence of the classical solutions $m = m(t, x)$ and $m = m^{(P)}(t, x)$ to the corresponding problem (2.7)-(2.10) with

$$\max\left\{\| m \|_{C^1(E)}, \| m^{(P)} \|_{C^1(E)}\right\} \le C\varepsilon, \tag{4.2}$$

where constant $C > C_P$.

We use the iteration method to prove (2.20). Assuming that for some $t_0 > 0$ and $N \in \mathbb{N}$, we have

$$\max_{i=1,2} \| m_i(t,\cdot) - m_i^{(P)}(t,\cdot) \|_{C^0} \leq C_S \varepsilon \delta^N, \qquad \forall t \in [t_0, t_0 + T_0],$$

we show that

$$\max_{i=1,2} \| m_i(t,\cdot) - m_i^{(P)}(t,\cdot) \|_{C^0} \leq C_S \varepsilon \delta^{N+1}, \qquad \forall t \in [t_0 + T_0, t_0 + 2T_0].$$

In order to this purpose, we let

$$\zeta(t) = \max_{1 \leq i \leq 2} \sup_{x \in [0,L]} \left| m_i(t,x) - m_i^{(P)}(t,x) \right|.$$

Obviously, $\zeta(t)$ is continuous with

$$\zeta(t_0 + T_0) \leq C_S \varepsilon \delta^N.$$

It is only necessary to show

$$\zeta(t) \leq C_S \varepsilon \delta^{N+1}, \qquad \forall t \in [t_0 + T_0, \tau] \tag{4.3}$$

under the assumption

$$\zeta(t) \leq C_S \varepsilon \delta^N, \qquad \forall t \in [t_0, \tau] \tag{4.4}$$

for each given $\tau \in [t_0 + T_0, t_0 + 2T_0]$.

Since both $m = m(t,x)$ and $m = m^{(P)}(t,x)$ are the solutions of (2.7), we get

$$(\partial_t + \lambda_i(m + m_\alpha)\partial_x) m_i = \frac{\alpha(t)}{2}(m_1 + m_2), \qquad i = 1,2, \tag{4.5}$$

$$\left(\partial_t + \lambda_i\left(m^{(P)} + m_\alpha\right)\partial_x\right) m_i^{(P)} = \frac{\alpha(t)}{2}\left(m_1^{(P)} + m_2^{(P)}\right), \qquad i = 1,2. \tag{4.6}$$

From the boundary conditions (2.9)-(2.10), we can directly get

$$m_1(t,L) - m_1^{(P)}(t,L) = 0,$$
$$m_2(t,0) - m_2^{(P)}(t,0) = 0,$$

then (4.3) obviously holds on the boundary.

In the domain E, we use (4.5)-(4.6) to get

$$\begin{aligned}
&(\partial_t + \lambda_i(m + m_\alpha)\partial_x)\left(m_i - m_i^{(P)}\right)\\
&= (\partial_t + \lambda_i(m + m_\alpha)\partial_x) m_i - \left(\partial_t + \lambda_i\left(m^{(P)} + m_\alpha\right)\partial_x\right) m_i^{(P)}\\
&+ \left(\lambda_i\left(m^{(P)} + m_\alpha\right) - \lambda_i(m + m_\alpha)\right)\partial_x m_i^{(P)}\\
&= \left(\lambda_i\left(m^{(P)} + m_\alpha\right) - \lambda_i(m + m_\alpha)\right)\partial_x m_i^{(P)}\\
&+ \frac{\alpha(t)}{2}\left(m_1 - m_1^{(P)} + m_2 - m_2^{(P)}\right), \qquad i = 1,2.
\end{aligned}$$

Then we integrate this equation along the i-th characteristic curve $x = f_i(t; t_*, x_*)$ $(i = 1,2)$ is defined by the following ODE system:

$$\begin{cases} \dfrac{d f_i(t; t_*, x_*)}{dt} = \lambda_i(m + m_{\alpha)}(f_i(t; t_*, x_*), t), \\ \qquad\qquad f_i(t_*; t_*, x_*) = x_*, \end{cases}$$

and using (1.5), (3.29), (4.2) and (4.4), we have

$$\zeta(t_*) \leq 2T_0 C_1 C \varepsilon C_S \varepsilon \delta^N + 2T_0 \varepsilon C_S \varepsilon \delta^N.$$

Here we note that due to (2.21), passing through each point $(t_*, x_*) \in [t_0 + T_0, \tau] \times [0, L]$, the backward characteristic curve $x = f_i(t; t_*, x_*)$ intersects the boundary $x = 0$ or $x = L$ at $t \in [t_0, \tau]$.

We can choose $\varepsilon_2 > 0$ so small that

$$2T_0C_1C\varepsilon + 2T_0\varepsilon \leq \delta,$$

which yield

$$\zeta(t_*) \leq C_S\varepsilon\delta^{N+1}.$$

By the arbitrariness of t_*, we get (4.3). Thus, we complete the proof of Theorem 2.2.

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