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A Study on Biodiversity in a Polluted Marine Ecosystem Using Hydrodynamic and Ecosystem Models

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Abstract

In this paper, a study is carried out to understand the biological and chemical changes down the ocean depth by considering a coupled nonlinear ecosystem (biodiversity) model containing a pollution function. Our research interest is to gain insight into temporal-spatial dynamics of phytoplankton, zooplanktons, fishes and birds in the ocean polluted by chemical substances. We determine the equilibrium points and the analytic solution to the model using Fourier transform method and numerical simulations for the model are carried out under certain conditions. The population of the species is found to be cyclic, oscillatory and show persistence, coexistence and chemotaxis phenomena at some periods during the simulation. The population of the species is global sector stable and the population ratio of the species varies on a spatial scale of 10 meters from the surface of the ocean to the bottom. From simulation results on species population ratio, the species tend to flourish in patches that provide nutrients, abundant oxygen and comfort from pollution effects.

Keywords

Biodiversity, Marine Ecosystem, Model, Oscillation and Simulation

1. Introduction

The anthropological activity of human beings in the marine ecosystem has had a great impact on marine biological population. Many marine lives are destroyed because of pollution of oceans with chemical substances from oil spillage and radioactive substances [1]. Toxic substances introduced into coastal water accumulate in the body of marine organisms through the food chain [2, 3]. Moreover, oil leaks from offshore facilities and broken tankers due to accidents result in oil pollution which cause volatile organic compounds to be emitted into the atmosphere [3, 4].

Eutrophication/hypertrophication are experienced in the coastal areas because of run-off of chemical substances such as fertilizer into the body of water. This leads to growth of algae and depletion of oxygen in the marine environment. This will eventually affect the survival of species in the water [5, 6].

In recent times, pollution of the sea and oceans with plastic products are on increase and possess dangerous effects on marine ecosystems. Therefore, there is the need to have marine friendly ecosystems by checkmating activities of man that are having a negative influence on marine ecosystems.

Mathematical modeling is one of key research tools in marine ecosystem study these days, because of the effect of pollution and climate change on marine ecosystems. There is much data acquired from marine exploration and predictive models are used for gaining insight to biodiversity, species-abundance and the effect of climate change on the species. Many models are on population level assessment; others are on ecological risk, physical characteristics such as salinity and temperature [7-10]. Chemical changes in the environment that may be dangerous to biomass in the given habitat in the long run [11-12]. Study of thermodynamic and ecological process occurring in the natural environment involving the sea, atmosphere and soil, their interactions and forecasts become relevant for sustainable development [13].

Mathematical models and empiric studies revealed that two potentially disruptive influences on ecosystems are instabilities caused by nonlinear feedback, and time lag in the interaction of biological species and the stochastic force by fluctuating environment [3, 7, 12]. Stochastic effect directly affects survival of species [12].

In oceanographic studies, mathematical modeling is a frontal tool for understanding the physical, chemical and biological changes down the ocean depth. There is vast data from the observatory system, in which predictive analyses of the marine data needed to be analyzed using machine learning and risk assessment models [8, 9]. Models give insight on temporal-spatial dynamics of the species in the ocean. Many ecosystems and hydrodynamic models are in the form of coupled ordinary differential equations and partial differential equations [2, 5, 14]. Some models are from the stochastic family [12].

Numerical simulation is one of the useful tools to predict the effects of anthropogenic sources on marine ecosystems. A numeric model used in numerical simulation contains not only a physical or hydrodynamic model but also an ecosystem model. A Hydrodynamic model studies the flow of chemicals in water, often capturing information on circulation of nutrients, velocity of the water, concentration of pollutants, physical properties of the water such as temperature, salinity and so on. Significant insights are gained on factors that affect the biological regiment of marine ecosystems and how climate and pollution affect the biodiversity in the marine habitat [7, 12]. Mangrove forest is found to store more carbon than other forest, influencing the fish and reducing global warming [15].

Population-level assessments (PLA) provide a better measure of response to chemicals than individual-level assessments by making use of ecological models to integrate potentially complex data related to the effects of chemicals on life history traits [6, 8, 9]. Ecological models and PLA provide relevant measures of ecological impact [9].

Ecological models are tools to estimate and manage the environmental fate and ecological risk of chemicals [8]. AQUATOX is a general ecological risk model capable of representing the combined environmental fate, effects of toxic chemicals and their impacts on aquatic ecosystems. AQUATOX combines aquatic ecosystem, chemical fate, and ecotoxicological constructs to obtain a truly integrative fate and effects model [8].

Currently, numerous physicochemical indicators including biochemical oxygen demand (BOD), concentration of oxygen (COD), nutrients, and toxic residues are being used to indicate disturbances during water quality assessment of aquatic ecosystems. Along with rapid development of interfacing and computational methods, behavioral data feasibly utilized for monitoring [5, 9].

The ecosystem models often consider the population of species in the niche, for example, primary food producers like phytoplankton and zooplankton, benthos etc. The detailed uses of the hydrodynamic models are in many articles and books in the literature (see for examples [14, 3, 10, 12]).

The motivation for this paper is to explore the use of prey-predator models to study predation of phytoplankton by zooplankton and the later predated upon by fish and the fish by seabird. Simulate the model when chemical substances pollute the ocean. We will investigate accidental discharge of chemicals into the ocean by spillages from crude oil and other chemical substances and the effect on growth of the species in the marine ecosystem. Moreover, investigate how it could possibly lead to loss of habitat of the species. The result from this paper will be useful for strategic conservation of resources in the marine ecosystem for sustainable development.

Figure 1. Marine biodiversity.

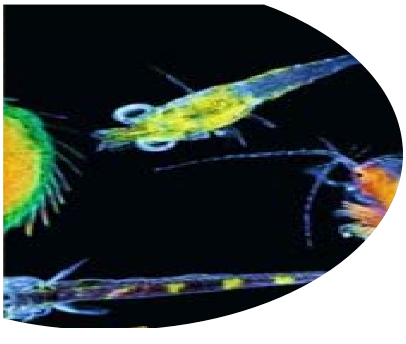

Figure 2. Zooplanktons in marine environment.

Figure 1 is a typical marine environment containing a variety of species such as seaweeds, phytoplankton, and small sea animals like zooplankton and plants and animals in the ocean floor. Figure 2 is on zooplankton in a given marine environment

2. Statement of the problem and preliminaries

2.1 Assumptions

The environment is complex with heterogeneous structure and species—abundance relation satisfied. The ecological conditions divided into patches which are not uniform within different ecological niches.

Throughout, we assume that the underlying differential equations used to formulate the model in the equation (1) are well poised, that is, the solution exists, continuously dependent on initial data and its solution is uniquely determined in a given interval.

We will study the final state of the solution of the system in the equation (1) when it starts with some initial boundary conditions $\boldsymbol{x}^{\mathbf{0}} = (\boldsymbol{x}_1^{\mathbf{0}}, \boldsymbol{x}_2^{\mathbf{0}}, \boldsymbol{x}_3^{\mathbf{0}}, \boldsymbol{x}_4^{\mathbf{0}})$. We note that it will generate $\mathbf{2^4 = 16}$ equilibrium points in the whole space $-\infty < \boldsymbol{x}_i < \infty, \forall i = 1,2,3,4$.

Figure 3. Penguin, jellyfish and plant in the marine ecosystem.

Figure 3: Top: Penguin birds at the seashore, below: Jellyfish and plant at the seashore.

Figure 4 is about air and water pollution by cars, trains, airplanes, ships and manmade activities from industrial sources. The pollution from these sources is leading to greenhouse effects and entropy traps.

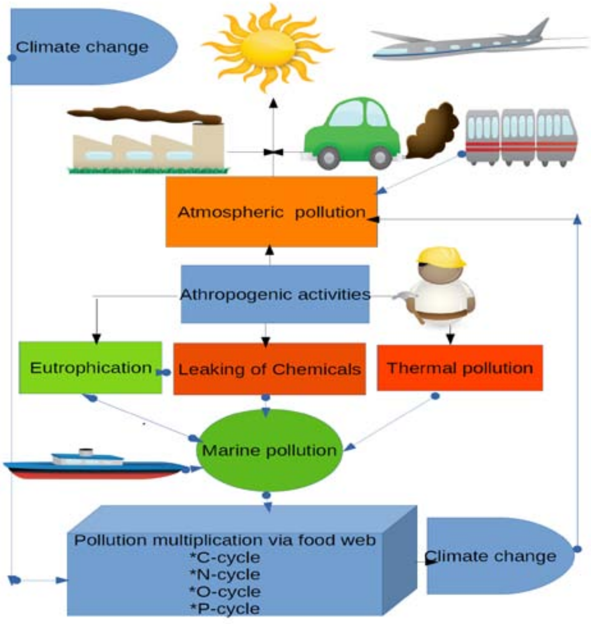

Figure 4. Modelling description diagram for marine ecosystem.

Pollution of air through incomplete combustion of hydrocarbons leads to acid rain. Acid rain affects agricultural productivity of land and marine life. Ships dump waste oil into the sea/ocean. Wastes from thermal stations are increasing the ocean temperatures that ultimately lead to some cataclysmic effect on marine life. Pollution of the ocean is interfering with CNOP –cycles and leading to adverse effect on climate and environment.

3. Methods

Consider nonlinear ecosystem (biodiversity) model (NBM)

$$\frac{\partial X_i(t,x)}{\partial t} = X^{\alpha}{}_i(t,x)\sum_{j=0}^{i} a_j X_j(t,x) + D_i\frac{\partial^2 X_i(t,x)}{\partial x^2} - d_i p(x) \tag{1a}$$

$$j = 1,2,\cdots i, i = 1,2,3,..,n$$

Subject to initial-boundary conditions

$$\begin{aligned} &X_1(0,t) = 0, X_1(L,t) = 0, X_1(x,t) = f_1(x)\\ &X_2(0,t) = l_1, X_2(L,t) = 0, X_2(x,t) = f_2(x)\\ &X_3(0,t) = l_2, X_2(L,t) = 0, X_3(x,t) = f_3(x)\\ &X_4(0,t) = l_3, X_4(L,t) = 0, X_4(x,t) = f_4(x)\\ &\vdots\\ &X_N(0,t) = l_N, X_N(L,t) = 0, X_N(x,t) = f_N(x) \end{aligned} \tag{1b}$$

Where $X_i(x,t), i = 1,2,\ldots,N$ are the population of the marine species at time t and spatial distant x from the water surface of the ocean, a_1 intrinsic growth rate of the specie and $a_i, i = 2,3,4,\ldots,N$ are coefficients of inter and interspecies competitive interaction consists of intrinsic power of interference (attack) to other individuals and intrinsic ability of defense against of the other species. $D_i, i = 1,2,3,4,\ldots,N$ Diffusion rates and $p(x)$ is pollution function obtained from some pollution model. α and $d_i, i = 1,2,3,4$ are some pollutants absorbed by the species. $i_i, L_i, i = 1,2$ depth constants and $f_i(x), i = 1,2,3,4,\ldots,N$ are continuous functions of x. $p(x)$ is the pollution function which is define as follows:

The species are linked together through food web and the populations of the prey and sufficient enough to sustain the predators. For definite application of the model, consider the nonlinear phytoplankton-zooplankton-fish-bird model which is member of the family of NBM with $n = 1,2,3,4$.

$$\left.\begin{aligned} \frac{\partial X_1(t,x)}{\partial t} &= X^{\alpha}{}_1(t,x)\sum_{i=0}^{n} a_i X_i(t,x) + D_1\frac{\partial^2 X_1(t,x)}{\partial x^2} - d_1 p(x)\\ \frac{\partial X_2(t,x)}{\partial t} &= X^{\alpha}{}_2(t,x)\sum_{i=0}^{n} b_i X_i(t,x) + D_2\frac{\partial^2 X_2(t,x)}{\partial x^2} - d_2 p(x)\\ \frac{\partial X_3(t,x)}{\partial t} &= X^{\alpha}{}_3(t,x)\sum_{i=0}^{n} c_i X_i(t,x) + D_3\frac{\partial^2 X_3(t,x)}{\partial x^2} - d_3 p(x)\\ \frac{\partial X_4(t,x)}{\partial t} &= X^{\alpha}{}_4(t,x)\sum_{i=0}^{n} d_i X_i(t,x) + D_4\frac{\partial^2 X_4(t,x)}{\partial x^2} - d_4 p(x) \end{aligned}\right\} \tag{2a}$$

Subject to initial-boundary conditions

$$\begin{aligned} &X_1(0,t) = 0, X_1(L,t) = 0, X_1(x,t) = f_1(x)\\ &X_2(0,t) = l_1, X_2(L,t) = 0, X_2(x,t) = f_2(x)\\ &X_3(0,t) = l_2, X_2(L,t) = 0, X_3(x,t) = f_3(x)\\ &X_4(0,t) = l_3, X_4(L,t) = 0, X_4(x,t) = f_4(x) \end{aligned} \tag{2b}$$

Where $X_1(x,t)$ is the population of the marine specie at time t and spatial distant $x$from the water surface of the ocean, a_1, b_1, c_1, d_1 are intrinsic growth rate of the species and $a_i, b_i, c_i, d_i, i = 2,3,4$ are coefficients of inter and interspecies competitive interaction consists of intrinsic power of interference (attack) to other individuals and intrinsic ability of defense against of the other species. $D_i, i = 1,2,3,4$ Diffusion rates; $i_i, L_i, i = 1,2$ depth constants and $f_i(x), i = 1,2,3,4$ are continuous functions of x.

The value of $p(x)$ can be obtained from real life data by using machine learning or fluid dynamics. α and $d_i, i = 1,2,3,4$ are some pollutants absorbed coefficients by the species.

For purpose of numerical simulation, we will make use of the pollution function $p(x)$ defined as follows:

Pollution function:

$$p(x)=\begin{cases}\frac{q\left(1-\frac{1}{2}\frac{\delta+\beta}{\beta}\right)e^{(\delta+\beta)x}}{k_1A}, x\geq 0\\ \frac{q\left(\frac{\delta+\beta}{\beta}\right)e^{(\delta+\beta)x}}{k_1A}, x<0\end{cases} \quad (3)$$

Where $\delta=\frac{v}{2D_p}, \beta=\frac{\sqrt{v^2+4D_pk_1}}{2_p}$, q the rate of pollutant discharged into the stream (kgm-1day -1); $k_1$the degradation rate coefficient at 20°C. D_p the dispersion in the direction of x and $v$velocity of water. The cross section of the stream in the ocean containing the pollutants is 2100m, $k_1=8.29, D_p=$ 43,200 m^2/day^{-1} and q=0.5. The source for the parameters is [5].

Remark 1

The value $p(x)$ can also obtain from real life data using machine learning or the use of other forms of fluid dynamics models. If $\alpha=1$, the model becomes the Lotka-Volterra model and when $\alpha=0$, it is simple diffusion model with pollution function.

Definition 1

First, we consider the conditions that guarantee the existence of a positive equilibrium point $x^*(x_i{}^*>0),\forall i$. Let $x^*=(x_1^*,x_2^*,x_3^*,x_4^*)$ be the equilibrium point to the equation (2a-2b). We define global sector stability of the solution of equation (1) as follows:

$\boldsymbol{P}=\{\boldsymbol{i}:\boldsymbol{x_i^*}>0\}, \boldsymbol{Q}=\{\boldsymbol{j}:\boldsymbol{x_j^*}=\boldsymbol{0}\}$ and

$\Omega=\{x:x_i>0$ for $i\in P, x_j\geq 0$ for $j\in Q\}$.

The nonnegative equilibrium x_i^* is globally sector stable if every solution of the equation (1a-1b), $X_i(t,x)$ which starts from Ω remains in it for all finite t and converges to x^* as $t\to\infty$. That is $\lim_{t\to\infty} X_i(t,x)=x_i^*$

4. Results

4.1 Equilibrium equations and Equilibrium sets

Without loss of generality take $D_i=0, i=1,2,3,4,$ and $p(x)=0$ for simplicity's sake, we have denoted $X_i(x,t)$ by $x_i=X_i(x,t), i=1,2,3,4$, then the equilibrium points for the model in equation (2a) are the points $\boldsymbol{x_i^*}, \boldsymbol{i}=\boldsymbol{1},\boldsymbol{2},\dots\boldsymbol{4}.$ such that

$$\left.\begin{array}{c}\sum_{i=1}^4 a_i x_i=0\\ \sum_{i=1}^4 b_i x_i=0\\ \sum_{i=1}^4 c_i x_i=0\\ \sum_{i=1}^4 d_i x_i=0\end{array}\right\} \quad (4)$$

We obtain solution to the equilibrium equations in the equation (4) and put them into equilibrium sets. There are sixteen possible equilibrium sets we will list eight sets only because of complexity in obtaining the solution to the equilibrium equations [16]. The equilibrium sets are:

$E_1=\{x_1=0,x_2=0,x_3=0,x_4=0\}$

$E_2=\{x_1=\frac{a_1b_2}{b_1a_2}, x_2=-\frac{a_1}{b_1}, x_3=0, x_4=0\}$

$E_3=\left\{x_1=\frac{a_1c_3}{c_1a_3}, x_2=0, x_3=\frac{a_1c_3}{a_3c_1}, x_4=0\right\}$,

$$E_4=\left\{x_1=\frac{a_1(a_2b_3-a_3b_2)}{a_2b_1c_3-a_2b_3c_1-a_3b_1c_2-a_3b_2c_1}, x_2=-\frac{a_1(a_2c_3-a_3c_2)}{a_2b_1c_3-a_2b_3c_1-a_3b_1c_2-a_3b_2c_1}, x_3=\frac{a_1(a_2b_3-a_3b_2)}{a_2b_1c_3-a_2b_3c_1-a_3b_1c_2-a_3b_2c_1}, x_4=0\right\}$$

$E_5=\left\{x_1=\frac{a_1d_4}{d_1a_4}, x_2=0, x_3=0, x_4=-\frac{a_1}{d_1}\right\}$

$$E_6 = \left\{x_1 = \frac{a_1(b_2d_4 - d_4d_2)}{a_2b_1d_4 - a_2b_3d_1 - a_4b_1d_2 - a_4b_2d_1}, x_2 = -\frac{a_1(a_2d_4 - a_4d_2)}{a_2b_1d_4 - a_2b_3d_1 - a_4b_1d_2 - a_4b_2d_1}, x_3 = 0, x_4 = \frac{a_1(a_2b_3 - a_3b_2)}{a_2b_1d_4 - a_2b_3d_1 - a_4b_1d_2 - a_4b_2d_1}\right\}$$

$$E_7 = \left\{x_1 = \frac{a_1(b_2d_4 - d_4d_2)}{a_2b_1d_4 - a_2b_3d_1 - a_4b_1d_2 - a_4b_2d_1}, x_2 = -\frac{a_1(a_2d_4 - a_4d_2)}{a_2b_1d_4 - a_2b_3d_1 - a_4b_1d_2 - a_4b_2d_1}, x_3 = 0, x_4 = \frac{a_1(a_2b_3 - a_3b_2)}{a_2b_1d_4 - a_2b_3d_1 - a_4b_1d_2 - a_4b_2d_1}\right\}$$

$$E_8 = \left\{x_1 = \frac{a_1(c_3d_4 - c_4d_3)}{a_2b_1d_4 - a_2b_3d_1 - a_4b_1d_2 - a_4b_2d_1}, x_2 = 0, x_3 = -\frac{a_1(a_3c_4 - a_4d_2)}{a_2b_1d_4 - a_2b_3d_1 - a_4b_1d_2 - a_4b_2d_1}, x_4 = \frac{a_1(a_3c_4 - a_4d_3)}{a_2b_1d_4 - a_2b_3d_1 - a_4b_1d_2 - a_4b_2d_1}\right\}$$

4.2 Analytic Solution to the model

We intend to study the behaviors of the four species in a given habitat in the marine ecosystem. We will investigate the behaviors of the species from the surface of water to 10 meters deep down the ocean.

We obtain the analytic solution to the model as follows:

Multiply the first equation in the equation (2a) by b_i and the second equation in the same equation i.e., equation (1) by a_i and sum the equations together. By collecting the like terms, we get

$$\sum_1^n b_i \frac{\partial X_1(x,t)}{\partial t} - \sum_1^n a_i \frac{\partial X_2(x,t)}{\partial t} = D_1 \frac{\partial X_1(x,t)}{\partial x^2} \sum_1^n b_i - D_2 \frac{\partial X_2(x,t)}{\partial x^2} \sum_1^n a_i = -\eta\left(\sum_1^n b_i - \sum_1^n a_i\right) \tag{5}$$

Now, let $a = \sum_1^n a_i$, $b = \sum_1^n b_i$, $c = \sum_1^n c_i$ and $d = \sum_1^n d_i$. Collecting the like terms in the equation (5) and separating the variables, we have the following equations:

$$\frac{\partial X_1(x,t)}{\partial t} = D_1 \frac{\partial^2 X_1(x,t)}{\partial x^2} + \frac{\lambda}{b} \tag{6}$$

$$\frac{\partial X_2(x,t)}{\partial t} = D_2 \frac{\partial^2 X_2(x,t)}{\partial x^2} + \frac{\eta(x,t)}{a}(b - a) - \frac{\lambda}{b} \tag{7}$$

$$\frac{\partial X_3(x,t)}{\partial t} = D_3 \frac{\partial^2 X_3(x,t)}{\partial x^2} - \frac{\mu}{b} \tag{8}$$

$$\frac{\partial X_4(x,t)}{\partial t} = D_4 \frac{\partial^2 X_4(x,t)}{\partial x^2} + \frac{\eta(x,t)}{c}(c - b) - \frac{\mu}{d} \tag{9}$$

For some constants λ and μ.

The solution to the equation (6) using Fourier series method with boundary condition $X_1(0,1) = 0, X_1(10,t) = 0, X_1(x,0) = u_0(x)$ is obtained as

$$\left.\begin{array}{l} X_1(x,t) = \frac{2}{10(D_1b)^2}\sum_{n=1}^{\infty}\left[\int_0^{10}\left(\left(\left(u_0(x)D_1b + \frac{\lambda x(x-10)}{2}\right)\sin\left(\frac{n\pi x}{10}\right)dx\right)\sin\left(\frac{n\pi x}{10}\right)e^{\frac{-D_1\pi^2n^2t}{100}}\right)D_1b - \theta_1\right] \\ \theta_1 := \lambda x(x-10) \end{array}\right\} \quad ..(10)$$

Choosing the parameters as $t = 0, b = 0.11, \lambda = 1, \mu = 2, D_1 = 0.1, n = 100$, $0 \le t \le 5$ and $0 \le x \le 10$ then we have the following graph in Figure 5:

The solution to the equation (5) using Fourier series method with boundary condition $X_2(0,t) = 1, X_2(10,t) = 0, X_2(x,0) = v_0(x)$ is obtained as

$$\left.\begin{array}{l} X_2(x,t) = \frac{1}{10(D_2)^2}\sum_{n=1}^{\infty}\left[\int_0^{10}\left(\left(\left(2v_0(x)D_2 + (xg + \frac{D_2}{5})(x-10)\right)\sin\left(\frac{n\pi x}{10}\right)dx\right)\sin\left(\frac{n\pi x}{10}\right)e^{\frac{-D_2\pi^2n^2t}{100}}\right)D_2 - \theta_2\right] \\ \theta_2 := 5(xg + \frac{D_2}{5})(x-10) \end{array}\right\} \tag{11}$$

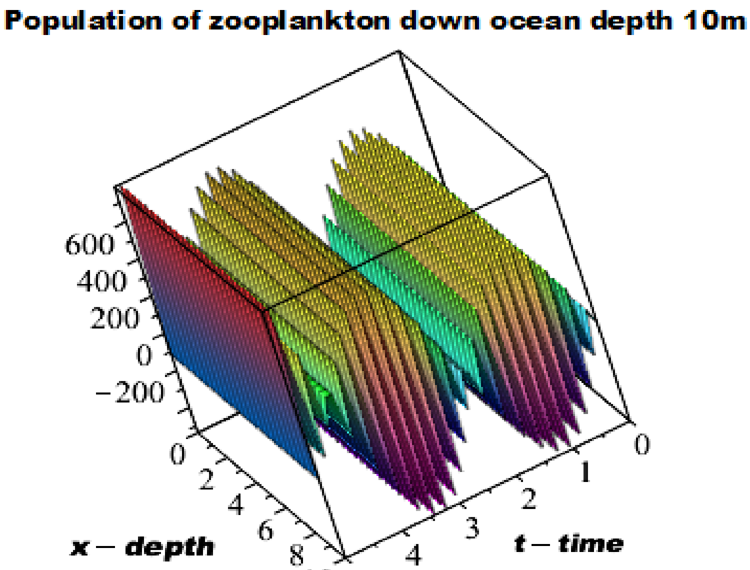

Figure 5. Population of zooplankton from the surface of the ocean to 10m down.

Choosing the parameters as $t = 0, g = 2, \lambda = 1, \mu = 2, D_2 = -0.1, n = 100$, $0 \le t \le 5$ and $0 \le x \le 10$ then we have the following graph in the Figure 6.

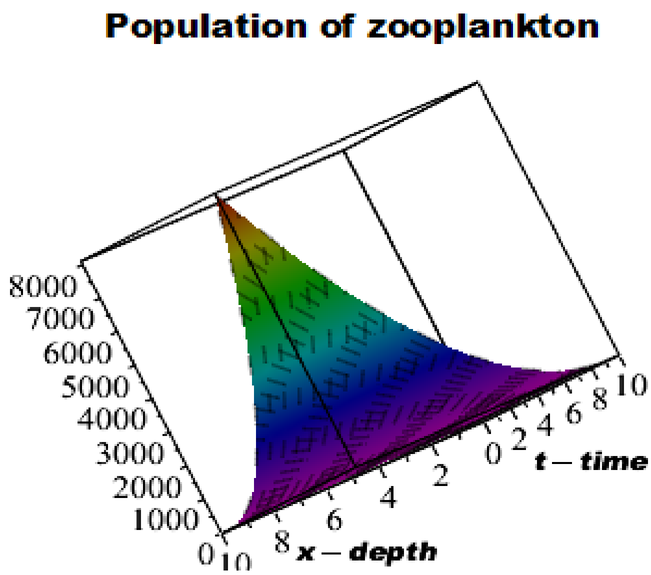

Figure 6. Population of zooplankton at 10m ocean depth.

The solution to the equation (6) using Fourier series method with boundary condition $X_3(0,t) = 1, X_3(10,t) = 0, X_3(x,0) = z_0(x)$ is

$$\left.\begin{aligned} X_3(x,t) &= \frac{1}{50(D_3 d)^2}\sum_{n=1}^{\infty}\left[\int_0^{10}\left(\left(\left((10 z_0(x) D_3 d + (x-10)(D_3 d - 5\mu x)\right)\right)\sin\left(\frac{n\pi x}{10}\right)dx\right)\sin\left(\frac{n\pi x}{10}\right)e^{\frac{-D_3\pi^2 n^2 t}{100}}\right)D_3 d - \theta_3\right] \\ \theta_3 &:= (x-10)(D_3 d - 5\mu x) \end{aligned}\right\} \quad (12)$$

Choosing the parameters as $t = 0, h = 1, \mu = 2, D_3 = -0.1, n = 100$, $0 \le t \le 5$ and $0 \le x \le 10$ then we have the following graph

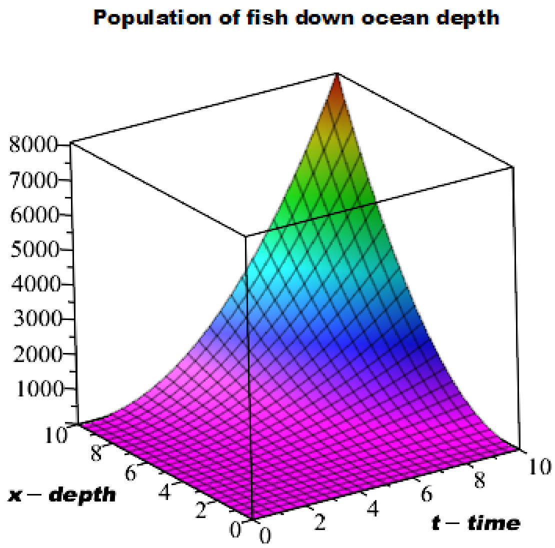

Figure 7. Population of fish at 10m ocean depth.

The solution to the equation (6) using Fourier series method with boundary condition $X_4(0,t) = 1, X_4(10,t) = 0, X_4(x,0) = w_0(x)$ is obtained

$$X_1(x,t) = \frac{1}{50(D_4c)^2}\sum_{n=1}^{\infty}\left[\int_0^{10}\left(\left(-5\sin\left(\frac{n\pi x}{10}\right)\left(-2w_0(x)D_4c + (x-10)(b-c+1)\mu x - \frac{D_4c}{5}\right)dx\right)\sin\left(\frac{n\pi x}{10}\right)e^{\frac{-D_4\pi^2 n^2 t}{100}}\right)D_4c - \theta_4\right]$$

$$\theta_4 = 5(x-10)(b-c+1)\mu x - \frac{D_4c}{5} \tag{12}$$

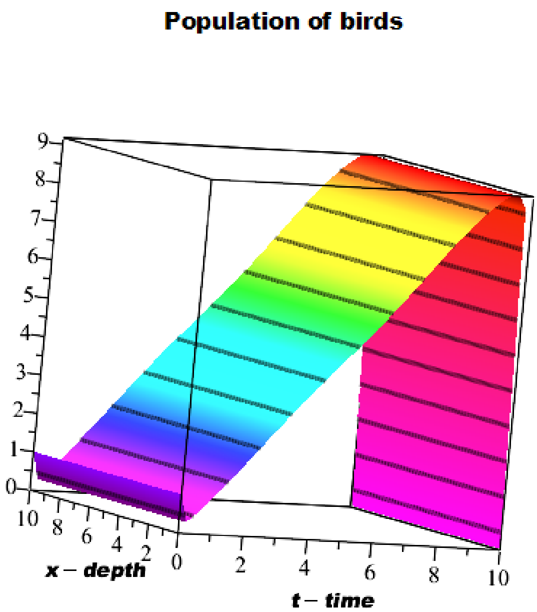

Figure 8. Population of seabird swimming at 10m ocean depth.

We investigate the asymptotic behavior of the populations of the species. We note that $\lim_{t\to\infty} X_i(t,x) = -\theta_i(x), i = 1,2,3,4$.

The population is nonnegative; therefore, we expect $\theta_i < 0, i = 1,2,3,4$ where $\theta_i = (\theta_1 \quad \theta_2 \quad \theta_3 \quad \theta_4)$.

Solving the individual inequality for $\theta_i < 0, i = 1,2,3,4.$ We have for phytoplankton

$$\theta_1(x) := \begin{cases} x \in (10,\infty) \cup (-\infty,0) & \lambda < 0 \\ \phi & \lambda = 0 \\ x \in (0,10) & \lambda < 0 \end{cases}$$

$$\theta_2(x) := \begin{cases} x \in (10,\infty) & 5xg + D_2 < 0 \\ \varphi, & 5xg + D_2 = 0 \\ x \in (-\infty,10) & 5xg + D_2 > 0 \end{cases}$$

$$\text{For fish } \theta_3(x) := \begin{cases} \{10 < x, x < v\} & 0 < a \wedge 10 < v \\ \{v < x, x < 10\} & 0 < a \wedge v < 10 \\ \{v < x, x < 10\} & a < 0 \wedge 10 < v \\ \{v < x, x < v\} & v - 10 = 0 \wedge a < 10 \\ \{v < x, 10 < v\} & a < 0 \wedge v < 10 \\ \varphi & \text{Otherwise} \end{cases}$$

$$\text{For seabird } \theta_4(x) := \begin{cases} \{10 < x, x < D_2\} & 0 < a \wedge 10 < D_2 \\ \{x < 10, D_2 < x\} & 0 < a \wedge D_2 < 10 \\ \{D_2 < x, x < 10\} & a < 0 \wedge 10 < D_2 \\ \{D_2 < x, x < D_2\} & D_2 - 10 = 0 \wedge a < 10 \\ \{x < D_2, 10 < x\} & a < 0 \wedge D_2 < 10 \\ \varphi & \text{Otherwise} \end{cases} \tag{13}$$

The only practical feasible situations for phytoplankton is the case $10 < x < \infty, 0 < x < 10, \lambda < 0$.

For zooplankton, the practical feasible situation is when $10 < x < \infty$, $5xg + D_2 < 0$ and $0 < x < 10$, $5xg + D_2 > 0$. For fish, the situation is very complex, but for practical situation, we must have that

$\{v < x, x < 10\} \cup \{a < 0 \wedge 10 < v\}$ and $\{v < x, x < 10\} \cup \{a < 0 \wedge v < 10\}$. For bird the situation is very complex, but for practical situation $\{x < 10, D_2 < x\} \cup \{0 < a \wedge D_2 < 10\}$ and $\{x < 10, D_2 < x\} \cup \{0 < a \wedge D_2 < 10\}$.

4.3 Global Sector Stability Analysis

The global sector stability required that the equilibrium point of the model in the equation (1) is nonnegative together with $x_i = x_i^*$, $i = 1,2,3,4$ where $x_i = X_i(t,x)$, $i = 1,2,3,4$ are solutions to the model and x_i^* coincide with the equilibrium points in the sets $E_i, i = 1,2,..,8$.

Therefore, for a practical feasible situation for phytoplankton, zooplankton, fish and bird to be global sector stable the solution to the model must converge to the equilibrium points i.e. $\lim_{t\to\infty} X_i(t,x) = x_i^*, i = 1,2,3,4$ as $t \to \infty$ and $x_i^* \in E_j$, where E_j are the equilibrium sets.

Remark 1

The pollution of the species will be uniformly stable for $\theta_i < 0, i = 1,2,3,4$ and exhibit oscillatory behaviors if $\theta_i > 0, i = 1,2,3,4$. This means that the population exhibits persistent behaviors for some time and eventually goes to extinction at some points. The species must have diverse means for survival; hence, prey must reproduce at the rate greater than rate of predation by the species at the higher trophic level.

4.4 Numerical Simulation

Let us consider the numerical simulation of the model in the equation (1) with parameters substituted into the equation as follows:

$$\left.\begin{aligned} \frac{\partial X_1(x,t)}{\partial t} &= 0.5X_1(x,t) - 0.25X_2(x,t) - 0.14X_3(x,t) - 0.025X_4(x,t) - \frac{1}{20}\frac{\partial^2 X_1(x,t)}{\partial x^2} - 1.16\times 10^{-5}x \\ \frac{\partial X_2(x,t)}{\partial t} &= -0.25X_1(x,t) + 0.5X_2(x,t) - 0.34X_3(x,t) - 0.25X_4(x,t) - \frac{\partial^2 X_2(x,t)}{\partial x^2} - 1.16\times 10^{-5}x \\ \frac{\partial X_3(x,t)}{\partial t} &= 0.5X_1(x,t) + 0.25X_2(x,t) + 0.24X_3(x,t) - 0.35X_4(x,t) - \frac{1}{40}\frac{\partial^2 X_3(x,t)}{\partial x^2} - 1.16\times 10^{-5}x \\ \frac{\partial X_4(x,t)}{\partial t} &= 0.25X_1(x,t) + 0.05X_2(x,t) + 0.05X_3(x,t) + 0.24X_4(x,t) - \frac{1}{30}\frac{\partial^2 X_3(x,t)}{\partial x^2} - 1.16\times 10^{-5}x \end{aligned}\right\} \tag{14a}$$

Subject to the Initial-boundary conditions:

$$IBC = \left.\begin{array}{c} X_1(0,t) = 0, X_1(10,t) = 0, X_1(x,0) = e^{1.2x} \\ X_2(0,t) = 1, X_2(10,t) = 0, X_2(x,0) = 1 - x \\ X_3(0,t) = 1, X_3(10,t) = 0, X_3(x,0) = x \\ X_4(0,t) = 0, X_4(10,t) = 0, X_4(x,0) = x \end{array}\right\} \quad (14b)$$

Here the investigation is on the population of the species in the polluted environment. The simulation is carried out from the surface to 10 meters below the ocean.

In Figure 9, the population of phytoplankton, a few meters down the ocean is very small because of pollution at the surface of the water. However, as the distance increases down the ocean, phytoplankton attains population peak of 1×10^7 at 10 meters from the surface of water and population could reach 1×10^8. For zooplankton, the population is cyclic and oscillatory because of persistence effect and this specie must increase its population to survive the predation effect by fishes and birds and the depletion of oxygen.

The population of fishes show two kinds of behavior, the linear growth at the surface at the range of $10^{-5} \le x \le 5 \times 10^{-5}$ fluctuating within $10^{-5} \le x \le 5 \times 10^{-2}$ and then attains transient value of 10^3. The second behavior is linear in the range $10^{-1} \le x \le 10^{-2}$ and attains the peak value at 10^6.

From Figure 9, we extract the following information and put into Table 1 below:

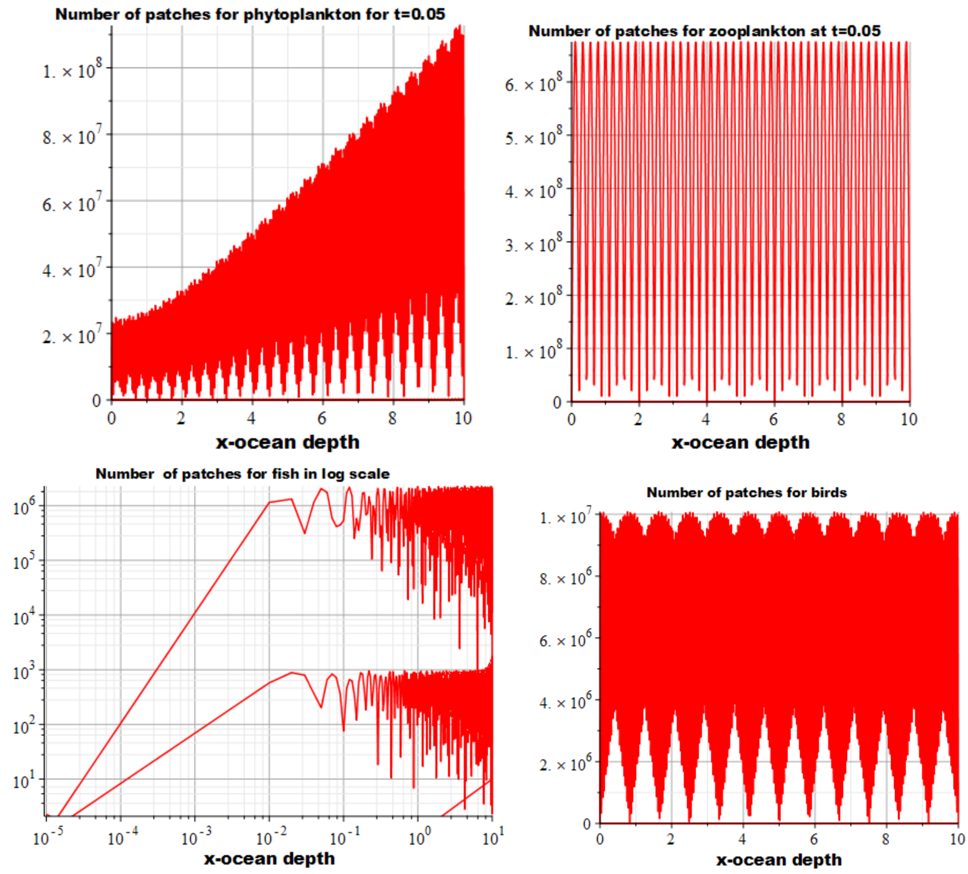

Figure 9. Numerical simulation of equations (13-14).

Table 1. Estimated extreme population of species and carriage capacity for the species

Specie	Maximum population	Minimum population	Simulated Carriage Capacity
Phytoplankton	1.7×10^7	2.0×10^6	2.0×10^8
Zooplankton	1.7×10^7	2.0×10^7	7.0×10^7
Fish	1.0×10^6	2.0×10^1	5.0×10^6
Birds	3.5×10^6	2.5×10^7	1.7×10^7

4.5 Species Population ratio

From graphs below, we found maximum (minimum) population ratios of the Species are displayed in Table 2.

Table 2 shows that the maxima population ratio of the species as obtained from Figures 10-12.

In the second figure in Figure 10, the ratio of pollution of fish to bird attains a maximum value of 5887.91 at x=8.22 meters from the surface of the ocean. In Figure 11, the ratio of pollution of zooplankton to phytoplankton attains a maximum value of 7.2833 at $x = 1.73$ meters from the surface of the ocean. In view of this small ratio, the species is most endangered because of heavy pollution by the chemical at the surface.

In Figure 12, the ratio of population of bird to fish attains a maximum value of 6242.88 at $x = 4.078$ meters from the surface of the ocean. The fish flourished at this distance, 6243 fishes to one bird. We can simply infer that at this point in the niche, the environment is enriched with nutrients, oxygen and supports coexistence of both fishes and birds. It is worthy of note that at several points down the ocean within 10 meters range, the ratio of population of fish to bird is very small. For example, at x=0.3 meter, the ratio tends to zero which means the pollution of fish is persistent and eventually tends to go into extinction. Alternatively, the population of birds is extremely large which, in real life context, is unlikely at that distance. Another reason for the smallness of the ratio is that the toxic substance from pollutants killed several fishes and the remnants were being preyed upon by the birds in spite of the pollution.

For pollution of phytoplankton and zooplankton, using population ratio, we can characterize the behaviors of the species at various spatial scales in similar way as done above.

Table 2. Species Maximum (Minimum) Population ratio

Species	Phytoplankton	Zooplankton	Fish	Bird
Phytoplankton	1.00	7.2833 (1.730)	6242.88 (4.078)	19.708 (0.991)
Zooplankton	135826.3 (8.006)	1.00	4275.05 (9.127)	4087.65 (2.039)
Fish	7.215 (1.642)	77893.63 (9.668)	1.00	5873.91 (6.327)
Bird	6242.88 (4.078)	6313.713 (9.127)	367.80 (8.22)	1.00

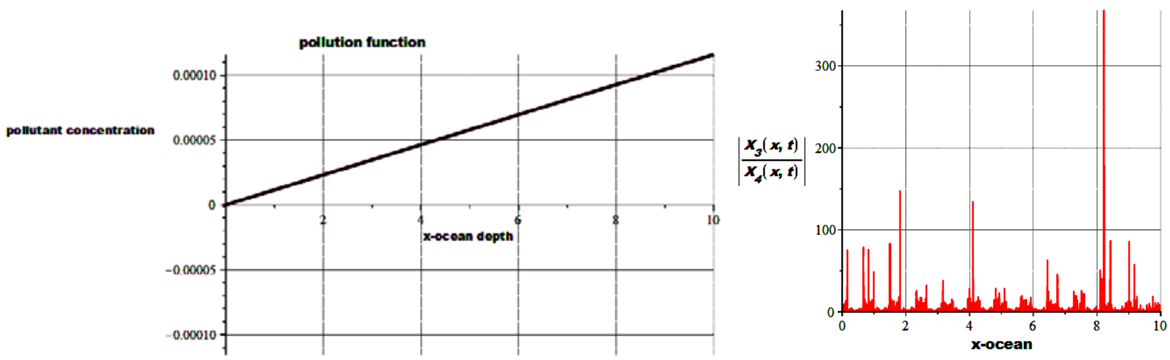

Figure 10. The First plot is on pollution function and the second on fish to bird population ratio.

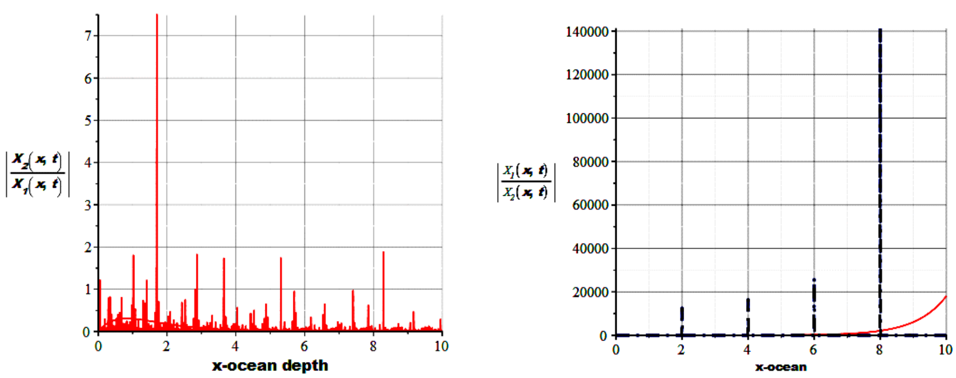

Figure 11. First plot is on zooplankton to phytoplankton population ratio and the second plot on Phytoplankton to zooplankton population ratio.

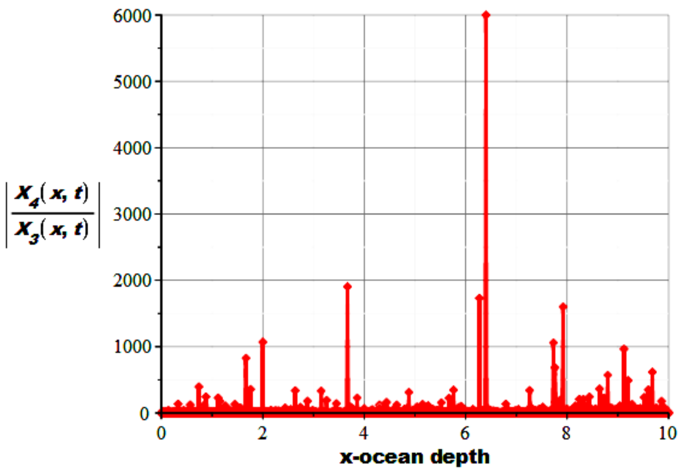

Figure 12. Plot on fish to bird population ratio.

5. Conclusion

We considered coupled ecosystem and hydrodynamic model in the form of prey-predator models for phytoplankton, zooplankton, bird and fish in a polluted environment. The habitats are destroyed by pollution; hence, pollution has a negative effect on marine ecosystems. Species must diversely survive effects of the pollution. We can infer from our simulation results on species population ratio, that the species exhibit persistence and coexistence phenomena down the ocean depth and tend to flourish in patches that provide nutrients, oxygen and scours from pollution effect.

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