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Optimal Control Analysis of COVID-19 Transmission Model with Physical Distance and Treatment

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Abstract

The transmission dynamics with optimal control of the novel COVID-19 pandemic is formulated and analyzed through a deterministic model using different human compartments. This research work investigated the effect of different control strategies in the form of physical distance and treatment. The basic reproduction number, $\mathcal{R}_*$ is calculated and used to perform sensitivity analysis to identify the most influential parameters in disease transmission. Optimal control theory and Pontryagin's maximum principle are used to optimize the model and obtain important optimality conditions to reduce disease burden. Optimal control analysis and numerical simulations reveal that the combined implementation of physical distance and treatment as interventions is more effective than other single control strategies discussed in this study and which yields a good result in reducing infection.

Keywords

COVID-19, Basic Reproduction number, Sensitivity Analysis, Optimal Control, Pontryagin's Maximum Principle

1. Introduction

Coronavirus disease 2019 (COVID-19) is a contagious disease caused by severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2), which is a new strain of coronavirus and was announced on March 11, 2020, a pandemic by the World Health Organization (WHO) [1]. Coronaviruses are a group of enveloped viruses with a single-stranded positive-sense RNA and crown-like viral particles from which the virus named as coronavirus [2]. The infection can be spread from an infected individual to a non-infectious person through the nose, mouth and eyes through droplets when someone sneezes or coughs [3]. Like the flu or cold, it has symptoms such as cough, fever and shortness of breath. It has an incubation period of 2-14 days. Nearly 80% of Covid patients have mild symptoms or sometimes are asymptomatic and recover from the disease within 2 weeks without receiving treatment. However, in people with chronic diseases and the elderly, severity of the disease and a high death rate are observed. The virus was detected in Wuhan City, Hubei Province, China in December 2019 and spread worldwide within 3 months due to its high infectivity [4].To lessen the spread of infection, different countries are adopting different strategies based on non-pharmacological measures such as regular hand washing with water and soap, wearing surgical masks, using hand sanitizers, and social distancing in the form of home quarantine, lockdown or isolation because there is no approved drug to abolish the virus; moreover, the treatment is helpful [5].

Mathematical models are the most efficient and reliable tools to formulate control measures to suppress and mitigate the impact of contagious diseases, epidemics and pandemics such as SARS, MERS and Ebola [6] and to determine the utility of the interventions. To study the dynamics of COVID-19, several mathematical models [7]-[13] have been developed.

The dynamics of transmission of COVID-19 has been analyzed in [14] with optimal control and cost-effectiveness. The

COVID-19 outbreak through a non-singular derivative is introduced in [15]. The fractional epidemic model in the Caputo sense is introduced with isolation, quarantine and environmental effects in [16]. In [17], an optimal control analysis of COVID-19 outbreak in United States is presented. The effects of pharmaceutical as well as non-pharmaceutical interventions on COVID-19 are discussed in [18]. In [19], an optimal control analysis of the COVID model is performed based on non-pharmaceutical interventions. The transmission dynamics of COVID-19 with classical approach including environment, vaccination and social distance as interventions are analyzed in [20]-[22]. A SEIARD epidemic model for Covid-19 in Mexico and Rohingya refugee camp in Bangladesh are discussed in [23, 31] with mathematical analysis and prognosis at the state level.

The main objective of the study is to enhance the above studies by formulating a new compartmental mathematical model including optimal control analysis. In this study, we first develop a SEIHR model without applying control. A sensitivity analysis is performed on the basic reproduction number $\mathcal{R}_*$ to identify the most dominant parameter of the proposed model. Based on this analysis, we incorporate two time-dependent optimal controls $w_1(t)$ for physical distance and $w_2(t)$ for treatment improvement to lessen treatment costs and number of infectious populations.

The rest of the work is organized as follows: the model conception without control and parameter details of the model are given in Section 2; a sensitivity analysis of the model is performed and existence of the solutions is discussed in Section 3 including in Appendix A and; the proposed model is transformed into an optimal control problem by including two optimal controls in Section 4, and this Section 5 is dedicated to the analysis of the optimal control problem; the analytical results are supported by numerical simulations and concluding observations are given in Section 6.

2. Model Formulation

We formulate a deterministic model for Covid-19transmission and the total human population, *N(t)* is divided into five subpopulations (Susceptible *(S(t))*, Exposed *(E(t))*, Infected *(I(t))*, Hospitalized *(H(t))*, Recovered *(R(t))*), in accordance with disease status. Susceptible human class consists of those individuals who can have contact with the infection at any time. Those who has exposed to the virus having no clinical symptoms, are kept in the exposed class for incubation period of the virus. After testing, those who are tested positive, that is the ratio of exposed (p) are moved to infected class (*I(t))*at a rate ξ_1, while the others who are tested negative are returned at a rate ξ_2 to susceptible class *(S(t))*. Infected individuals who face severe health problems and need to hospitalized for ventilation or other treatment are transferred to the hospitalized class *(H(t))*at a rate γ. After recovery from the disease, individuals from *I(t)* and *H(t)* are moved to *R(t)*, the recovered class.

In different stages of infection, considering the time dependent changes, we formulate the following mathematical model:

$$\begin{aligned}
\frac{dS}{dt} &= \Lambda + (1-p)\xi_2 E - \beta(I+H)S - \mu S \\
\frac{dE}{dt} &= \beta(I+H)S - (1-p)\xi_2 E - p\xi_1 E - \mu E \\
\frac{dI}{dt} &= p\xi_1 E - (\gamma + \mu + \tau_1)I \\
\frac{dH}{dt} &= \gamma I - (\mu + \mu_1 + \tau_2)H \\
\frac{dR}{dt} &= \tau_1 I + \tau_2 H - \mu R
\end{aligned} \tag{1}$$

and the initial conditions are, $S_0 \geq 0$, $E_0 \geq 0, I_0 \geq 0, H_0 \geq 0$ and $R_0 \geq 0$.

In the model (1), Λ represents the recruitment rate of the population, μ is the natural death rate for all classes and μ_1 is the disease induced death rate. We consider, the infected and hospitalized individuals are accountable for disease transference and β is effective contact rate of these individuals with susceptible. On the other hand, τ_1 and τ_2 are recovery rate from infection of the class *I(t)* and *H(t)*.

The corresponding values of the different parameters of the model (1) is given in Table 1.

Table 1. Description and values of the model parameters

Parameter	Description	Value
Λ	Recruitment rate	0.0015875 day^{-1}[10]
μ	Natural death rate	0.000034 day^{-1}[11]
μ_1	Disease induced death rate	0.0068 day^{-1}[11]
ξ_1	Transmission rate from class *E(t)* to *I(t)*	0.1429 day^{-1}[11]
ξ_2	Transmission rate from class *I(t)* to *S(t)*	0.07143 day^{-1}[11]
τ_1	Recovery rate of infected individuals	0.03521 day^{-1}[11]
τ_2	Recovery rate of hospitalized individuals	0.04255 day^{-1}[11]
β	Effective contact rate	0.25$person^{-1}day^{-1}$[11]
p	Proportion of exposed individuals	0.2 [assumed]
γ	Transmission rate from class *I(t)* to *H(t)*	0.13266 day^{-1}[11]

3. Model Analysis

The positivity and boundedness of solutions of model (1) are presented in **Appendix A** and basic reproduction number is calculated there. For simplicity, we have called the reproduction number is defined here.

$$\mathcal{R}_* = \frac{p\beta\Lambda\xi_1(\mu+\mu_1+\gamma+\tau_2)}{\mu(\mu+p\xi_1+(1-p)\xi_2)(\mu+\gamma+\tau_1)(\mu+\mu_1+\tau_2)}$$

3.1 Sensitivity Analysis

The sensitivity analysis of $\mathcal{R}_*$ in relation to the model parameters plays an important role in this research work. Sensitivity analysis represents a qualitative measure of dominant parameters and gives a quantitative assessment of the influence of each parameter on the output of the model, while the other parameters are kept constant and allow us to determine the significant parameters that play an important role in disease transmission as well as disease control. Using the formula developed in [24, 30], a sensitivity analysis is performed on $\mathcal{R}_*$ and the standard sensitivity index of $\mathcal{R}_*$ is given by:

$$\Upsilon_\kappa^{\mathcal{R}_*} = \frac{\kappa}{\mathcal{R}_*}\frac{\partial \mathcal{R}_*}{\partial \kappa}$$

where, κ refers to the parameters of the model. By using the parameter values from Table 1, the partial differentiation of $\mathcal{R}_*$ regarding the parameters of the model is evaluated and the sensitivity indices are provided in Table 2, which gives information about the influence of parameters on the disease transference and domination.

Table 2. Sensitivity indices of $\mathcal{R}_*$ to the parameters of the model (1)

Parameter	Value	Sensitivity index
μ_1	0.0068	-0.100343
ξ_1	0.1429	+0.904781
ξ_2	0.07143	-0.904328
τ_1	0.03521	-0.209703
τ_2	0.04255	-0.62788
β	0.25	1
γ	0.13266	-0.0613694

The parameters with positive indices mean that increasing these parameters increases $\mathcal{R}_*$, and for parameters with negative indices there is an inverse relationship. From Table 2, the positive indexed parameters are β and ξ_1, which means that increasing (or decreasing) these parameters increases (or decreases) the value of $\mathcal{R}_*$. Further, the negative indexed parameters, $\mu_1, \xi_2, \tau_1, \tau_2$ and γ, indicate that increasing (or decreasing) these parameters will decrease (or increase, respectively) the value of $\boldsymbol{\mathcal{R}}_*$. The introduction of control variables interferes with the highly sensitive parameters. But some highly sensitive parameters like human recruitment rate (Λ), natural mortality rate (μ)etc. cannot be intervened because they are beyond human control.

4. Optimal Control Analysis

A new optimal control problem is formulated in this section to mitigate the spread of COVID-19 by introducing time dependent controls,$w_1(t)$ and $w_2(t)$ in the model (1). The control $w_1(t)$ is used for physical distance to reduce the efficacious contacts of infected and hospitalized individuals with susceptible individuals. Physical distancing is the practice of wearing mask and keeping distance with others at least 3 feet or we can say avoid crowds. The control $w_2(t)$ is used for the enhanced treatment of hospitalized individuals. The optimal control problem is given as follows

$$\begin{aligned}
\frac{dS}{dt} &= \Lambda + (1-p)\xi_2 E - (1-w_1(t))\beta(I+H)S - \mu S \\
\frac{dE}{dt} &= (1-w_1(t))\beta(I+H)S - (1-p)\xi_2 E - p\xi_1 E - \mu E \\
\frac{dI}{dt} &= p\xi_1 E - (\gamma + \mu + \tau_1)I \\
\frac{dH}{dt} &= \gamma I - (\mu + \mu_1 + (1+w_2(t))\tau_2)H \\
\frac{dR}{dt} &= \tau_1 I + (1+w_2(t))\tau_2 H - \mu R
\end{aligned} \tag{2}$$

and the nonnegative initial conditions are, $S_0 \geq 0$, $E_0 \geq 0, I_0 \geq 0, H_0 \geq 0$ and $R_0 \geq 0$.

For reducing the transmission of COVID-19 infection, the following objective function needs to be minimized.

$$J(w_1, w_2) = \int_0^{t_f} [a_1 E + a_2 I + a_3 H + \frac{1}{2}(a_4 {w_1}^2 + a_5 {w_2}^2)]dt \tag{3}$$

where, the weight constants $a_1, a_2$and a_3 are associated with infectious population and a_4, a_5 are associated with cost of interventions and t_f represents the final time. Here, all control efforts namely $w_1(t)$ and $w_2(t)$ are considered to be bounded as well as Lebesgue measurable on the interval $[0, t_f]$. Here, we consider quadratic objective functional because of nonlinear intervention among the population [8-9]. If $w_1(t) = w_2(t) = 1$, then 100% control effort is applied in reduction of effective contact and enhanced treatment respectively and if $w_1(t) = w_2(t) = 0$, then there is no control effort exist to apply. The main goal is to find optimal control variables ${w_1}^*(t)$ and ${w_2}^*(t)$for reduction of effective contact and enhanced treatment respectively, such that

$$J({w_1}^*, {w_2}^*) = \min_{\mathcal{D}} J(w_1, w_2) \tag{4}$$

and the control effort set, $\mathcal{D}$ is defined by,

$$\mathcal{D} = \{(w_1, w_2): [0, t_f] \to [0,1], (w_1, w_2) \textit{ is a Lebesgue measurable}\}$$

The Lagrangian $(\mathcal{L})$ and Hamiltonian $(\mathcal{H})$ correspond to the optimal control problem is defined as

$$\mathcal{L} = a_1 E + a_2 I + a_3 H + \frac{1}{2}(a_4 {w_1}^2 + a_5 {w_2}^2) \tag{5}$$

and

$$\begin{aligned}
\mathcal{H} = {} & a_1 E + a_2 I + a_3 H + \frac{1}{2}(a_4 {w_1}^2 + a_5 {w_2}^2) \\
& + \psi_S[\Lambda + (1-p)\xi_2 E - (1-w_1(t))\beta(I+H)S - \mu S] \\
& + \psi_E[(1-w_1(t))\beta(I+H)S - (1-p)\xi_2 E - p\xi_1 E - \mu E] \\
& + \psi_I[p\xi_1 E - (\gamma + \mu + \tau_1)I] + \psi_H[\gamma I - (\mu + \mu_1 + (1+w_2(t))\tau_2)H] \\
& + \psi_R[\tau_1 I + (1+w_2(t))\tau_2 H - \mu R]
\end{aligned} \tag{6}$$

where, $\psi_S, \psi_E, \psi_I, \psi_H$ and ψ_R are adjoint variables.

To derive the solution of the COVID-19 optimal control problem (2), we use Pontryagin's maximum principle [25]. Now, using equation (6), we sate the following theorem.

Theorem 1. The solutions S^*, E^*, I^*, H^* and R^* of the optimal control system (2) and optimal controls ${w_1}^*, {w_2}^*$ that minimizes the objective functional $J(w_1, w_2)$ over $\mathcal{D}$. Then, there exists adjoint variables $\psi_S, \psi_E, \psi_I, \psi_H$ and ψ_R along with transversality conditions $\psi_S(t_f) = \psi_E(t_f) = \psi_I(t_f) = \psi_H(t_f) = \psi_R(t_f) = 0$ such that

$$\frac{d\psi_S}{dt} = \psi_S\left(\left(1 - w_1(t)\right)\beta(I^* + H^*) + \mu\right) - \psi_E\left(\left(1 - w_1(t)\right)\beta(I^* + H^*)\right)$$

$$\frac{d\psi_E}{dt} = -a_1 - \psi_S(1-p)\xi_2 + \psi_E\left((1-p)\xi_2 + p\xi_1 + \mu\right) - \psi_I p\xi_1$$

$$\frac{d\psi_I}{dt} = -a_2 + \psi_S\left(1 - w_1(t)\right)\beta S^* - \psi_E\left(1 - w_1(t)\right)\beta S^* + \psi_I(\gamma + \mu + \tau_1) - \gamma\psi_H - \tau_1\psi_R \tag{7}$$

$$\frac{d\psi_H}{dt} = -a_3 + \psi_S\left(1 - w_1(t)\right)\beta S^* - \psi_E\left(1 - w_1(t)\right)\beta S^* + \psi_H(\mu + \mu_1 + (1 + w_2(t))\tau_2) - \psi_R(1 + w_2(t))\tau_2$$

$$\frac{d\psi_R}{dt} = \mu\psi_R$$

and the optimality conditions are given by,

$${w_1}^*(t) = max\left\{0, \min\left\{1, \frac{(\psi_E - \psi_S)\beta(I^* + H^*)}{a_4}\right\}\right\} \tag{8}$$

$${w_2}^*(t) = max\left\{0, \min\left\{1, \frac{(\psi_H - \psi_R)\tau_2 H^*}{a_5}\right\}\right\} \tag{9}$$

Proof. We can easily prove the convexity of the integrand of J regarding to w_1 and w_2 and boundedness of the solutions of the system (2). We can also prove that regarding to the state variables, the system has Lipschitz property. With the help of these properties and using corollary 4.1 of [26], the existence of the optimal control is proved. Using the Pontryagin's Maximum Principle, we get,

$$\frac{d\psi_S}{dt} = -\frac{\partial\mathcal{H}}{\partial S}, \psi_S\left(t_f\right) = 0,$$
$$\frac{d\psi_E}{dt} = -\frac{\partial\mathcal{H}}{\partial E}, \psi_E\left(t_f\right) = 0,$$
$$\frac{d\psi_I}{dt} = -\frac{\partial\mathcal{H}}{\partial I}, \psi_I\left(t_f\right) = 0, \tag{10}$$
$$\frac{d\psi_H}{dt} = -\frac{\partial\mathcal{H}}{\partial H}, \psi_H\left(t_f\right) = 0,$$
$$\frac{d\psi_R}{dt} = -\frac{\partial\mathcal{H}}{\partial R}, \psi_R\left(t_f\right) = 0,$$

evaluating at the optimal control derives the adjoint system. The control set, $\mathcal{D}$ on which the optimality conditions are defined,

$$\mathcal{D} = \left\{\left(w_1(t), w_2(t)\right) | 0 \leq (w_1(t), w_2(t) \leq 1\right\} \tag{11}$$

as

$$\frac{\partial\mathcal{H}}{\partial w_1} = a_4 w_1 + \psi_S\beta(I^* + H^*)S^* - \psi_E\beta(I^* + H^*)S^* = 0$$

$$\frac{\partial\mathcal{H}}{\partial w_2} = a_5 w_2 - \tau_2 H^*\psi_H + \tau_2 H^*\psi_R = 0 \tag{12}$$

Solving (12) we get,

$${w_1}^*(t) = \frac{(\psi_E - \psi_S)\beta(I^* + H^*)}{a_4}$$

$$w_2^*(t) = \frac{(\psi_H - \psi_R)\tau_2 H^*}{a_5}$$

and the compact form of the optimal control efforts are given by equations (8) and (9).

Therefore, the equations (2), (7) with optimality conditions (8), (9) and the nonnegative initial conditions and the transversality conditions, $\psi_S(t_f) = \psi_E(t_f) = \psi_I(t_f) = \psi_H(t_f) = \psi_R(t_f) = 0$ form the optimality system.

Since the state variables and the adjoint functions are bounded and the state as well as adjoint system has Lipschitz structure regarding to the corresponding variables, we get the unique solutions of the optimality system for the interval [0, t_f]. Therefore, we can say that, the boundedness and uniqueness of the solutions to the discussed optimality system exists for $t \in [0, t_f]$.

5. Numerical Simulation and Discussion

Numerical simulations of the proposed COVID-19 model are performed, in this section to investigate the effect of control strategies on transmission of disease and control costs. Values of the parameters of the COVID-19 model are given in Table 1. The initial conditions of the state variables are $S_0 = 0.60$, $E_0 = 0.06, I_0 = 0.05, H_0 = 0.02$ and $R_0 = 0.15$ and the weight constants $a_1 = 20, a_2 = 20, a_3 = 20, a_4 = 0.4$ and $a_5 = 0.2$. By considering three different cases based on two controls,w_1(social distancing and wearing mask to reduce effective contacts) and w_2 (treatment of hospitalized infected individuals), we simulate the control model to investigate the effectiveness of each control strategy that comprises of

1) implementation of single control strategy based on $w_1 \neq 0$, in this case $w_2 = 0$,
2) implementation of single control strategy based on $w_2 \neq 0$, where $w_1 = 0$ and
3) implementation of two controls simultaneously, and in this case $w_1 \neq 0$ and $w_2 \neq 0$.

Figure (1)-(4) shows the changes in the density of exposed, infected, hospitalized and recovered subpopulations respectively without applying any control and with control based on considering three different scenarios and Figure (5) shows the different control profiles. Figure 1 depicts that in the first case to reduce effective contact using physical distancing that is $w_1 \neq 0$ and$w_2 = 0$, and in the third case that is when both controls applied simultaneously ($w_1 \neq 0$ and $w_2 \neq 0$), the number of exposed individuals decrease rapidly compared to the second case, whenever $w_1 = 0$ and without control.It means making distance between infected and hospitalized individuals with susceptible, minimize the possibility of getting infected of susceptible individuals. A notable decrease is witnessed in infected subpopulation, same as exposed subpopulation for first ($w_1 \neq 0$ and $w_2 = 0$) and third strategy ($w_1 \neq 0$ and $w_2 \neq 0$) and slight decrease is observed for second ($w_1 = 0$ and $w_2 \neq 0$) strategy. The details are shown in Figure 2. For both exposed and infected subpopulation only enhanced treatment of hospitalized individuals ($w_1 = 0$ and $w_2 \neq 0$) does not have noticeable impact on reduced infection rate. It means that infection will persist in the community. So, for both exposed and infected subpopulation, strategy (i) and (iii) are effective and helpful for reducing the infection from the community.

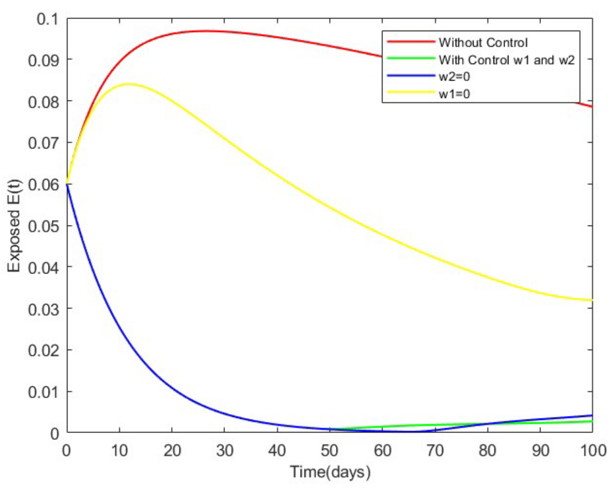

Figure 1. Density of Exposed subpopulation (E(t)) without control and with control.

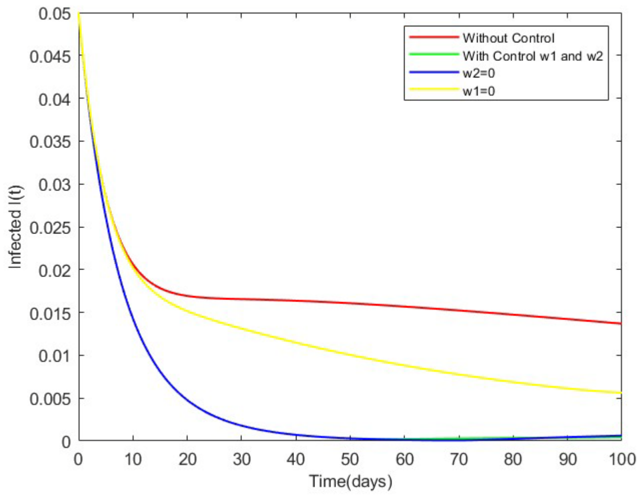

Figure 2. Density of Infected subpopulation (I(t)) without control and with control.

The biological impact of all these three strategies for hospitalized individuals are shown in Figure 3.For the third strategy where, physical distancing control and treatment control are taken into account ($w_1 \neq 0$ and $w_2 \neq 0$), the number of hospitalized individuals is decreasing significantly in compare to the first strategy where only physical distancing control ($w_1 \neq 0$ and $w_2 = 0$) and second strategy where only treatment control ($w_1 = 0$ and $w_2 \neq 0$) are taken into account. If physical distancing in form of isolation and wearing mask are not implemented strictly then only giving treatment to hospitalized individuals can't reduce the infection from community. So, for hospitalized individuals third strategy is more effective than other strategies.

Figure 4 shows the change of density of recovered subpopulation without and with different control strategies. Since for third strategy (in which both controls are implemented simultaneously) exposed, infected and hospitalized individuals decrease rapidly than any other strategy, the recovered individuals also decrease notably for third strategy. There is an increase is seen in the recovered subpopulation without any control and while applying second control strategy, this is because of existence of high infection in community.

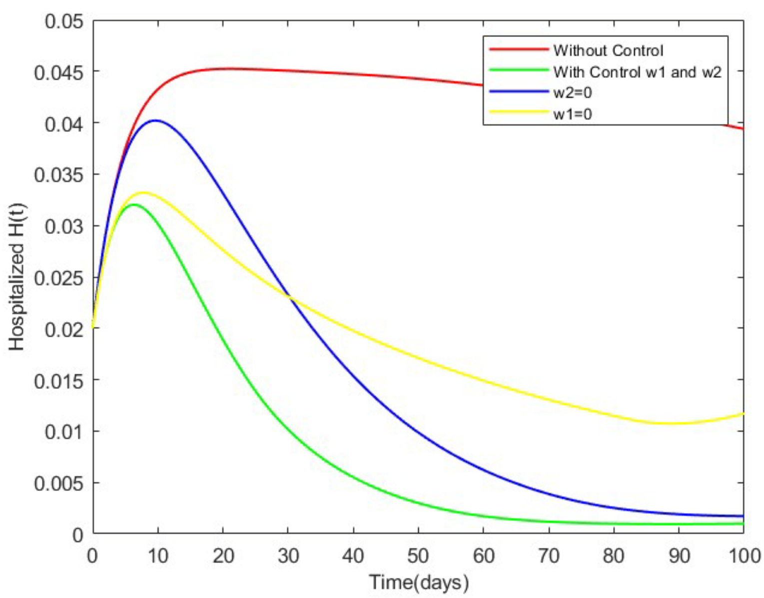

Figure 3. Density of Hospitalized subpopulation (H(t)) without control and with control.

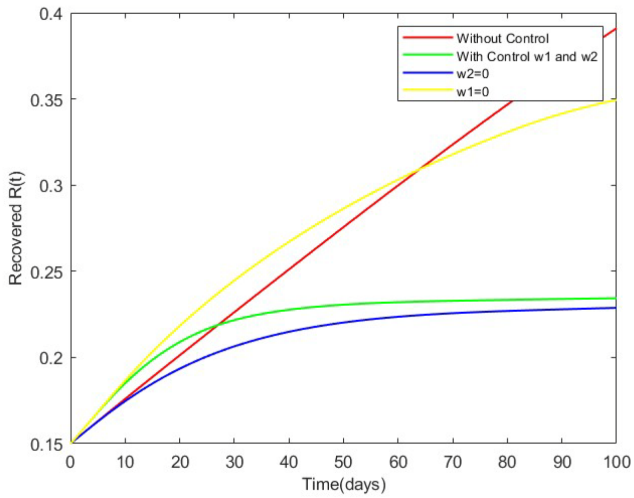

Figure 4. Density of Recovered subpopulation (R(t)) without control and with control.

Figure 5(a) shows the control profiles of implementation of both (w_1, w_2) controls simultaneously, where the combined implementation of controls takes about 100% input of physical distancing (w_1) for 3-4 weeks and 99% input of treatment (w_2) for 6-7 weeks before reach their lower bound, that produces better result than other two strategies.

For single control strategy (i), Figure 5(b) shows that when physical distancing is considered, it takes about 100% input of physical distancing for 9-10 weeks and Figure 5(c) shows that, while treatment is considered as control, it takes about 100% input of treatment for 11-12 weeks before reach their lower bound.

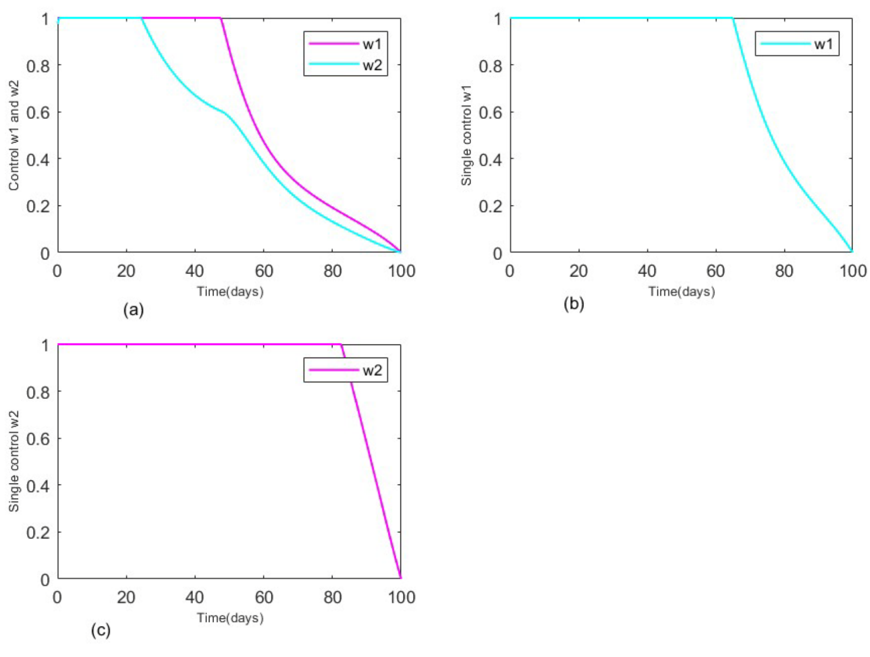

Figure 5. Different control efforts (a) two control strategies, $\mathbf{w_1}$ (physical distancing to reduce effective contacts) and $\mathbf{w_2}$(treatment of hospitalized infected individuals) (b)single control strategy, $\mathbf{w_1 \neq 0, w_2 = 0}$ (c) single control strategy, $\mathbf{w_2 \neq 0, w_1 = 0}$.

Figure 6 shows the effect of the control weights a_4=100 and $a_5 = 80$ on the exposed, infected, hospitalized and recovered individuals and in this case, we considered without any control and combined control strategy ($w_1 \neq 0$ and $w_2 \neq 0$). When the weight of the corresponding control cost is increased, a slight decrease is seen in the number of exposed, infected and hospitalized individuals compared to without any control scenario and no change is observed in the recovered class due to the very small initial implemented controls whose values are $w_1 = 0.063$ and $w_2 = 0.02$.

From Figure 7, we observed different scenario for control weights a_4=0.04 and $a_5 = 0.2$. The number of exposed, infected and hospitalized individuals decrease rapidly and there is increase is shown in the recovered class. In this case the implemented initial controls are$w_1 = 1$ and $w_2 = 0.99$. Considering all these observations, we can say that utilizing the combined control strategies with low control cost weights that is high control efforts is more successful as compared to single control strategies and helpful to lessen the disease burden from the community.

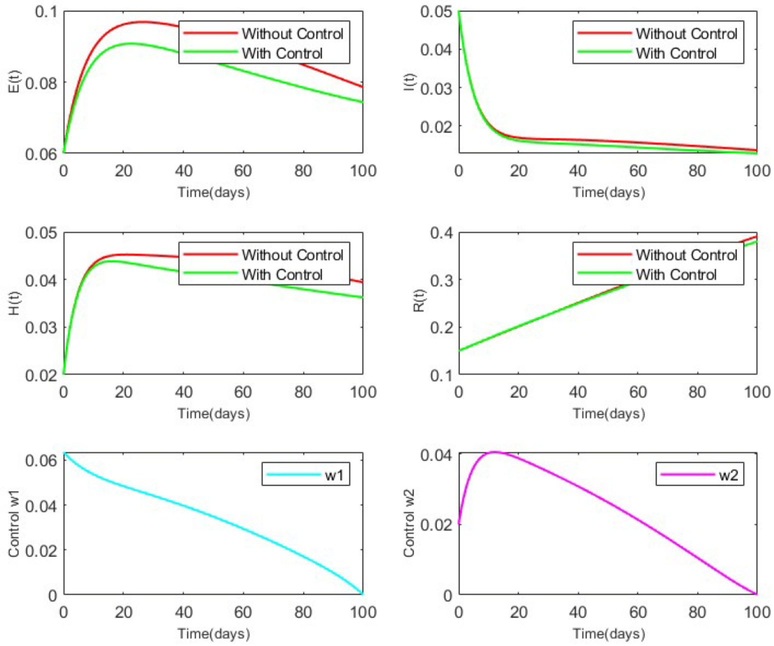

Figure 6. Effect of different control weights ($a_4 = 100, a_5 = 80$).

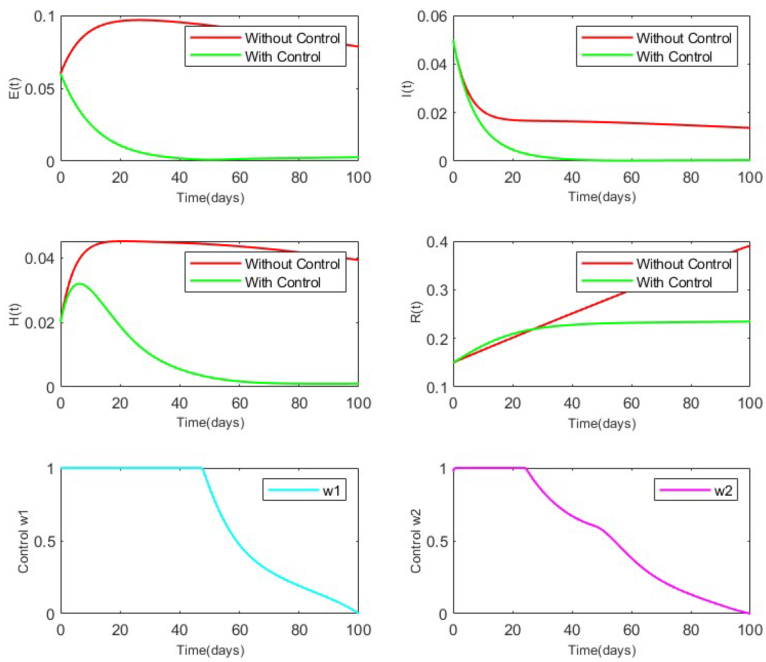

Figure 7. Effect of different control weights ($a_4 = 0.4, a_5 = 0.2$).

6. Conclusion

In this study, a COVID-19 model is formulated and analyzed. Applying some appropriate control measures and with the help of optimal control theory, the model is optimized. The sensitivity analysis is performed and depending on the most important parameters, physical distance and treatment are taken into account as the interventions. Hamiltonian and Lagrangian are formulated to explore the existence of the optimal control and Pontryagin's maximum principle is used to find the necessary optimality conditions. To curtail the infection three different control strategies are considered and we observed that the combined implementation of interventions is more effective to curtail the infection and reduce the disease burden. From this study we also observed that the weight of cost of interventions has reversal relation with the control efforts. If the corresponding weight of cost of interventions, physical distance and treatment, are lower (or higher) then the implemented levels of interventions are higher (or lower respectively.).

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Appendix A

Positivity and Boundedness of solutions

Theorem A.1. The closed set $\varphi(t) = \left\{\left(S(t), E(t), I(t), H(t), R(t)\right) \in \mathbb{R}_+^5 : N(t) \leq \frac{\Lambda}{\mu}\right\}$ with initial conditions $\varphi(0) \geq 0$ is positively-invariant and attracting with respect to the model (1).

Proof. Adding all the equations of model (1), we get

$$\frac{dN}{dt} = \Lambda - \mu N - \mu_1 H \leq \Lambda - \mu N \tag{A1}$$

Using standard comparison theorem [27], we get

$$N(t) \leq N(0)e^{-\mu t} + \frac{\Lambda}{\mu}(1 - e^{-\mu t})$$

Therefore, $N(t) \leq \frac{\Lambda}{\mu}$ if $N(0) \leq \frac{\Lambda}{\mu}$. Hence the region φ is positively invariant. Again from (A1), $\frac{dN}{dt} < 0$, if $N(t) > \frac{\Lambda}{\mu}$. So, the solutions enter the neighborhood φ in finite time, otherwise $N(t) \to \frac{\Lambda}{\mu}$ and infected classescloser to zero. Hence the region φ is attracting as well as all solutions of the system in $\mathbb{R}_+^5$ ultimately enter the neighborhood, φ. Thus, the model (1) is mathematically and epidemiologically well-posed in the neighborhood, φ [29],

Basic Reproduction Number, $\mathcal{R}_*$

The model (1) has a Disease-Free Equilibrium (DFE), $\mathcal{E}_* = (S_*, E_*, I_*, H_*, R_*) = \left(\frac{\Lambda}{\mu}, 0, 0, 0, 0\right)$.

The basic reproduction number, $\mathcal{R}_*$ is calculated by using next generation method [28].

$$\mathcal{R}_* = \frac{p\beta\Lambda\xi_1(\mu + \mu_1 + \gamma + \tau_2)}{\mu(\mu + p\xi_1 + (1-p)\xi_2)(\mu + \gamma + \tau_1)(\mu + \mu_1 + \tau_2)}$$

where, $\mathcal{R}_* = \rho(BD^{-1})$, ρ is spectral radius and the matrices B, new infection terms and D, the transition terms are given by,

$$B = \begin{bmatrix} 0 & \beta\frac{\Lambda}{\mu} & \beta\frac{\Lambda}{\mu} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{and } D = \begin{bmatrix} \mu + p\xi_1 + (1-p)\xi_2 & 0 & 0 \\ -p\xi_1 & \mu + \gamma + \tau_1 & 0 \\ 0 & -\gamma & \mu + \mu_1 + \tau_2 \end{bmatrix}.$$