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Estimating Option Prices with Discrete Dividend Payment Using Finite Difference Method and Monte Carlo Simulation: A Comparative Study

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Abstract

Valuation of the option prices through numerical methods and real option valuation has had a significant influence on the way the traders price the financial derivatives over the past years. The Black-Scholes (BS) model is an essential model that plays an important role in pricing option prices. In this paper, the numerical solution of the Black-Scholes model for pricing European call options with discrete dividend payment using the Monte Carlo (MC) and Finite-Difference Method (FDM) has been presented. The explicit, implicit, and Crank Nicolson finite difference schemes have been used in this study. All of these approaches, including MC Simulation, are applied to the same example to assess their efficiency. The results obtained using these methods have been compared to the option values derived using the option pricing formula. Numerical results reveal that the Crank Nicolson Finite Difference Scheme (CNFDS) converges faster and offers more accurate results than the other two Finite Difference Schemes (FDSs) and the MC simulation.

Keywords

Black-Scholes model, Monte Carlo simulation, Option pricing, European options, Dividend payment, Finite difference method

1. Introduction

Several studies have been conducted, and numerous models for option valuation have been designed in considering the rising importance of the options. Among them, the BS model is one of the most widely used approaches in option price contacts. Black and Scholes developed the infamous BS equation which has been used as a crucial tool for the pricing of several derivatives nowadays.

The BS model is a stochastic differential equation that determines option pricing based on the underlying asset's price at any particular time till maturity. In the concept of Black-Scholes-Merton for option pricing [1, 2], a comprehensive review of two numerical methods is presented: FDMs studied by Brennan and Schwarz [3] and the MC approach developed by Boyle [4]. These methods support much of the infrastructure in which various improvements to the field have been developed.

In the continuous BS model, the FDM seeks to discretize the differential operators and resolve algebraic equations to find the optimal solution.The numerical solution of the BS model using FDM is reported by various authors [11, 12, 13, 14]. Hor *et al.* [9] use FDM to calculate American option prices with dividend payments. Seda Gulen proposed three combined methods for determining the BS model solution for the European put option. Fourth-order FDS for spatial discretization is combined with strong stability preserving Runge Kutta, fourth-order Runge Kutta, and therefore the One-step approach respectively for time discretization [6].

MCS is also a very important technique and has been successfully used for option pricing.The MC approach generates a probabilistic solution to option pricing models by simulating the random behavior of stock prices. A new technique is introduced by Fu and Hu for estimating option value with its sensitivity to various parameter of the model for MCS [15]. Jabbour *et al*. [8] show that option prices using MCS are statistically same as BS prices.

In recent years, a reliable hybrid FDM with a MC boundary condition has been proposed by Jeong *et al.* [5]. A comparative study between MCS, Binomial tree model, and BSM model for pricing option is performed by Bendob and Bentouir [16]. Fadugba *et al*. [7] performed a comparative study for pricing European options without dividend payment using the FDM and the MC method and shows that Crank Nicolson method provides more accurate result and converges faster than MCS method for pricing European options.

In this study, we have performed a comparative study of the performance of the approaches under consideration for pricing European call options with discrete dividend payments. These schemes are closely related but they differ significantly in terms of stability and accuracy, which are thoroughly examined here.

2. Mathematical Formulation

The stock price process for a non-dividend paying stock is

$$dS = S\mu dt + \sigma S dz \tag{1}$$

where μ is a constant drift rate, dz is the winer process, and σ is the constant volatility. By Ito's lemma, the price of the derivate, follows the following process

$$df = \left(\frac{\partial f}{\partial S}\mu S + \frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right)dt + \frac{\partial f}{\partial S}\sigma S dz \tag{2}$$

The winner process, dz,in the preceding equation can be eliminated by constructing a portfolio consisting of short one derivative and long an amount $\frac{\partial f}{\partial S}$of the underlying assets atthe same time. From Black and Scholes [2] and Merton [1], a portfolio's return must be the same as other short-term risk-free securities in order to preserve its risk-free status. As a result, the widely used BSM model is

$$\frac{\partial f}{\partial t} + rs\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 f}{\partial S^2} = rf \tag{3}$$

where r is the risk-free interest rate. A stock starting at price S with a dividend yield at rate q is equivalent to a stock starting at price Se^{-qT} with no dividend [1]. So, if a dividend yield of q is included, the differential equation (3) become

$$\frac{\partial f}{\partial t} + (r-q)s\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 f}{\partial S^2} = rf \tag{4}$$

The above equation can be solved to determine the price of a derivative for the stock price in the ranges of $0 < S < \infty$ and time $0 < t < T$, where T is the derivative's life in years. The price of the option at time T is simply its payoff. As a result, at time Step T, the boundary condition is

$$\begin{cases} \text{Max}(S(T) - k, 0) & \text{for Call option} \\ \text{Max}(K - S(T), 0) & \text{for Put option} \end{cases}$$

3. Method of Solution

3.1 Finite difference method

Equation (4) is a linear partial differential equation with variable coefficient and can be solved using FDM method (Explicit, Implicit and Crank Nicolson Scheme). The domain of the stock prices for equation (4) is discretized with constant increment ΔS from S_{min} to S_{max} as

$$S_{min} + j\Delta S \text{for} j = 0,1,2,\ldots m$$

Where S_{min} is the stock price that is reasonably low and $S_{\max}$ is the stock price that is reasonably high for the option to be significantly in or out of the money depending on the kind of option, and m is the number of partitions. The option's time is discretized into n intervals with $\Delta t = T/n$ as

$$i\Delta t, i = 0,1,2,\ldots n$$

The values of the European option at maturity time, $i = n$, is

$$f_{n,j} = \begin{cases} \text{Max}(S_{min} + j\Delta s - k, 0) & \text{for Call option} \\ \text{Max}(K - S_{min} - j\Delta S, 0) & \text{for Put option} \end{cases} \quad \text{for } j = 0,1,\dots,m$$

The call option is substantially out of the money, while the put option is significantly in the money at S_{min} boundary, and vice versa at S_{max} boundary. So, the option values at S_{min} boundary are

$$f_{i,0} = \begin{cases} \text{Max}(S_{min} - k, 0) & \text{for Call option} \\ \text{Max}(K - S_{min}, 0) & \text{for Put option} \end{cases} \text{for } i = 0,1,\dots,n$$

And the option values at S_{max} boundary are

$$f_{i,m} = \begin{cases} \text{Max}(S_{max} - k, 0) & \text{for Call option} \\ \text{Max}(K - S_{max}, 0) & \text{for Put option} \end{cases} \quad \text{for } i = 0,1,\dots,n$$

For Implicit Forward Difference Method (IFDM), forward difference formula for time t and central difference formula for s are used to discretize equation (4) and noting that $S = j\Delta S$ gives

$$\frac{f_{i+1,j} - f_{i,j}}{\Delta t} + (r-q)j\Delta S \frac{f_{i,j+1} - f_{i,j-1}}{2\Delta S} + \frac{1}{2}\sigma^2 j^2 (\Delta S)^2 \frac{f_{i,j+1} - 2f_{i,j} + f_{i,j-1}}{\Delta S^2} = rf_{i,j}$$

For $j = 1,2,\dots m-1$ and $i = 0,1,2,\dots n-1$. Rearrange and simplify to get,

$$\alpha_j f_{i,j-1} + \beta_j f_{i,j} + \gamma_j f_{i,j+1} = f_{i+1,j} \tag{5}$$

Where,

$$\alpha_j = \frac{1}{2}(r-q)j\Delta t - \frac{1}{2}\sigma^2 j^2 \Delta t$$

$$\beta_j = 1 + \sigma^2 j^2 \Delta t + r\Delta t$$

$$\gamma_j = -\frac{1}{2}(r-q)j\Delta t - \frac{1}{2}\sigma^2 j^2 \Delta t$$

For each node points along time t, starting from the point with $i = n-1$ corresponding to time $T - \Delta t$ towards backward direction, $m-1$ simultanious equation is solved for $m-1$ unknown option values and eventually the option prices are obtained at every node. This formula is represented in matrix form as

$$Af^i = f^{i+1} + B$$

where

$$\text{A}=\begin{bmatrix} \beta_1 & \gamma_1 & 0 & \cdots & 0 \\ \alpha_2 & \beta_2 & \gamma_2 & \cdots & 0 \\ 0 & \alpha_3 & \beta_3 & \gamma_3 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & \cdots & \alpha_{m-1} & \beta_{m-1} \end{bmatrix},\ f^i=\begin{bmatrix} f_{i,1} \\ f_{i,2} \\ \vdots \\ \vdots \\ f_{i,m-1} \end{bmatrix}, \text{ and B}=\begin{bmatrix} -\alpha_1 f_{i,0} \\ 0 \\ \vdots \\ 0 \\ -\gamma_{m-1} f_{i,m} \end{bmatrix}$$

In Explicit Finite Difference Method (EFDM), the values of $\frac{\partial f}{\partial S}$ and $\frac{\partial^2 f}{\partial S^2}$ at point (i,j) are assumed to be the same as at point $(i+1,j)$. One of the advantages of ERDM is that $(m-1)$ simultaneous equations have not been solved at each time steps. The difference equation is

$$\frac{f_{i+1,j} - f_{i,j}}{\Delta t} + (r-q)j\Delta S \frac{f_{i+1,j+1} - f_{i+1,j-1}}{2\Delta S} + \frac{1}{2}\sigma^2 j^2 (\Delta S)^2 \frac{f_{i+1,j+1} - 2f_{i+1,j} + f_{i+1,j-1}}{\Delta S^2} = rf_{i,j}$$

Rearranging terms to get

$$f_{i,j} = \alpha_j^* f_{i+1,j-1} + \beta_j^* f_{i+1,j} + \gamma_j^* f_{i+1,j+1} \tag{6}$$

Where,

$$\alpha_j^* = \frac{1}{1 + r\Delta t}\left(-\frac{1}{2}(r-q)j\Delta t + \frac{1}{2}\sigma^2 j^2 \Delta t\right)$$

$$\beta_j^* = \frac{1}{1 + r\Delta t}(1 - \sigma^2 j^2 \Delta t)$$

$$\gamma_j^* = \frac{1}{1 + r\Delta t}\left(\frac{1}{2}(r-q)j\Delta t + \frac{1}{2}\sigma^2 j^2 \Delta t\right)$$

From equation (6), the value of option at point (i, j) is calaulated with the help of three known values of option at point $(i+1, j-1), (i+1, j)$ and $(i+1, j+1)$.

The forward and backward difference formulae are first order accurate in time with a truncation error of order $O(\Delta t)$ but central difference formula is second order with truncation error of order $O(\Delta S^2)$ [17]. The two points forward difference approximation of $\frac{\partial f}{\partial t}$ at point (i, j) can be viewed as a three-point central difference approximation about the mid-point of $(i+1, j)$ and (i, j) using a step size of $\Delta t/2$. So, the difference quotient

$$\frac{f_{i+1,j} - f_{i,j}}{\Delta t}$$

can be considered a central difference formula for $\frac{\partial f}{\partial t}$. The central difference approximation of $\frac{\partial f}{\partial S}$ and $\frac{\partial^2 f}{\partial S^2}$ must also be evaluated at the mid point. This is achieved by averaging the formulas at points (i, j) and $(i+1, j)$. Then the equation (4) is approximated as,

$$\frac{f_{i+1,j} - f_{i,j}}{\Delta t} + (r-q)j\Delta S \frac{1}{2}\left(\frac{f_{i,j+1} - f_{i,j-1}}{2\Delta S} + \frac{f_{i+1,j+1} - f_{i+1,j-1}}{2\Delta S}\right) + \frac{1}{2}\sigma^2 j^2 (\Delta S)^2 \frac{1}{2}\left(\frac{f_{i,j+1} - 2f_{i,j} + f_{i,j-1}}{\Delta S^2} + \frac{f_{i+1,j+1} - 2f_{i+1,j} + f_{i+1,j-1}}{\Delta S^2}\right) = r\left(\frac{f_{i,j} + f_{i+1,j}}{2}\right)$$

Simplify to get,

$$-\alpha_j' f_{i,j-1} - (1+\beta_j') f_{i,j} + \gamma_j' f_{i,j+1} = \alpha_j' f_{i+1,j-1} - (1-\beta_j') f_{i+1,j} - \gamma_j' f_{i+1,j+1} \quad (7)$$

where,

$$\alpha_j' = \frac{1}{4}(r-q)j\Delta t - \frac{1}{4}\sigma^2 j^2 \Delta t$$
$$\beta_j' = \frac{1}{2}\sigma^2 j^2 \Delta t + \frac{1}{2} r\Delta t$$
$$\gamma_j' = \frac{1}{4}(r-q)j\Delta t + \frac{1}{4}\sigma^2 j^2 \Delta t$$

It is noted from the equation (7) that the option prices $f_{i,j}$ is not given directly in terms of known option prices one time step earlier but is a function of unknown option prices at adjacent position as well. This requires solving a set of system of equation at each time step. This formula can be represented as the matrix form

$$A f^i = B f^{i+1} + C \text{ for each } i = 0,1, \ldots n-1$$

Where $A = \begin{bmatrix} -(1+\beta''_1) & \gamma_1' & 0 & \cdots & 0 \\ -\alpha'_2 & -(1+\beta'_2) & \gamma'_2 & \cdots & 0 \\ 0 & -\alpha'_3 & -(1+\beta'_3) & \gamma'_3 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & \cdots & -\alpha'_{m-1} & -(1+\beta'_{m-1}) \end{bmatrix}$

$B = \begin{bmatrix} -(1-\beta'_1) & -\gamma'_1 & 0 & \cdots & 0 \\ \alpha'_2 & -(1-\beta'_2) & -\gamma'_2 & \cdots & 0 \\ 0 & \alpha'_3 & -(1-\beta'_3) & -\gamma'_3 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & \cdots & \alpha'_{m-1} & -(1-\beta'_{m-1}) \end{bmatrix}$, $f^i = \begin{bmatrix} f_{i,1} \\ f_{i,2} \\ \vdots \\ \vdots \\ f_{i,m-1} \end{bmatrix}$

and $C = \begin{bmatrix} \alpha_1' f_{i+1,0} + \alpha_1' f_{i,0} \\ 0 \\ \vdots \\ 0 \\ \gamma_{m-1}' f_{i+1,m} + \gamma_{m-1}' f_{i,m} \end{bmatrix}$

3.2 Monte Carlo simulation

The stock price behavior follows the geometric Brownian motion [1, 2]. That is

$$dS = S\mu dt + \sigma S dz \quad (8)$$

where μ is the expected risk-free return and for dividend paying stock, it is $(r-q)$, dz is the winer process, and σ is the volatility. The life of the derivative is partitioned into N subintervals of length Δt to simulate the path followed by S. The change in z, Δz, over a short period of time, Δt, is given by $\Delta z = \epsilon\sqrt{\Delta t}$, where ϵ is a random variable that follow standard normal distribution with mean 0 and standard deviation 1[10]. So, equation (8) for discrete time interval is approximated as

$$S(t+\Delta t) - S(t) = S(t)\mu\Delta t + \sigma S(t)\epsilon\sqrt{\Delta t} \quad (9)$$

where $S(t)$ is the value of the stock at time *t*.The value of the stock price at time$t+\Delta t$, $S(t+\Delta t)$, is determined from the value of $S(t)$. Similarly, the value is calculated at every time step by sampling repeatedly ϵ from $\phi(0,1)$. Then the payoff is calculated by the formula

$$\begin{cases} \text{Max}(S(T)-K,0) & \text{for Call option} \\ \text{Max}(K-S(T),0) & \text{for Put option} \end{cases}$$

The preceding procedure is repeated in order to generate a significant number of sample payoff values. The expected payoff in a risk-free condition is derived by taking the mean of these values and discounting it at the risk-free rate to provide an estimation of the option price.

3.3 Black-Scholes pricing formulas

The pricing formulas of a derivative on a stock paying a dividend yield at rate q were first derived by Merton by replacing S by Se^{-qT} [1]. The prices of a European call, *c*, and a European put, *p*, are as described in the following:

$$c = Se^{-qT}N(d_1) - Ke^{-rT}N(d_2) \quad (10)$$

$$p = Ke^{-rT}N(-d_2) - Se^{-qT}N(-d_1) \quad (11)$$

where

$$d_1 = \frac{(\ln\left(\frac{S}{K}\right) + \left(r - q + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{(\ln\left(\frac{S}{K}\right) + \left(r - q - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

4. Results and Discussions

The price of a two-month European call option on a stock is determined when the current stock price is 930, the risk-free interest rate is 8% per year, the exercise price is 900, and the volatility of the index is 20% per year. Dividend yields of 0.2% and 0.3% are expected in the first and second months, respectively [10]. MATLAB 2018 has been used to generate the computer code for the computations.

By the BS call option pricing formula (10), option price for the given problem is 51.8329567964908. The approximate results of European call option with discrete dividend by using the EFDM, IFDM and CNFDM for $\Delta t = 1.67 \times 10^{-4}$ and $\Delta S = 2.67$ are 51.8008444738828, 51.8321130479744, and 51.8328516285982, respectively. The approximate result of European call option with discretedividend by MC simulation for number of paths, $N = 25000$ is 51.8328969551723. In the case of EFDM the numerical resultis correct to the result of option pricing formula up to one decimalplace whereas it is correct up to threedecimal places for IFDM, CNFDM and MC simulation.The three-dimensional graphs for the EFDM, IFDM and CNFDM generated by MATLAB is shown in the Figure 1. The impacts of the number of grid points are now being studied, and the number of stock price grids, M, is being increased for fixed time grid points, N=1000. Since FDM generate output only at the nodal point, Newton interpolation polynomial is used to find the option prices at the desired point.

The values of the European call option with dividend payment calculated using the three FDSs are shown in Table 1. The option price converges to the option price derived by the pricing formula as the stock price grid increases for the EFDM of equation (6). The results obtained using the IFDM scheme in equation (5) for the European call option with dividend payment suggest that this scheme converges faster and performs better than the EFDM scheme. The option price gradually converges as the stock price grid increases and offers more accurate values to the call option when the stock price grid M is 200. The results of the CNFDM scheme indicate that the option price converges significantly faster than EFDM and IFDM and generates a much more appropriate result when the stock price grid M is 100.

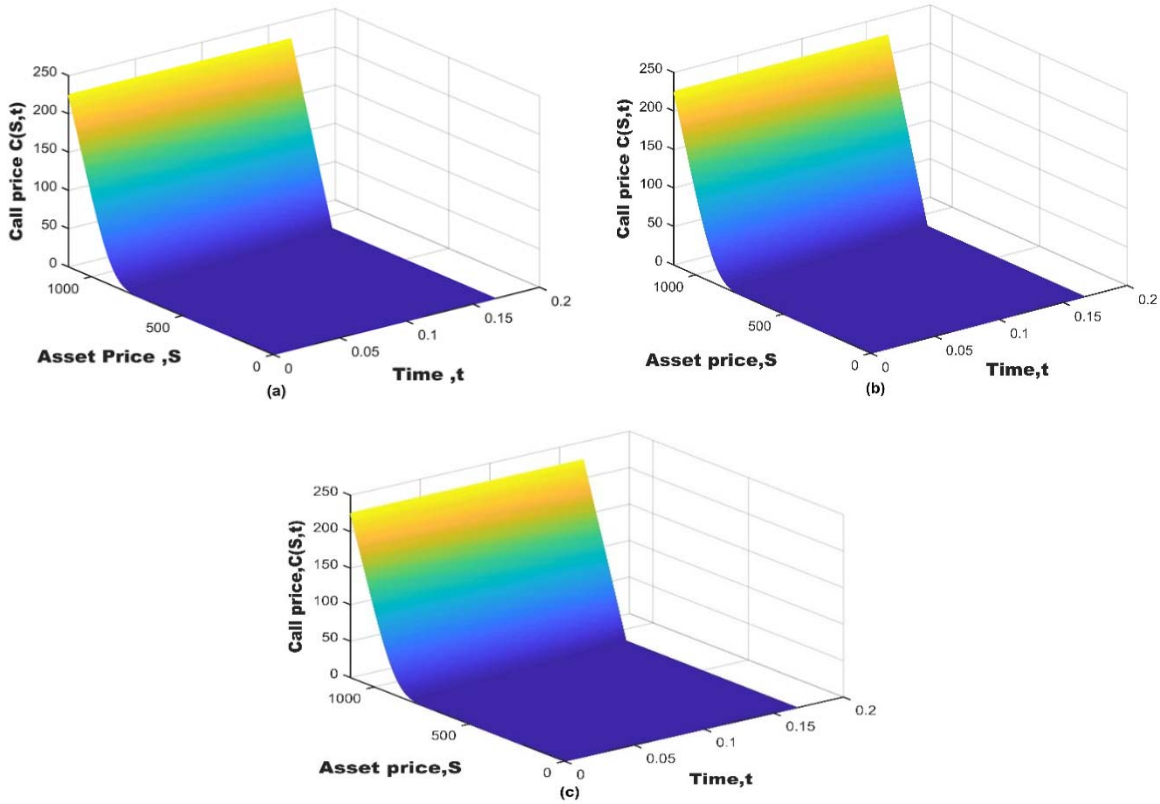

Figure 1. Solution of the model for stock price grid points, M=375, and time grid points, N=1000 by (a) EFDM; (b) IFDM; (c) CNFDM.

Table 1. The results of three FDS when N=1000 is fixed and M changes

N	M	IFDM	EFDM	CNFDM	EXACT	Absolute Error		
						IFDM	EFDM	CNFDM
1000	50	51.72264	51.50819	51.72568	51.83296	0.110316	0.324766	0.107275
	75	51.80900	51.67452	51.81212	51.83296	0.02396	0.158433	0.020836
	100	51.82968	51.73224	51.83285	51.83296	0.003279	0.100718	0.000105
	125	51.83480	51.75887	51.83801	51.83296	0.001847	0.074088	0.005053
	150	51.83516	51.77331	51.83838	51.83296	0.002198	0.059644	0.005427
	175	51.83389	51.78201	51.83714	51.83296	0.000936	0.050942	0.00418
	200	51.83211	51.78766	51.83537	51.83296	0.000844	0.045298	0.002413
	225	51.83023	51.79153	51.83350	51.83296	0.002723	0.041429	0.000544
	250	51.82842	51.79429	51.83169	51.83296	0.00454	0.038662	0.001266
	275	51.82672	51.79634	51.83000	51.83296	0.006235	0.036616	0.002954
	300	51.82517	51.79790	51.82845	51.83296	0.00779	0.035059	0.004503
	325	51.82375	51.79911	51.82704	51.83296	0.009208	0.033848	0.005917
	350	51.82246	51.80007	51.82575	51.83296	0.010498	0.032887	0.007202
	375	51.82129	51.80084	51.82458	51.83296	0.011671	0.032112	0.008373

Figure 2 presents the graphical representation of Table 1. Figure 2 demonstrates that option values converge to the analytic BS solution as stock price grid M increases for all FDS.When the stock price grid M increases, the price converges to an exact solution faster than the EFDM for IFDM and CNFDM and provides a better result when M = 200 for IFDM and M = 100 for CNFDM. Figure 3 demonstrates that, in comparison to the other two approaches, EFDM produces relatively higher absolute errors. The findings of the IFDM and CNFDM schemes are quite similar to the analytic BS solution, indicating that both approaches have a negligible absolute error. Furthermore, when the stock price grid M increases, Figure 3 reveals that the CNFDM scheme has less absolute error than the EFDM and IFDM schemes, implying that CNFDM converges to the analytic BS solution faster than EFDM and IFDM schemes. As a result, it can be revealed that the CNFDM scheme performs better than the EFDM and IFDM schemes in terms of accuracy.

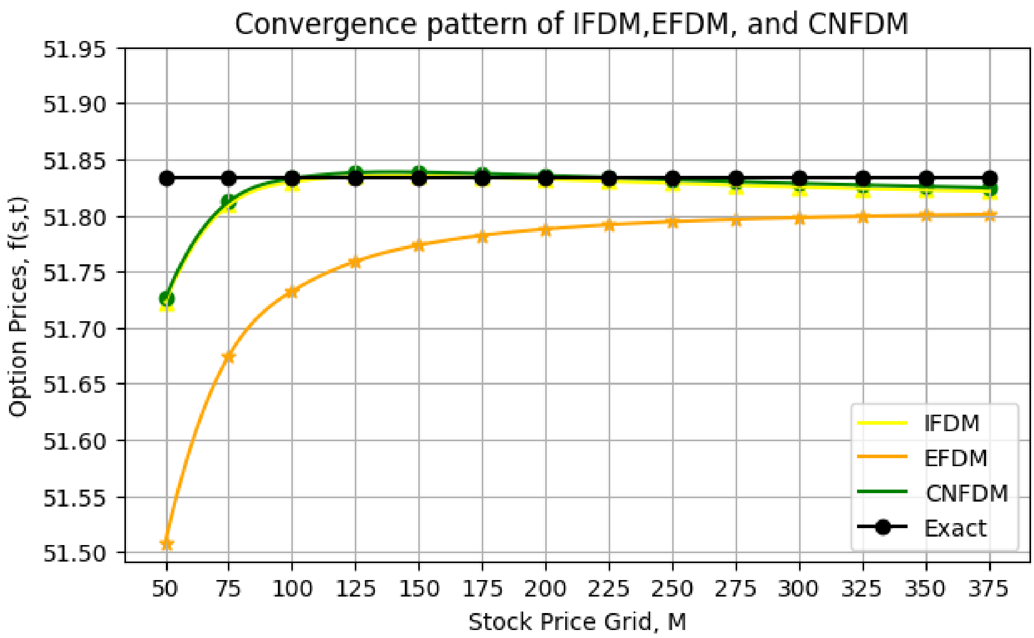

Figure 2. Convergence pattern of EFDM, IFDM, and CNFDM for pricing European Call option with dividend payment.

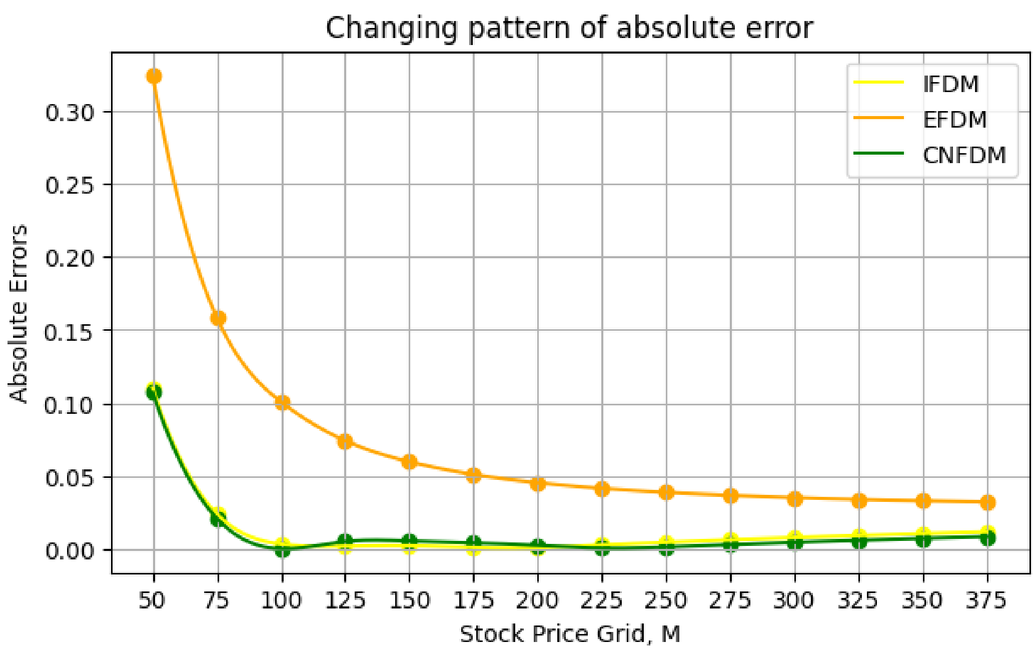

Figure 3. Comparison of absolute error among EFDM, IFDM and CNFDM for a fixed number of paths.

We have observed how option values converge to analytic BS solutions when Monte Carlo Simulation (MCS) is used to calculate option values. Table 2 shows the option values derived using MCS for the different number of paths (N), and the option value converges to the analytic BS solution as the number of paths increases. Figure 4 is a representation of Table 2. Figure 4 shows that when the number of paths N is 25000, the option value derived using MCS is closely similar to that obtained using analytic BS.

Table 2. The results of MCS when number of time steps M is 1000

N	MCS	Pricing	Absolute Error
100	53.0929151675755	51.8329567964908	1.2599583710847
500	52.4107555629895	51.8329567964908	0.5777987664987
1000	52.3715771135895	51.8329567964908	0.5386203170987
2000	52.2516184065818	51.8329567964908	0.4186616100910
3000	52.1480619935201	51.8329567964908	0.3151051970293
5000	51.9562816190120	51.8329567964908	0.1233248225212
10000	51.8694195271477	51.8329567964908	0.0364627306569
15000	51.8456921351546	51.8329567964908	0.0127353386638
20000	51.8304556966917	51.8329567964908	0.0025010997991
25000	51.8328969551723	51.8329567964908	0.0000598413185

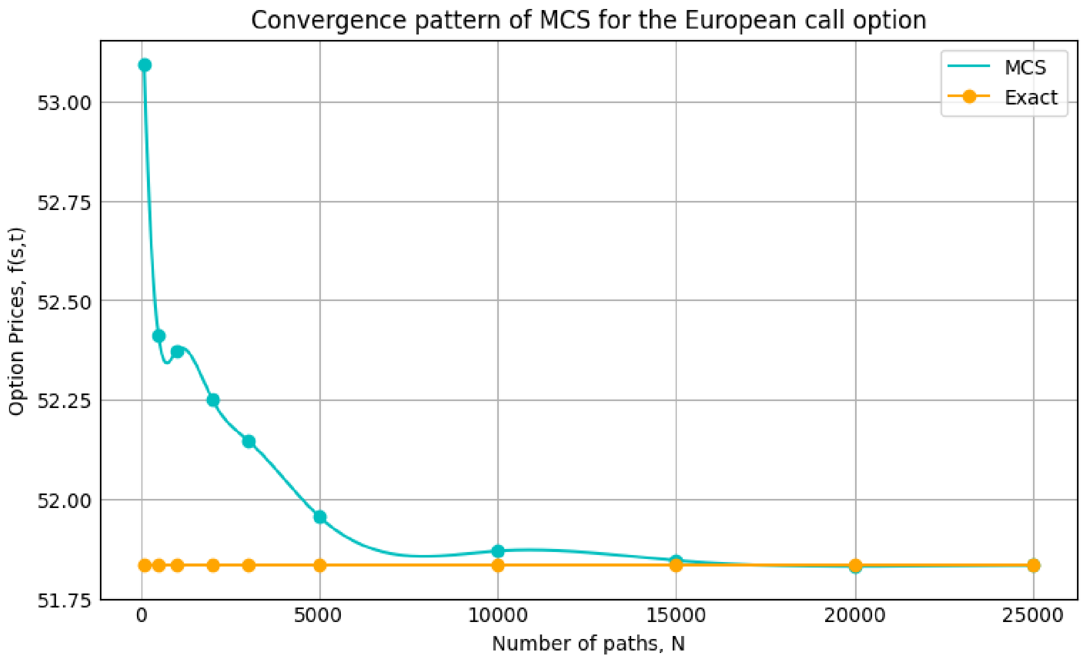

Figure 4. Convergence pattern of MCS for the European call option.

Figure 5 shows that as the number of paths increases, the absolute error of MCS converges to zero. Thus, MCS gives more accurate result for higher number of paths. Tables 1 and 2 show that the option value generated using CNFDM for N = 100 is quite similar to that obtained using MCS with 25000 paths, while MCS takes more time to resolve option price than CNFDM. Thus, it can be decided that the CNFDM pricing European call option with dividend payment is better than the MCS pricing European call option with discrete dividend payment which is similar to the findings of Fadugba *et al*. [7].

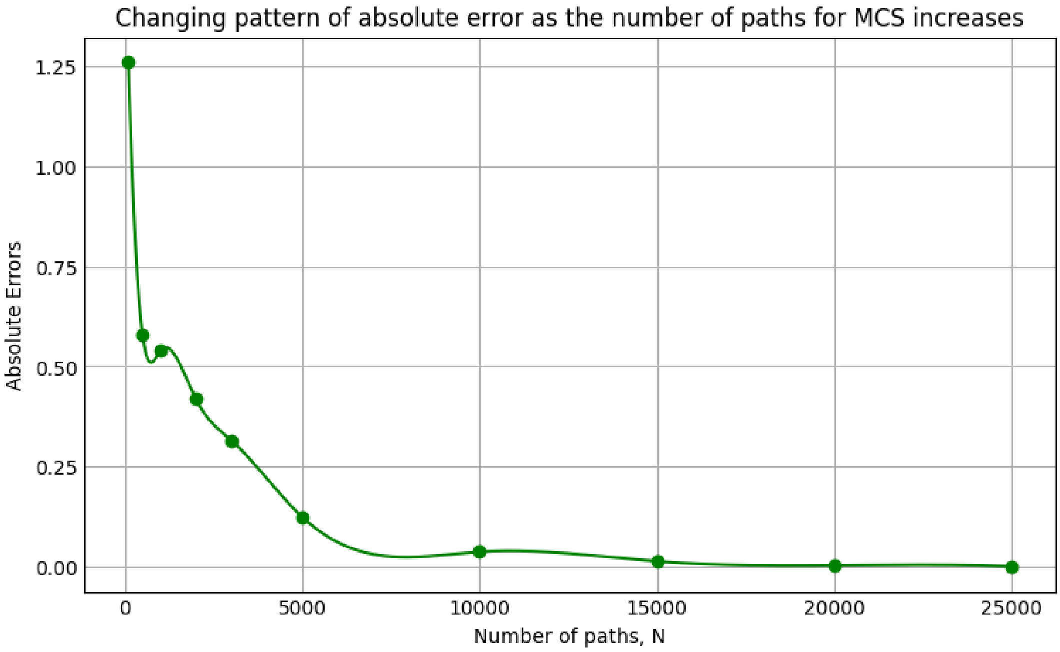

Figure 5. Changing pattern of absolute error as the number of paths for MCS increases.

5. Conclusions

In general, each of the numerical approaches, the FDM and the MCS, has benefits and drawbacks.For IFDM and CNFDM, a large system of linear equations has to be solved at each time step, which is comparatively difficult and time-consuming. However, assuming a suitable choice of Δt and Δs, these two techniques converge faster and generate more accurate results. On the other hand, the EFDM approach converges slowly, but an equation system does not need to be solved at each time step. In contexts of pricing European options, the MCS method is very flexible. It converges gradually and delivers good results for a large number of paths, which increases the computational cost. Finally, after assessing the benefits and drawbacks of each scheme, it can be decided that the FDM performs better compared to the MCS for pricing the European option with dividend payment. Moreover, the CNFDM scheme is better than the other two FDM schemes studied in this paper for European option pricing.

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