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Reference Point-based Evolutionary Multi-objective Optimization for Reversible Logic Circuit Synthesis

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Abstract

In this paper, Reversible logic circuit synthesis is formulated as a quantum cost-minimization problem with equality constraint. A new reference-point based evolutionary multi-objective method R-EMO-RLC is specially designed to attack the equality constraint. First, the reference point is determined dynamically according the distribution of solutions. Then, a new crowding comparative operator is fabricated to adapt the uncertainty of constraint violation and objective value aroused by variable length encoding. Experimental results show that R-EMO-RLC can increase the feasible ratio and obtain savings in quantum cost for some benchmarks from recent publications comparing with previously known circuits. This paper aims at research on RLC synthesis using evolutionary algorithm. The problem is formulated as a QC- minimization problem with equality constraint. It is different from the existing evolutionary solutions for RLC synthesis, where the evaluation of an individual is according to the satisfaction on the benchmark specification or a weighted sum of the correctness and the cost of the circuit.

Keywords

Reversible logic circuit, equality constraint, reference point multiobjective optimization

1. Introduction

Research on reversible logic circuits (RLC) has received a lukewarm attention by advances in quantum computing, nanotechnology and low-power design. Therefore, RLC synthesis has been intensively studied.

A gate (circuit) is called reversible if there is a one-one correspondence between its inputs and outputs. Reversible Logic Circuit Synthesis is to find a series of reversible gates from a general reversible gates library so that it satisfies the reversible benchmarks and is with minimum gate count or quantum cost.

Existing RLC synthesis methods are often classified into three categories: deterministic approaches, heuristic techniques and metaheuristic ones. Some deterministic approaches, such as transformation methods [1, 2], binary decision diagram based algorithms [3, 4], cycle-based decomposition methods [5], can found a solution efficiently even for a large scale problem, but the results often needs post-optimization further [6, 7]. Other deterministic algorithms often use exhaustive search to find an optimum solution. They are often characterized by the high computational cost and cannot solve the problems with more than 4-qubit, such as the GC- minimization methods for all 3-bit functions [8] and for all 4-bit functions from CNP(CNOT,NOT, Peres) library [9]. There are no deterministic methods for obtain QC-optimal circuits. The heuristic algorithm [10] constructs a priority-based search tree by employing the Reed-Muller expansion of a reversible function and rapidly prune the search space by the heuristic information. The search is biased toward GC-minimization or QC-minimization through adjust the coefficient of priority computation. The solutions obtained are often not optimal owning to the greedy nature of the algorithm. Evolutionary algorithms are already used in reversible circuit synthesis [11] [12] owing to its global search ability. But only small scale problems with low complexity are tested in these algorithm and no superior solutions were obtained.

Considering the feature of RLC synthesis, existing evolutionary multi-objective based techniques for constraint handling are not appropriate for the problem. First, owing to the scarcity of infeasible reparation mechanism, it is impossible to employ the two stage strategy [13], in which the optimization of objective value is conducted after the CV has been optimized. Second, the variable length encoding of RLC is required for the unknown of the length of the optimal solution. We adopt a non-destructive crossover operator and size limit method to automatically grow the chromosomes in a slow way to avoid chromosome bloat. In this process, the increasing of the objective value (quantum cost of the circuit) often accompany with the decreasing of constraint violation (CV). It's hard to tell which one is better between solutions non-dominated or even dominated each other when they are vey close. We want a constraint solving method which prefers the decreasing of CV as a whole and is with a new solutions ranking method different from domination relationship.

Reference-point multi-objective optimization idea is originally used to find a preferred solution [14] or preferred set solutions [15, 16] near the reference point. We make use of the idea that the preference information given by reference point can guide the search towards better solutions corresponding to the preferences. However, if absence of the priori unknown of the searching space, it is difficult for decision maker to give a reasonable reference point and it is not involved in [15, 16]. Aiming at the demand of our constraint solving, the two main contributions of our algorithms are: a dynamic reference point setting method according to the current distribution of solutions and a new crowding comparative operator which give priority to the distance ranking then to non-dominated ranking.

The paper is organized as follows. Section II recalls the basic concepts of reversible logic and the model of RLC synthesis. Section III simply introduces the original reference- point based evolutionary method. Section IV gives the detailed description of our reference-point based evolutionary multi- objective for RLC synthesis (R-EMO-RLC). Section V presents comparative results to circuits derived from general Toffoli gates. Section VI summarizes the paper with conclusions and suggestion for further research.

2. Reference Point-Based NSGA-II Algorithm

NSGA-II [17] is a classic multiojective algorithm which was able to matintain a better spread of solutions and converge better in the obtained nondominated front. Crowding comparative operator (٢_{μ}) of NSGA-II defines a partial order of solutions to provide a basis for the two selection stage blocked in Fig. 1. If individual i has less nondomination rank (i_{rank}) than individual j, or if i is equal to j in nondomination rank but has larger crowding distance ($i_{cdistance}$) than j, then

$$\mathrm{i}\text{٢}_{\mu} \quad \mathrm{j}.i\text{٢}_{\mu} j \quad if\,(i_{rank} \quad j_{rank}) \qquad (1)$$
$$or\ ((_{irank} \quad j_{rank}\,)\ and\ (i_{cdistance}\, j_{cdistance}\,))$$

Crowding distance operator ($i_{cdistance}$) means the density around the solution i and is used to obtain solutions distributed more homogeneously.

Reference point-based evolutionary multiobjecitve optimization algorithm R-NSGA-II [15, 16] substitutes the crowding comparative operator ٢_{μ} in NSGA-II with reference point comparative operator ٢_{μ} The reference point comparative operator is designed to obtain soluitons close to reference point on the pareto front. It adopts a new the reference point, instead of the crowding distance.

Reference point rank *rdistance* is obtained as follow. For each reference point, the normalized Eucidean distance of each solution of the front is caculated and the solutions are sorted in ascending order. The solution closest to the reference point is assigned a rank of one. Then such operations are conducted for all reference points, the minimum of the assigned ranks is the final rank.

That is, between two solutions with differing nondomination rank *rank*, reference point comparative operator ($\prec nr$) prefers solutions with the lower nondomination rank *rank.* Otherwise, if both solutions belong to the same front, it prefers the solution that has lower reference point rank *rdistance*.

$$i\ \text{٢}_{\mu} \quad j \quad if\,(i_{rank} \quad j_{rank}\,) \qquad (2)$$
$$or\ ((i_{rank} \quad j_{rank}\,)\ and\ (i_{rdistance} \quad j_{rdistance}\,)$$

No matter Crowding comparative operator (٢_{μ}) or the reference point comparative operator ٢_{μ}, they all prefer the solutions with lower non-domination rank. The difference is that the former distribute the solutions more homogeneous, the latter likes solutions close to some reference point.

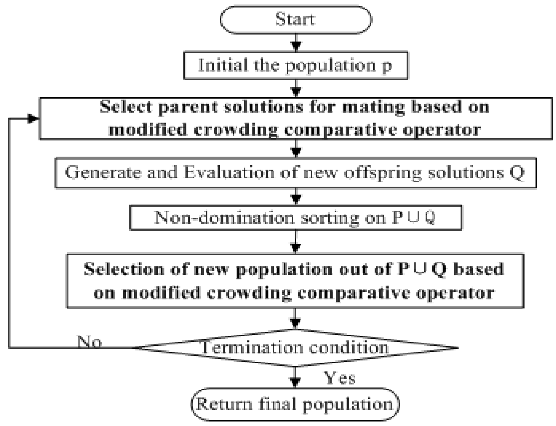

Figure 1. The flowchart of R-NSGA-II.

3. Reference Point-Based Moea for RLC Synthesis

Reference point-based method is used as an equality constraint solving method for RLC synthesis. We have no idea about the location of optimum. The only definite thing is that we want the CV being decreased to zero. If we set a reference point with CV of zero, it will be far away from the individuals in the initial population and the arbitrarily selected objective value of a reference point will mislead the evolution process. It is very critical knowing about how to setting the reference point.

3.1 The setting of reference point

In R-NSGA-II, the reference point is randomly selected to show the convergence capability toward it. The setting of reference point is not involved in it. In a specific application, the decision maker should clearly know what he wants. In our R-EMO-RLC, without the knowledge about the optimum, we have no choice but to generate the reference point according to the distribution of current solutions. The reference point should guide the search and population toward CV decreasing with QC increasing not too much.

After the population was conducted a non-dominated sorting, we can obtain the first two solutions (*QC1,CV1*) and (*QC2*, *CV2*) from CV ascending perspective in the front with non-domination rank 1. They are corresponding to (0.3,0.6) and (0.5, 0.5) in Fig. 2 respectively. The first solution has the least value of CV by far. The second solution has the suboptimal CV but smaller QC than the first solution. From which we can construct the reference point according to:

$$\text{xQC1(1-)xQC2,CV1).} \tag{3}$$

Where belongs to [0, 1]. Therefore, the reference point has the minimal CV by far and a smaller QC than that of the first solution in non-domination rank1. The larger the value of, the closer the reference point is apart from the first solution, accordingly, the larger the QC of the reference point. That is, the larger the CV, the faster the speed of CV decreasing and QC increasing.

The normalization process is applied to QC. The original CV obtained by (1) is divided by the number of terms in the original PPRM of the reversible function, so the value is between 0 and 1 in Fig.2.

Combining with the subsequent new crowding comparative operator, the reference point-guided search can be realized.

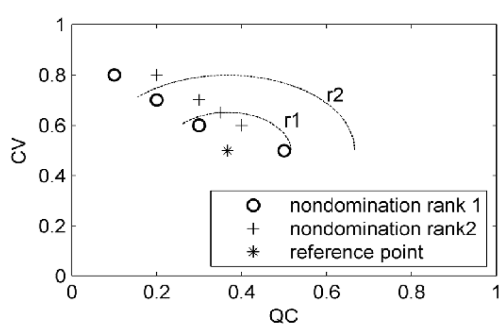

Figure 2. The setting of reference point.

3.2 The new reference point comparative operator

In our algorithm, we want to conserve solutions to guide the search toward the direction of CV decreasing, so the two comparative operators ٢ $_{\mu}$ and ٢ $_{\mu}$ are not suitable for our RLC synthesis problems. We designed a new reference point comparative operator ٢ $_{\mu}$ here to satisfy our requirement.

With too small CV or too large QC are not desired during the evolution. So we not give the priority to the nondo First, the solutions with too small QC but too large CV or mination ranking *rank* of solutions in the new partial relation٢ $_{\mu}$.

Secondly, we can not definitely determine which one is better actually between two infeasible solutions close to each other, for they are only partial solutions, not complete.

Considering the above analysis, we first calculate the distance between each solution and the reference point and sort the solutions in distance ascending order, and then we classify the solutions into different groups according to the distance. The solutions in the first group with the distance less than $r1$ are assigned distance rank 1, and the solutions locate between $r1$ and $r2$ are assigned distance rank 2 and so on, see Fig.2. Hereto, each solution has a non-domination rank *rank* and a distance rank *distancer*. Now we define our reference point comparative operator (٢ $_{\mu}$), a partial order of solutions, as:

$$\text{i٢}_{\mu} j \text{ if}(i_{dis\ tan} < j_{dis\ tan})$$
$$or\ ((i_{distancer} \quad j_{distancer})\ and\ (i_{rank} \quad j_{rank})) \tag{4}$$

That is, between two solutions with differing reference point distance ranks, we prefer the solution with the lower (better) rank. Otherwise, if both solutions belong to the same distance group, then we prefer the solution that is located in a better non-domination front.

3.3 R-EMO-RLC

The main steps of R-EMO-RLC are as follow:

Step 1. Initialize the population $P(t)$, t =1;

Step 2. Compute the nondomination rank for each individual in $P(t)$.

Step 3. Calculate the reference point according to (5).

Step 4. Obtain the distance rank or group of each individual.

Step 5. Apply tournament selection based on reference-
point crowding operator $\prec r$.

Step 6. Crossover and mutation operators are used to create a offspring population $Q(t)$.

Step 7. Nondomination sorting the $P(t) \cup Q(t)$.

Step 8. Update the reference point according to (5).

Step 9. Obtain the distance rank according to the updated reference point for individuals in $P(t) \cup Q(t)$.

Step 10. The solutions in the combined population are selected and put into $P(t+1)$. If the size of first group $G1$ is smaller than the population size N, we definitely choose all members of the set $G1$ for the new population $P(t+1)$. The remaining members of P(t+1) are chosen from subsequent distance groups in the order of their ranking. Thus, solutions from $G2$ are chosen next, followed by solutions from $G3$, and so on. This procedure is continued until no more sets can be accommodated. Say that the set G_k is the last distance group beyond which no other groups can be taken in. In general, the count of individuals in all groups from $G1$ to Gk would be larger than the population size. To choose exactly N population individuals, we alternately apply the tournament selection based on CV and the same selection based on QC among the individual in G_k, until the number of individuals in $P(t+1)$ reaches N. The double tournament selection can improve the diversity among a distance group and keep pace with the essence of uncertainty of evaluation for a partial solution.

Step 11. $t = t+1$;

Step 12. If the stop condition does not be satisfied, go to step 5, or else the algorithm is terminated.

Apart from the usual evolutionary parameters, R-EMO- RLC introduces another two parameters, is the parameter related to the definition of reference point, the G_n means the number of different groups.

4. Conclusion

Owing to the scarcity of the infeasible reparation mechanism and the difficulty of construct feasible solutions, R-EMO-RLC is designed to attack the equality constraint using reference multi-objective optimization method. The dynamic reference point is generated and a new reference point comparative operator is designed to guide the search toward the CV decreasing. The experimental results showed that the two mechanisms are effective in improving the feasible ratio and solution quality.

One possible further research direction may be improving the computational complexity of distance comparative operator which involves the time consuming calculating of nondomination rank. The impact of the value of parameters and G_k deserves to be studied further.

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