Journal of Applied Mathematics and Computation, 2023, 7(2), 280-287
https://www.hillpublisher.com/journals/JAMC/
ISSN Online: 2576-0653
ISSN Print: 2576-0645

Ergodic Characteristics of Payment Points under the Slow Second Cashier Problem

Sulaiman Sani*, Mlamuli W. Malindzisa

Department of Mathematics, FOSE, University of Eswatini, Kwaluseni Campus-Kwaluseni, Kingdom of Eswatini.

How to cite this paper: Sulaiman Sani, Mlamuli W. Malindzisa. (2023) Ergodic Characteristics of Payment Points under the Slow Second Cashier Problem. *Journal of Applied Mathematics and Computation*, 7(2), 280-287.
DOI: 10.26855/jamc.2023.06.009

Received: June 3, 2023
Accepted: June 29, 2023
Published: July 30, 2023

***Corresponding author:** Sulaiman Sani, Department of Mathematics, FOSE, University of Eswatini, Kwaluseni Campus-Kwaluseni, Kingdom of Eswatini.

Abstract

We study the slow second cashier problem of payment points with two cashiers and provide the analysis that minimizes both the violation of the First Come First Served payment discipline (FCFS) and ergodic characteristics vital for operations management respective of decision making when it is operationally optimal to balance the work rate of the slow second cashier. Initially, a non-FCFS payment schedule whose operational characteristics identify the most suitable payment point and present vital performance measures for balancing payments is constructed using ergodic theory coupled with the Markov property of the payment system. Our result shows that in a given payment period, there exists real ergodic points to which one of these points is operationally optimal. In addition, these points contain vital information needed for the overall management of payment systems globally. Finally, we state vital characteristics of the optimal ergodic point for the sake of operations management of payment centers.

Keywords

Ergodic point, Optimal Ergodic point, Non-FCFS, Null ergodic point, Payment points

1. Introduction

Payment schedules are key elements of banking halls, manufacturing centers and numerous other centers of finance. They play important roles in defining quality of service (QoS) to customers ([1-3]). For instance, if a supermarket pay point decides to allow customers jump the payment line, customer-dissatisfaction may grow in time especially in the presence of impatient customers. [4] has shown that generally if customers feel their waiting times are increasing sequel to violation of payment schedules, then total revenue is affected negatively. To demonstrate this effect, consider a supermarket with two cashiers as follows; cashier-1 and cashier-2 such that cashier-1 has a significantly higher work rate compared to the work rate of cashier-2(slow second cashier problem) owing to varying levels and degrees of experience among others. Suppose that paying customers join a single payment line working under the FCFS schedule. If customer-1 is served by cashier-2 and customer-2 by cashier-1, then there is a positive probability that customer-2 finishes payment ahead of customer-1. In this case, the FCFS schedule is violated owing to random choice of cashiers and can lead to reduced QoS with direct linings on investment resources ([5-6]).

The randomized inaction exemplified above creates both the reason and the need to modify traditional payment schedules for finance centers strong enough to balance work rates in multi-cashier payment systems to prevent customer impatience growing beyond tolerance limits and ensuring that investment resources are optimally utilized ([7-8]). There are several benefits constructing such payment schedules will bring to business centers in operational management. First, constructing improved non-FCFS paying schedules will make pay points more proactive in identifying issues on time and by so doing, improves pay point QoS. Again, it can help payment stations keep track of goods and services, the action that can save investment resources directly. Third, constructing newer non-FCFS payment schedules opens the slowing reality of the second cashier gateway in designing trainings for an improved efficiency that guarantees higher profit margins in payment points. It can define how much training a prospective slow cashier shall require as a function of his current work rate for the overall benefits of the payment point. Fourth, new non-FCFS payment schedules provide bases for rewarding highly work-rated cashiers for improved effectiveness in service delivery in accordance with the work-reward paradigm presented in [9-11] as emphasized by modern day methods of managing financial centers globally.

One way to balance pay point services to cultivate the gains stated above is through the application of the ergodic hypothesis suggesting that customers of pay points eventually will pass through the space allocated for payment purposes in time. Thus, the existence of critical points in the path of payments is guaranteed. Here, the application of the Poisson Arrivals See Time Averages (PASTA) can be useful in predicting the characteristics of payment points and can provide balancing fulcrums for deriving enhanced QoS that maximizes the use of investment resources and minimizes losses ([12-14]). Again, if one mixes PASTA with the theory of point processes that characterized functional limit theorems (FLTs) in financial engineering, then the natural question of balancing QoS with investment resources can be transformed into a limiting problem of finding strong stability points of ergodic processes and their numbered iterations [15]. This way, the characterizing exercise that identifies optimally strong ergodic iterations of payment points through their ergodic behaviors provides the means for adjusting payment services through schedules for the overall development of financial centers and their real time behaviors.

In this respect, "*the slow second cashier problem*" studied in this work has a long history in theoretical and practical finance and operational research. For instance [16] studied the time-dependent behavior of two-heterogeneous system with impatient customers where customers refuse to join the system or join but only to leave after a certain period of time owing to the slow second cashier problem and derived both the transient and the steady state performance measures of the pay point. [17] studied a class of the slow second cashier problem called "*multiple vacation systems*" and derived performance measures subject to cashier's unavailability. The work shows that operationally, there exists working regimes when it is better not to install the slow cashier at all owning to the diminishing of the combined work rates of the cashiers in the systems.

[18] studied the slow second cashier problem and provided a generalization for *n*-heterogeneous cashiers concluding that, there exists a given arrival rate below which the slow cashier should not be used and vice versa. [19] analyzed the case of two heterogeneous cashiers under the cost of offering payment service at each payment point and proved that the most efficient cashier is the one with the minimum total average cost at a given payment point. [20] studied a two cashier heterogeneous system where the slower cashier has a threshold for receiving payment. That is, if there are less than $K>0$ customers in the system, the slower cashier is left idle and once the system reaches K customers, the slower cashier then starts to work until steady state.

The case studied in this work is closely related with that of [20] in formation only that, our mode of analysis is completely different both in approach and in presentation. In our case, we incorporated the specific assumption that the slow cashier comes out of vacation whenever there are three or more customers in the system. This way, the rate at which the slow cashier returns to work is heavier than the case identified above and can be said to follow a different distribution in analysis. Again, our approach simplifies the operational management and adoption making business life easier and convenient for owners and investors alike. The next section presents the methodology employed in this work.

2. Modeling and Methods

We consider a payment point with a fixed cashier (cashier-1) and a standby cashier (cashier-2) as in Figure 1. Customers enter the payment system as Poisson arrivals at a rate λ (Figure 1) to pay for items through a single line and payaccording to the FCFS payment schedule if and only if cashier-1 is the only cashier in the payment point (Figure 2(1)). If a customer paid through cashier-1, the payment time follows the exponential distribution with a mean rate μ_f. Suppose that customer-2 arrives during the payment time of customer-1 as in Figure 2(2). Then the said customer (customer-2) waits for customer-1 to finish payment so that he pays with cashier-1. Thus, each time there are two payment customers in the system, cashier-1 is the only cashier in the payment point. Suppose customer-3 arrives when customer-1 and customer-2 are in the payment system. Then, cashier-2 is installed and customer-3 pays through cashier-2 as shown in Figure 2(3). Cashier-2 has an exponential work rateμ_s such that $\mu_s < \mu_f$. Subsequent arrivals wait for their paymentturn depending on which cashier is available first and according to the non-FCFS payment schedule described in the Figures below.

The state of the payment point can be described by the number of payment customers anytime in the payment point. Suppose that one chooses the departure epochs to study the customer process in view of the Poisson arrivals assumption employed and upon departures from both cashiers in view of varying work rates, then the PASTA principle holds good. By the arguments in [21-22], the customer process described above forms an irreducible and aperiodic Markov chain as time grows large. Again, the time process becomes a stationary process as time grows large [23-24]. Let P_J: $j=0, 1, 2,$

3,...denote the stationary probability that there are *j* customers in the payment system. By the ergodic theory of [25-26], there exist a finite set of coupled Kolmogorov difference-differential equations satisfied by the customer process described in Figure 2 above. This coupled system describes the customer flow balance of payment customers in the payment point as arrivals and service completions occur simultaneously. Using the ergodic theorem and the flow path of the customer process, the below coupled difference equations are satisfied,

$$\lambda P_0 = \mu_f P_1; j = 1 \tag{1}$$

$$(\lambda + \mu_f)P_1 = \lambda P_0 + \mu_f P_2; j = 2 \tag{2}$$

$$(\lambda + \mu_f + \mu_s)P_2 = \lambda P_1 + (\mu_f + \mu_s)P_3; j = 3 \tag{3}$$

$$(\lambda + \mu_f + \mu_s)P_j = \lambda P_{j-1} + (\mu_f + \mu_s)P_{j+1}; j \geq 4 \tag{4}$$

Equations 1-4 describe how the payment system maintains any *j*= 0,1,2,3,… customers with probability P_j as arrivals and departures occur simultaneously in the payment point.

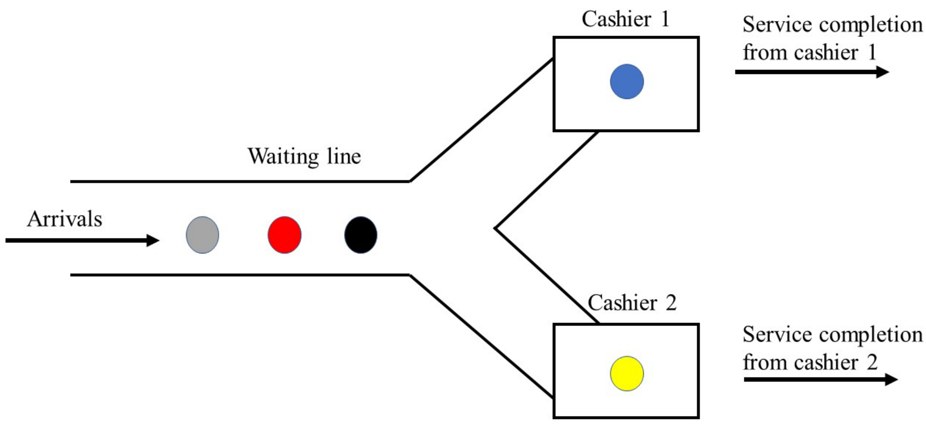

Figure 1. Customer-Cashier Sequence.

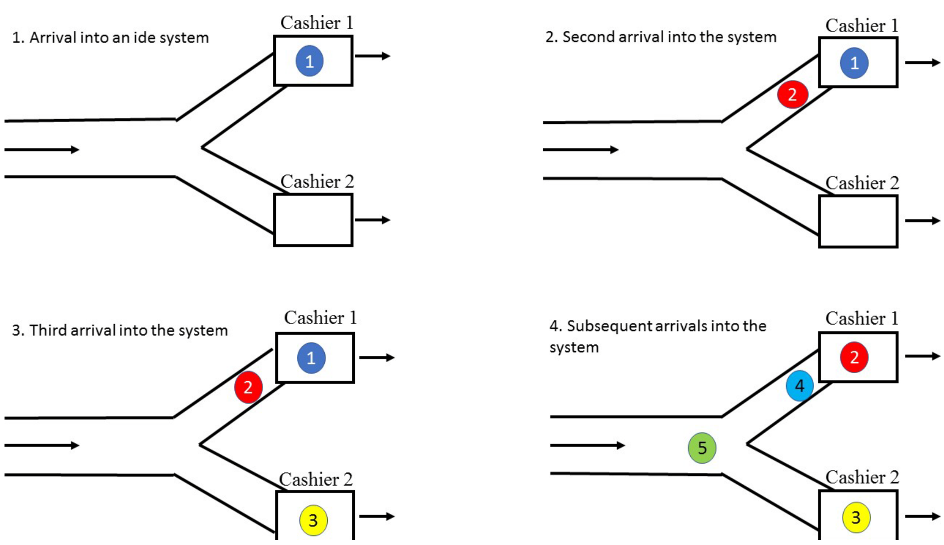

Figure 2. Non-FCFS Paying Schedule employed.

3. Ergodic Characteristics

Lemma 1:

Let $\rho = \frac{\lambda}{\mu_f + \mu_s} < 1$ and $\rho_f = \frac{\lambda}{\mu_f}$ so that the customer system is stable [27]. The stationary probability P_j

That there are *j* payment customers is given by

$$P_j = \left(\frac{\lambda + \mu_s}{\mu_f + \mu_s}\right)^{j-2} \left(\frac{\lambda}{\mu_f}\right)^2 P_0; \qquad j \geq 2$$

Proof:

Substituting (1) into (2) yields;

$$\lambda P_1 = \mu_f P_2 \rightarrow P_2 = \left(\frac{\lambda}{\mu_f}\right) P_1 \rightarrow P_2 = \left(\frac{\lambda}{\mu_f}\right)^2 P_0 \tag{5}$$

Again, substituting *(2)* into *(3)* gives;

$$P_3 = \left(\frac{\lambda+\mu_s}{\mu_f+\mu_s}\right) P_2 = \left(\frac{\lambda+\mu_s}{\mu_f+\mu_s}\right)\left(\frac{\lambda}{\mu_f}\right)^2 P_0 \tag{6}$$

Finally, the Lemma follows by substituting $j = 4, 5, \ldots, j(4)$, using both the Markov property of the system states and simplifying until arriving for the case for j-customers in the system.

$$P_j = \left(\frac{\lambda+\mu_s}{\mu_f+\mu_s}\right)^{j-2} \left(\frac{\lambda}{\mu_f}\right)^2 P_0; \qquad j \geq 2 \tag{7}$$

Note that the ratio$\left(\frac{\lambda+\mu_s}{\mu_f+\mu_s}\right) = \frac{\lambda}{\mu_f+\mu_s} + \frac{\mu_s}{\mu_f+\mu_s} \rightarrow \frac{\lambda}{\mu_f+\mu_s}$ for smaller values of$\mu_s$compared to those of μ_f. For operational convenience, this negligible rate can be ignored so that (6) becomes

$$P_j = \left(\frac{\lambda}{\mu_f+\mu_s}\right)^{j-2} \left(\frac{\lambda}{\mu_f}\right)^2 P_0; \qquad j \geq 2 \tag{8}$$

Lemma 2:

Suppose the work rate of cashier-2 deteriorates such that (8) holds. Then the pay point will behave like a one-cashier pay point.

Proof: If (8) holds, then μ_s is not needed in determining the overall work rate of the pay point, so that $\mu_s = 0$ under this condition. From (8), one obtains that

$$P_j = \left(\frac{\lambda}{\mu_f}\right)^{j-2} \left(\frac{\lambda}{\mu_f}\right)^2 P_0 = \left(\frac{\lambda}{\mu_f}\right)^j P_{0,} \tag{9}$$

which is the ergodic characteristic probability function for a one-cashier payment point [28].

Lemma 3:

The stationary expected number of customers *E[j]* in the payment point is given by

$$E[j] = \frac{P_0\rho_f\left(1-3\rho_f^2+2\rho_f^3\right)}{\left(1-\rho_f\right)^2} + \frac{P_0\rho_f^2\rho(3-2\rho)}{(1-\rho)^2} \tag{10}$$

Here, an arbitrary payment customer finds the payment point idle with probability P_0 given by

$$P_0 = \frac{1-\rho}{1-\rho+\rho_f-\rho_f\rho+\rho_f^2} \tag{11}$$

Proof:

Denote by $M(z)$ the generating function for the number of payment customers at the payment point such that

$$M(z) = \sum_{j=0}^{\infty} P_j z^j; \qquad z \leq 1 \tag{12}$$

In view of (8) on (12), one obtains that

$$M(z) = P_0\left[\left(1 + \rho_f z + \left(\rho_f z\right)^2\right) + \rho\rho_f^2 z^3(1 + \rho z + (pz)^2 + (pz)^3 + \cdots)\right] \tag{13}$$

Upon the application of certain properties of geometric series, we have

$$M(z) = P_0\left[\left(\frac{1-\left(\rho_f z\right)^3}{1-\rho_f z}\right) + \left(\frac{\rho\rho_f^2 z^3}{1-\rho z}\right)\right] \tag{14}$$

Differentiating (14) once with respect to *z* gives

$$M'(z) = \frac{P_0\rho_f\left(1-3\rho_f^2 z^2+2\rho_f^3 z^3\right)}{\left(1-\rho_f z\right)^2} + \frac{P_0\rho_f^2\rho\left(3z^2-2\rho z^3\right)}{(1-\rho z)^2} \tag{15}$$

The first part of the Lemma follows after evaluating (14) at *z=1*.

For the second part of the Lemma, apply the normalizing condition respective of (8). Consequently,

$$P_0 + \rho_f P_0 + \sum_{j=2}^{\infty} P_n = 1 = P_0 + \rho_f P_0 + \sum_{j=2}^{\infty} \rho^{j-2}\rho_f^2 P_0 \quad (16)$$

Which simplifies to

$$P_0\left[1 + \rho_f + \rho_f^2\left(\frac{1}{1-\rho}\right)\right] = 1 \quad 17)$$

Equation (11) follows directly upon simplifying (17).

Corollary 1: The stationary expected waiting time *E[W]* of payment customers is given by

$$E[W] = \frac{P_0\rho_f\left(1-3\rho_f^2+2\rho_f^3\right)}{\left(1-\rho_f\right)^2\lambda} + \frac{P_0\rho_f^2\rho(3-2\rho)}{(1-\rho)^2\lambda}\ ; \quad \rho < 1 \quad (18)$$

Proof:

Corollary 1 follows directly by the application of Little's theorem on Lemma 3 and simplifying the resulting expression.

4. Numerical Analysis

For some numerical illustrations of the ergodic characteristics studied in this work respective of the payment point analysis carried out, we numerically compare the performance of the payment point studied in this work as in (10) with that of Gross (2008) where the work rates of the two cashiers are identical. The purpose of this comparison is to present both the gains and the effects of installing the slow second cashier respective of the overall performance of the payment system. Again, we wish to understand the cost of running the payment point in question in the light of the case where both cashiers work with equally identical rates for operational management reasons. For some chosen values of λ, μ_f and $\mu_s$respectively, *E[j]* (constructed) was computed and compared with *E[j]* (Gross (2008)) working under the FCFS with two identical cashiers. This exercise brings out the operational gains in terms of financial management of payment points working under the FCFS by providing valid positions on when the FCFS is to be switched off to the non-FCFS payment schedule constructed and analyzed in this work.

CASE 1:

When $\lambda$and $\mu_f$increase monotonically and $\mu_s$fixed.

This case depicts a payment epoch where higher sales are made continuously and cashier-2 keeps increasing its work rate to accommodate high customer traffic in the light of fixed work rate of cashier-2. Table 1 summarizes the numerical values of *E[j]* obtained.

Table 1 summarises the *E[j]* for both models under selected values of $\boldsymbol{\lambda}$ simulated.It can be seen that there is no significant difference in the performance of the payment point respective of the increasing work rate of cashier-1and the arrival rate of payment customers in both models. Interestingly, there exists two ergodic points at $\boldsymbol{\lambda = 80}$ **and** $\boldsymbol{\lambda = 160}$where the performance of the two payment points coincides perfectly and stochastically continous; indicating that if cashier-1 raises its work rate significantly in time compared with cashier-2, the increasing arrivals into the payment point will henceforth not affect the stability and the overall functioning of the payment point. This result further implies that under increasing work rate of cashier-1, it is less expensive to provide paymentt service under the constructed payment schedule analyzed in this work in light of the Gross (2008) model used under the FCFS with two identically work rated cashiers. Thus, it is operationally better to use the mixed cashier arrangementpresented in this work than the homogenous cashier system of Gross (2008)under this condition for proper management of investment resources.

CASE 2:

When λ and $\mu_f$decrease monotonically and $\mu_s$fixed.

This payment epoch depicts the sphere of payment with light tailed characteristics coinciding with the wearing and tearing of cashier-1. Table 2 summarizes the *E[j]* for both models in this case. It can be seen that there are more customers in the system compared with case 1 implying that the wearing and tearing of cashier-1 has grossly affect the functioning of the payment point in terms of the number of customers remaining in the pay point. Again, the *E[j]* for both models decreases uniformly until the point when $\lambda = 120$ customers. At this point, the payment point achieves temporary ergodicity and at $\lambda = 90$, the payment point attains permanent ergodicity given all arrival rates of customers in the payment point. Thus, under the case 2 analyzed, the payment point has two ergodic points vital for proper management of payments.

Table 1. ***E*[*j*] for increasing arrivaland cashier-1 work rate**

arrival rate(λ)	*E*[*j*](Gross (2008))	*E*[*j*](constructed)
60	3	4
70	3	4
80*	3	3
90	2	3
100	2	3
110	2	3
120	2	3
130	2	3
140	2	3
150	2	3
160*	2	2
170	2	2
180	2	2
190	2	2

Table 2. Expected number of customers for decreasing arrival rates

arrival rate(λ)	*E*[*j*](Gross (2008))	*E*[*j*](constructed)
190	12	36
180	8	16
170	6	10
160	4	7
150	4	5
140	3	4
130	2	3
120*	2	2
110	2	2
100	1	2
90*	1	1
80	1	1
70	1	1
60	1	1

On the other hand, the result displayed in Table 2 is consistent with those in Table 1under increasing arrival rate and work rate of cashier-1 around the center. In this respect, one concludes that cashier-1 work ethics is the principal driver of payment points under the stability conditions employed in this work. The more industrious cashier-1 works (increasing ρ_f), the more stable (ρ) payment points are expected to behave in serviceand by extension, the better the operational management of the payment points.

CASE 3:

When λincreases monotonically and μ_f and μ_s simultaneously increase.

Here, the payment point is depicted to be under high sales throughout and both cashiers step up their work rates. More clearly, it depicts a payment epoch where cashier-2 is replaced by another slow cashier with a higher work rate. More precisely, the same situation depicts a case where cashier-2 work rate is improved by an additional training that jerks its work rate higher in light of increasing work rate of cashier-1. Table 3 summarized the *E[j]* obtained under this payment situation.

Table 3. *E*[*j*] for increasing service rates

Service rates of the two systems increased	*E*[*j*](Gross (2008))	*E*[*j*](constructed)
920	2	3
980*	2	2
1040	2	2
1100	1	2
1160*	1	1
1220	1	1
1280	1	1
1340	1	1
1400	1	1
1460	1	1
1520	1	1
1580	1	1
1640*	0	0
1700	0	0

In this case, the payment point achieves three ergodic points namely; temporary ergodic point at $\lambda = 980$ then permanent ergodic point at$\lambda = 1160$ and finally, null ergodic point at $\lambda = 1640$. This increasing number of ergodic points implies the place of improving the work rate of slow cashiers in payment points generally as a route to improving paying services thereby reducing the cost of payments. Additionally, it shows that under raising work rates of both cashiers, the payment point achieves advantageous similarity to the case where the cashiers have identical work rates in light of operational management of financial resources. This further suggests that the problem of slow rated cashiers has optimal ergodic root if and only if the work rate of cashier-2 is improved. This position holds good in view of the null ergodic point discovered in case 3 only and presented in this work.

5. Concluding Remarks

We present the ergodic characteristics of payment points working under the slow cashier-2 problem using ergodic theory and point processes analysis leading to some interesting results vital for operational management of payment points in shops and other financial systems. The work shows that the work rate of the faster cashier in light of the arrival rate of customers into the payment point determines both the stability of the payment point and the number of ergodic points. The crème of the work shows that improving the work rate of the slow cashier is paramount to achieving the null ergodic point needed for effective management of pay point businesses in general. There is a scope in automating the payment schedule analyzed in this work in view of its immense capacity to reduce cost of service without compromising QoS to customers of globally. One may also choose to generalize the characteristics of payment points with several slow cashiers and one fast cashier for reasons to do with operational management of global payment points and centers.

Acknowledgements

The authors are grateful to the Management of the Journal of Applied Mathematics and Computations (JAMC) and her Editorial board for inviting this article. We are also grateful to all the sources of literature used and to the anonymous reviewers and editors of this work.

References

[1] Adrian F., Treglown, L. and George H. (2020). The Psychology of Queuing. Psychology, 480-498.

[2] Sudhesh, R., Savitha, P. and Dharmaraja, S. (2017). Transient Analysis of a Two-Heterogeneous Servers Queue with System Disaster, Server Repair and Customers' Impatience. TOP, 179-205.

[3] Tolutope O., F., Adeniyi M. M. and Aremu M. (2016). Understanding the Queuing Theory for Improved Service Delivery: An Entrepreneurial-Mindset Approach. Ethiopian Journal of Business and Economics, 34-47.

[4] Taylor, S. (1994). Waiting for Service: The Relationship between Delays and Evaluation of Service. Journal of Marketing, 56-69.

[5] Green, L. (2006). Queuing Analysis in Healthcare. Patient Flow: Reducing Delay in Healthcare delivery, 281-307.

[6] Rubinovitch, M. (1984). The Slow Server Problem: A Queue with Stalling. Discussion Papers 592, Center for Mathematical Studies in Economics and Management Science, Northwestern University.

[7] Sani, S. and Daman, O.A. (2015). The M/G/2 Queue with Heterogeneous Servers under a Controlled Service Discipline: Stationary Performance Analysis. IAENG International Journal of Applied Mathematics, 45(1), 1-12.

[8] Sivasamy, R. Daman, O.A. and Sani, S. (2015). An M/G/2 where Customers are served Subject to a Minimum Violation of the FCFS Queue Discipline. European Journal of Operational Research, 240, 140-146.

[9] Cynthia E. and Frank K. D. (2011). Reward as an Employee Tool for Employee Performance. M.Sc. Thesis submitted to the School of Management, Tekniska Hokskola Blenkinge Bth, 1-78.

[10] Stajkovic, A. D., and Luthans, F. (2003). Behavioral Management and Task Performance in Organizations: Conceptual Background, Meta-Analysis, and Test of Alternative Models. Personnel Psychology 56(1), 155-194.

[11] Waugh, C. E. and Gotlib, I. H. (2008). Motivation for reward as a function of required effort: Dissociating the 'liking' from the 'wanting' system in humans. In Motivation and Emotion, 32, 323-330.

[12] Ronald W. W. (1982). Poisson Arrivals See Time Averages. Operations Research, 30(2), 223-231.

[13] Shahbazov, V. S. (2007). Minimizing Loss Probability in Queuing Systems with Heterogeneous Servers. Iranian Journal of Science and Technology, 199-206.

[14] Strauch R.E. (1970). When a Queue Looks The Same to an Arriving Customer as to an Observer. Management Science, 17, 140-141.

[15] Laszlo, G., Attila, L. and Miklos, R. (2023). On the Strong Stability of Ergodic Iterations, https://doi.org/10.48550/arXiv.2304.04657.

[16] Ammar, S. I. (2014). Transient Analysis of Two-Heterogeneous Servers Queue with Impatient Behavior. Journal of the Egyptian Mathematical Society, 90-95.

[17] Madheswari, B. K. (2005). An M/M/2 Queuing System with Heterogeneous Servers and Multiple Vacations. Mathematical and Computer Modeling, 1415-1429.

[18] Cabral, F. B. (2005). The Slow Server Problem for Uniformed Customers. Queuing Systems, 353-370.

[19] Efrosinin, D. and Rykov, V. (2008). On Performance Characteristics for Queuing Systems with Heterogeneous Servers. Automation and Remote Control, 1, 64-82.

[20] Kalaiselvi, R. K. (2019). Heterogeneous Server Queue with a Threshold on Slow Server. International Journal of Recent Technology and Engineering, 2277-3878.

[21] Cynthia E. and Frank K. D. (2011). Reward as an Employee Tool for Employee Performance. M.Sc. Thesis submitted to the School of Management, Tekniska Hokskola Blenkinge Bth, 1-78.

[22] Ekaterina, P. and Svetlana, M. F. (2022). Infinite-Server Resource Queuing System with Different Poisson Process and Renewal Arrivals. Mathematics, 10, 2962-2978.

[23] Krishnamoorthi, B. (1963). On Poisson Queue with Two Heterogeneous Servers. Operations Research, 11(3), 321-330.

[24] Dharmaraja, S. and Rakesh K. (2015). Transient Solution of a Markovian Queuing Model with Heterogeneous Servers and Catastrophes. OPSERCH, 52(4), 810-826.

[25] Jean, M. O. (1985). Ergodic Theory and Statistical Mechanics. Lecture Notes in Mathematics, First Edition, LNM, volume 1115, Springer Berlin, Heidelberg publication.

[26] Medhi, J. (2003). Stochastic Models in Queuing Theory. 2nd Edition, Academic Press, California, USA.

[27] Gross, D. (2008). Fundamentals of Queuing Theory. New Jersey: John Wiley and Sons.

[28] Shakir, M. and Manoharan, P. (2019). Analysis of an M/M/1 Queue with Working Vacation and Vacation Interruption. Applications and Applied Mathematics: An International Journal (AAM), 14(1), 19-33.