Analysis of Two-phase Flow through a Rectangular Curved Duct

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Abstract

The present work mainly focuses on analyzing an unsteady laminar incompressible two-phase flow in a rectangular curved duct. The corresponding governing equations are represented by the Navier-Stokes equations and Level set equation with the boundary conditions. Fluid flow through curved rectangular ducts is influenced by the centrifugal action arising from duct curvature and has a unique behavior different from fluid flow through straight ducts. Centrifugal force-induced secondary flow vortices and produce spiraling fluid motion within curved ducts. This paper shows the vector plot of the field flow, velocity contours, axial flow velocity and fluid volume fractions visualization graphically. The effect of curvature, Dean number and aspect ratio is also displayed. A comparison of two-phase flow between different fluids is also shown. The results reveal that the unstable behavior of the flow reduces for the increasing values of curvature, Dean number, and high viscosity flow.

Keywords

Rectangular Curved Duct, Dean number, Two-phase flow, Finite element Method

1. Introduction

Multiphase flow has great importance in experimental research and has broad applications. Two-phase flow is essential in hydraulic conveying, liquid mixing, liquid separations, liquid extraction, steam generators, jet engines, condensers, and distillation processes in the pipeline. Xu et al. [18] experimentally studied Gas-liquid two-phase flow regimes in rectangular channels with mini/micro gaps. Crandall et al. [19] compared experimental results and the numerical result of two-phase flows in a porous micro-model. Garg et al. [20] and Jason [21] investigated the fully developed flow field of two vertically stratified fluids in a curved channel of the rectangular cross-section. A numerical investigation of two-phase flows through the enhanced microchannel was investigated by Chandra et al. [22]. Al-Jibory et al. [23] discussed an experimental and numerical study for two-phase (water-air) in rectangular ducts with compound tabulators. Okechi and Asghar [24] studied two-phase flow in a groovy curved channel.

The Level Set Method (LSM) is a useful tool in physics, engineering, materials science, computer graphics, and beyond. Phase transformations and multiphase flow have been the most commonly used of the level set method. Osher and Sethian [25] introduced LSM that creates new algorithms for following fronts propagating with curvature-dependent speed derived from the Hamilton Jacobi equation. Furthermore, this level set approach has been applied to incompressible two-phase flow since the article of Sussman et al. [26]. Olsson and Kreiss [27] discuss the conservative LSM two-phase flow. Datta et al. [28] studied analytical, and level set method method-based study for two phases stratified flow in a plane channel and a square duct. LSM for computational multi-fluid dynamics was studied by Sharma [29].

From the open literature, the numerical investigation of two-phase flow with gravity through the curved duct has not been found yet. This paper uses the finite element method to solve Navier-Stokes equations with boundary conditions in the above problem. The vector plot of the field flow, velocity contour, the volume fraction of fluid on the domain for different times are depicted to understand the effect curvature, Dean number, aspect ratios, and particle concentration on each domain. The results are also compared for different fluids in the outer domain.

2. Mathematical Model

A laminar viscous incompressible unsteady three-dimensional two-phase flow is considered here. The flow passes through a curved duct with a rectangular cross-section. The height and the width of the cross-section are \( h \) (m) and \( d \) (m) respectively. For consideration \( h = 4 \) m and \( d = 4 \) m are taken fixed for square duct. Let \( O \) is the center of the curvature and \( L \) (m) is the radius of curvature of the duct shown in Figure 1. The analysis uses a mixture of water and Engine oil as the immiscible working fluid, which is sustained together into the curved channel path. In the curved channel inlet, Engine oil enters the outer domain, and water enters the inner domain with different velocities. It is considered that though the inlet velocity is different the Reynolds number remains the same for the fluid in both domains. All physical properties of the assumed fluids are constant.

![Figure 1. (a) Co-ordinate system, (b) cross sectional view.](image)

The mathematical model can be expressed by the governing equations as follows Kucuk [30] and Gyves [31]:

Continuity equation:

\[
\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0
\]  

(1)

Momentum equations:

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} - \frac{w^2}{x+L} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nabla \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{1}{x+L} \frac{\partial u}{\partial x} - \frac{u}{(x+L)^2} \right]
\]  

(2)
\[
\frac{\partial}{\partial t} \mathbf{v} + \mathbf{u} \cdot \nabla \mathbf{v} = -\frac{\partial p}{\partial y} + \nabla \left[ \frac{\partial^2 v}{\partial x^2} + \frac{1}{x+L} \frac{\partial v}{\partial x} + \frac{\partial^2 v}{\partial y^2} \right] \tag{3}
\]

\[
\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = -\frac{1}{\rho} \left( \frac{\partial p}{\partial x} + \frac{\partial w}{\partial x} + \frac{\partial^2 w}{\partial y^2} + \frac{1}{x+L} \frac{\partial w}{\partial x} - \frac{w}{(x+L)^2} \right) - g \tag{4}
\]

where \( u, v, \) and \( w \) are velocity components in \( x, y \) and \( z \) directions, respectively, \( \rho \) is density, \( v \) is kinematic viscosity, and \( L \) is Radius of curvature. The model neglects all terms of the order \( \frac{1}{L} \) and \( \frac{1}{L^2} \), except the centrifugal force term as in Gyves [32].

The boundary conditions at the channel and core walls and for inlet & outlet,

\[ (u, v, w) = 0 \quad \text{at } r = L, \quad r = L + d, \quad h = 0, \quad \text{and } h = h \tag{5} \]

At the inlet-1 \[ u = u_1 \bar{n} \]

and at the inlet-2 \[ u = u_2 \bar{n} \]

on the outlet \[ P = P_0 \] \tag{6}

The Dean number is typically denoted by

\[ De = Re \left( \frac{d}{L} \right)^{1/2} \tag{7} \]

Where \( Re \) is the Reynolds number, \( d \) is a typical length scale associated with the channel cross-section, \( L \) is the radius of curvature of the path of the channel.

Where Reynolds number \( Re \) is defined by

\[ Re = \frac{\rho du}{\mu} \tag{8} \]

Where \( \rho \) and \( \mu \) are density and dynamical viscosity of the fluid. Since the governing equations are non-dimensional and \( \rho, d \) and \( \mu \) are considered constant, so Dean number \( (De) \) as well as Reynolds number \( (Re) \) depend on value of \( u \).

The level set function \( \emptyset \) can be represented by the following equation: Olsson et al [33].

\[ \frac{\partial \emptyset}{\partial t} + \mathbf{u} \cdot \nabla \emptyset = \gamma \nabla \cdot \left( \epsilon \nabla \emptyset - \emptyset (1 - \emptyset) \frac{\nabla \emptyset}{|\nabla \emptyset|} \right) \tag{9} \]

Where \( \mathbf{u} \) is the fluid velocity. The \( \epsilon \) parameter determines the thickness of the layer of the interface. The \( \gamma \) parameter determines the amount of reinitialization and \( \emptyset \) is the level set function varies from zero to one. For Engine oil \( \emptyset = 0 \) and for water \( \emptyset = 1 \).

The level Set function \( \emptyset \) is defined by

\[ \emptyset(x) = \begin{cases} 0 & x \in \text{phase} - 1 \\ 1 & x \in \text{phase} - 2 \end{cases} \tag{10} \]

For calculating surface tension, the interface normal and curvature are obtained according to the sign function

\[ \bar{n} = \frac{\nabla \emptyset}{|\nabla \emptyset|} \text{ at } \emptyset = 0 \quad \text{and} \quad k = \nabla \cdot \frac{\nabla \emptyset}{|\nabla \emptyset|} \text{ at } \emptyset = 0 \tag{11} \]

The level set function is used to determine the density and dynamic viscosity globally by

\[ \rho = \rho_{Eo} + (\rho_w - \rho_{Eo}) \emptyset \tag{12} \]
\[ \mu = \mu_{E_0} + (\mu_w - \mu_{E_0}) \varnothing \]  

where \( \rho_{E_0}, \rho_w, \mu_{E_0} \) and \( \mu_w \) are the density and dynamic viscosity of engine oil and water respectively.

3. Numerical Solution

The finite element method is a numerical technique used to solve the problem. The assembly of finite elements discretizes a domain of interest. Approximating functions in finite elements are determined in terms of nodal values of a physical field. A continuous physical model is transformed into a discretized finite element model with the help of unknown nodal values. We can recover values inside finite elements using the nodal values.

The Finite element meshing of the computational Domain is displayed in Figure 3. A grid refinement test has been performed until the results show insignificant change for further refined mesh size.

![Figure 3. Mesh generation of the 3D domain.](image)

<table>
<thead>
<tr>
<th>Mesh Size</th>
<th>Extremely Coarse</th>
<th>Extra Coarse</th>
<th>Coarse</th>
<th>Coarser</th>
<th>Normal</th>
<th>Fine</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of elements</td>
<td>2256</td>
<td>6448</td>
<td>18196</td>
<td>61314</td>
<td>131956</td>
<td>389530</td>
</tr>
<tr>
<td>Average Velocity</td>
<td>0.220908</td>
<td>0.143967</td>
<td>0.129722</td>
<td>0.106895</td>
<td>0.101056</td>
<td>0.101037</td>
</tr>
</tbody>
</table>

From the above Table 1, it is observed that results for average velocity magnitude have no significant change up to four decimal places for Normal and Fine mesh size. Therefore, Normal mesh size is chosen to find the grid independent solution and to save computational time.

Earlier, the solution of the vector plot of flow parameters in a square curved duct was established by Norouzi and Biglari [16]. They studied single-phase flow and used the perturbation method to solve Governing equation with the boundary condition, and their result is shown in Figure 4(a). In the present study, their result has been reproduced by using the finite element method, which is shown in Figure 4(b). It is observed that both results are almost the same. Therefore, the present numerical method is in good agreement with the work presented in [16].

![Figure 4. Comparison of numerical solution of vector plot of flow at Re = 50 and \( \delta = 0.1 \).](image)
4. Results and Discussion

In a curved duct, the centrifugal action manifests two key effects. It generates a positive radial pressure field directed towards the outer duct wall. The centrifugal force drives the fluid radially from the inner to the outer duct wall within the positive pressure field, setting up lateral fluid circulation called secondary flow. The secondary fluid motion becomes vigorous and the radial pressure field intensifies when axial flow increases. Assisted by fluid viscosity, this positive pressure field adversely affects the secondary fluid flow moving towards the outer duct wall to slow it down. Consequently, near the outer duct wall, a stagnant flow region is formed.

The solution for the unsteady laminar incompressible two-phase fluid flow through a three-dimensional rectangular curved channel has been displayed here. The results in terms of axial flow velocity, velocity contour and vector plot of flow field have been discussed for the various radius of curvature ($20 m \leq L \leq 100 m$), Dean number ($180 \leq De \leq 1620$), aspect ratio (1:1 to 1:6), particle concentration of outer domain ($0.2 \leq \varnothing \leq 1$) and several time steps ($0^{th}$s to $300^{th}$s). Also, six different fluids (Engine Oil, Kerosene, Ethylene glycol, Heptane, Glycerol, Transformer oil) have been tested in the outer domain. All the figures are taken at the cut plane of $YZ$-plane at $x = 0$.

Figure 5(a) shows volume fraction visualization for different moment. At time $t = 0^{th}$s the multiphase fluid situated at different domain. Engine oil enters the outer domain (blue colour), and water enters the inner domain (red colour) and yellow define the interface of the domains. After few time the multiphase flow will be mixed. High viscosity fluid will situated at lower portion and low viscosity fluid will be situated at upper portion. Also observed that at time $t = 60^{th}$s and $120^{th}$s the interface will become periodic and when time $t \geq 300^{th}$s the mixed fluid become steady state.

![Figure 5(a) Volume fraction visualization at different time](image1)

**Figure 5(a). Volume fraction visualization at different time.** When $De = 180$, $L = 40 m$ and $\varnothing = 0.0$ at outer domain for aspect ratio (1:1).

Figure 5(b) shows the velocity contour of flow. It is observed that at time $t = 60^{th}$s there are four additional vortex

![Figure 5(b) Velocity contour of flow](image2)
occurs inside of the main vortex locating on the upper region of the duct cross-section, it suggest that the Dean’s flow is in steady state. At time $t = 120^{th}$s and $240^{th}$s there are six to ten contours and also shows that the axial flow is shifted near the inner and outer wall of the duct. That is Dean’s flow is chaotic. But when time $t = 150^{th}$s, $180^{th}$s, and $300^{th}$s there are twelve to fourteen contours and also shows that the axial flow is scattered in both domain. That is Dean’s flow is also chaotic.

Figure 5(b). Velocity contour at different time. When $De = 180$, $L = 40m$ and $\varnothing = 0.0$ at outer domain for aspect ratio (1:1).

Figure 10(c) shows a vector plot of the flow field. It is observed that at time $t = 60^{th}$s there are a single vortex of solution positioned at centre of duct. At time $t = 120^{th}$s there are two vortexes of solutions for secondary flow and some parallel lines along the wall. Pair of symmetric vortex are in opposite direction. Also At time $t = 150^{th}$s there are two vortexes of solutions for secondary flow. Pair of symmetric vortex are in opposite direction.
At time $t = 180^{th}$ and $240^{th}$, there are four vortexes of solution for secondary flow. Each pair of vortexes are in opposite direction. It is also observed that at time $t \geq 300^{th}$, there are six symmetric vortex and pair of symmetric vortex solution are in opposite directions.

From Figure 5(d), it shows that at time $t = 0^{th}$ axial flow velocity is in straight line, because there have no velocity at this time. At time $t = 60^{th}$ and $240^{th}$ axial flow velocity of mixed fluid is in hyperbolic shape and create multiple orbit and velocity of low viscosity fluid is higher than high viscosity fluid. Also observed that at time $t = 120^{th}, 180^{th}$ and $300^{th}$ axial flow velocity of mixed fluid is in hyperbolic shape and create only two orbit and velocity of low viscosity fluid is higher than high viscosity fluid.
Figure 6(a) demonstrates the effect of radius of curvature on velocity contour. When the radius of curvature $L = 20m$ and $40m$ it shows that there are eight to ten contours and that the axial flow is shifted near the outer wall of the duct. That is Dean’s flow is chaotic. But when $L = 60m$ there are four to six contours, it also shows that the axial flow is shifted near the duct’s outer wall. That is Dean’s flow is also chaotic. Again when $L = 80m$ contours are the tendency to gather in centers are seen and when $L = 100m$ there are four additional contour occurs inside of the main contour locating on the upper region of the duct cross-section, it suggest that the Dean’s flow is in steady state. Comparing with result, when $L = 200m$, additional contour are vanishing inside of the principal contour. Which shows that Dean’s flow behavior is becoming straight closed duct.

![Figure 6(a). Effect of radius of curvature on velocity contour. When $De = 180, t = 300ths$ and $\varnothing = 0.0$ at outer domain for aspect ratio (1:1).](image)

From figure 6(b) shows the effect of radius of curvature on vector plot of flow field. when $L = 20m$ and $40m$ there have six symmetric vortex solutions for secondary flow. Each pair of vortex are in opposite direction. When $L = 60m$ there are three asymmetric vortexes solutions and for $L = 80m$ there are two symmetric vortexes solution for secondary flow. And each pair of vortexes is in opposite direction. But when $L = 100m$ there is a single vortex solution.
Figure 6(b). Effect of radius of curvature on vector plot of flow field. When $D_D = 111, t = 300^{th}$ and $\emptyset = 0.0$ at outer domain for aspect ratio (1:1).

It can also say that when $L \geq 100m$ only single vortex are found oscillate on the centerline. Which shows that the flow behavior will become like a parallel channel. Comparing with result, when $L = 200m$ vortex solution are becoming vanishes.

From Figure 6(c), it shows that the effect of radius of curvature on axial flow velocity. When $L = 20m$ axial flow velocity of mixed fluid is in hyperbolic shape and create multiple orbit and velocity of high viscosity fluid is higher than low viscosity fluid. When $L = 40m$ axial flow velocity of mixed fluid is in hyperbolic shape and create two orbit and velocity of low viscosity fluid is higher than high viscosity fluid. When $L < 60m$, centrifugal force was high and $L \leq 60m$ fully depoled flow is created.
Again when $L = 60 \text{m}, 80 \text{m}$ and $100 \text{m}$ axial flow velocity of mixed fluid is in hyperbolic shape and create two orbit and velocity of low viscosity fluid is higher than high viscosity fluid. It also observed that axial velocity profile is increases due to inceases of radius of curvature. Comparing with result, when $L = 200 \text{m}$ axial flow velocity is hyperbolic shape with two orbit and velocity of high viscosity fluid is higher than low viscosity fluid.

![Figure 7(a). Effect of Dean number on velocity contour. When $L = 40 \text{m}, t = 300^{th}$s and $\emptyset = 0.0$ at outer domain for aspect ratio (1:1).](image)

Dean number effects on velocity contour shown in Figure 7(a). From figure, when $De = 45$ and $180$ there are eight to ten contours and that the axial flow is shifted near the outer wall of the duct. That is Dean’s flow is strongly chaotic. When the Dean number $De = 540$ and $900$ it shows that there are four to six contours and that the axial flow is shifted near the outer wall of the duct. That is Dean’s flow is chaotic. But when the Dean number $De = 1260$ and $1620$ it shows that there are two to four contours and that the axial flow is shifted near the inner wall of the duct. That is Dean flow is becoming steady state.

From Figure 7(b) when $De = 45, 180, 540$ and $900$ there are six symmetric vortexes solution for secondary flow and each pair of vortex are in opposite direction. Similarly when $De = 1260$, there are two symmetric vortexes of solution and two asymmetric vortexes of solution for secondary flow and each pair of vortex are in opposite direction. Again for $De = 1620$, there are a single vortex solution position in the inner domain.
Figure 7(b). Effect of Dean number on vector plot of flow field. When $L = 40m, t = 300^{th}s$ and $\theta = 0.0$ at outer domain and for aspect ratio (1:1).

Dean number effects on axial flow velocity is shown in Figure 7(c). Axial flow velocity of mixed fluid is in hyperbolic shape and create multiple orbit and velocity of high viscosity fluid is higher than low viscosity fluid for $De = 45$ and 180. Also shows that velocity of low viscosity fluid is higher than high viscosity fluid for $De = 45$ but velocity of high viscosity fluid is higher than low viscosity fluid for $De = 180$. When $De = 540$ axial flow velocity is zigzag line and velocity of high viscosity fluid is higher than low viscosity fluid. Again when $De = 900, 1260$ and 1620 axial flow velocity is in curved line shape and velocity of low viscosity fluid is higher than high viscosity fluid.

Figure 7(c). Effect of Dean number on axial flow velocity. When $L = 40m, t = 300^{th}s$ and $\theta = 0.0$ at outer domain and for aspect ratio (1:1).

Figure 8 depicts a comparison among different fluids for the same velocity inlet on two domains for axial flow velocity,
velocity contour and vector plot of flow field on the cut plane. Six different fluids; engine oil, kerosene, ethylene glycol, heptane, transformer oil and glycerol are used in the outer domain with water in the inner domain. Comparing of two-phase flow between different fluids, it is observed that the viscosity of kerosene is very low, and the viscosity of glycerol is very high viscosity (Table 1). Figure 8(a) shows that the mixed fluid flow for water-heptane, water-kerosene, and water-engine oil creates eight to ten contours and the axial flow is shifted near the duct’s inner and outer wall. That is Dean’s flow is chaotic. Again water glycerol, water-transformer oil creates four to six and shows that the axial flow is shifted near the center of the duct. That is Dean's flow is also chaotic. Finally, water-ethylene glycol creates only two contours and shows that the axial flow is shifted near the top and bottom of the duct. That is Dean's flow is becoming steady state.

![Figure 8(a). Comparison among different fluid for velocity contour. When $\text{De} = 180, \phi = 0.0, t = 300^\circ \text{s and } L = 40\text{m at outer domain. For aspect ratio (1:1).}]

From Figure 8(b) there are six symmetric vortexes of solution for for water-heptane, water-kerosene, water-glycerol, water-transformer oil and water-engine oil mixed fluid and each pair of vortexes are in opposite direction. Reversely water-ethylene glycol have two symmetric vortexes and two asymmetric vortexes. Also shows that symmetric vortexes are in opposite direction.
Figure 8(b). Comparison among different fluid for vector plot of flow field. When $\text{De} = 180, \phi = 0.0, t = 300^{th}$s and $L = 40m$ at outer domain. For aspect ratio (1:1).

Figure 8(c) it shows that mixed fluid flow for water-heptane, water-kerosene, water-glycerol and water-engine oil is in hyperbolic shape having with multiple orbit. Built axial flow velocity of water-transformer oil mixed fluid is in curved line. Also shows that velocity of high viscosity fluid is higher than low viscosity fluid for water-heptane, water-kerosene, water-glycerol, water-transformer oil and water-engine oil. Reversely the mixed flow for water ethylene-glycol also in hyperbolic shape having with only two orbit and velocity of low viscosity fluid is higher than high viscosity fluid.

Figure 8(c). Comparison among different fluids for axial flow velocity. When $\text{De} = 180, \phi = 0.0, t = 300^{th}$s and $L = 40m$ at outer domain. For aspect ratio (1:1).

The effect of aspect ratio on axial flow velocity are shown in Figure 9(a). From Figure 9(a), shows that for aspect ratio (1:1), (1:2), (1:3), (1:4), (1:5) and (1:6) are in hyperbolic shape having with multiple orbit. Also observed that axial ve-
locity for high viscosity fluid is higher than low viscosity fluid.

Figure 9(a). Aspect ratio effect on axial flow velocity. When \( L = 40m, De = 180, t = 300^{th} \text{s} \) and \( \emptyset = 0.0 \) at outer domain.

In Figure 9(b), the aspect ratio effect on velocity contour is obtained. When aspect ratio (1:1) and (1:2), it shows that there are six to eight contours and that the axial flow is shifted near the outer of the duct. That is Dean’s flow is chaotic. For aspect ratio (1:3) and (1:4), it shows that there are two to four contours and that the axial flow is shifted near the center of the duct channel. That is Dean’s flow is also chaotic. But when the aspect ratio is from (1:5) and (1:6), it shows that there are only two contours and that the axial flow is shifted near the center of the duct channel and the inner wall. That is, Dean's flow is becoming steady state. Because fluid flow gets enough space to flow, the centrifugal force drives the fluid radially from the inner to the outer duct wall, setting up lateral fluid circulation.

In figure 9(c), the aspect ratio effect on the vector plot of the flow field is obtained. When aspect ratio (1:1) and (1:2), there is four symmetric vortexes solution for secondary flow and each pair of vortex are in opposite direction. When aspect ratio (1:3) and (1:4) there are two symmetric vortexes and two asymmetric vortexes of solution for secondary flow. Symmetric vortex pair are in opposite direction and direction of asymmetric pair of vortices are in normal to the upper wall. Similarly, when aspect ratio (1:5) and (1:6), there are two asymmetric vortices for secondary flow placed on the upper and lower wall, and their direction is opposite.

The velocity magnitude of the average surface on a cut plane for the effect of curvature, Dean number, particle concentration and comparison among different fluid is shown in figure 10. The figure 10(a) shows the effect of curvature. It shows that velocity profile is increasing after increasing of radius of curvature. Also shows that for small radius of curvature the velocity profile is periodic and too large radius of curvature the velocity behavior is like the straight channel. Figure 10(b) shows the effect of Dean number on the velocity magnitude of average surface on the cut plane that when dean number increases the velocity is also increased. Because as the dean number increases the inlet velocity is also increasing, affecting the velocity profile. The velocity magnitude of the average surface on cut plane for the effect of particle concentration \( \emptyset = 0.0 \) and 1.0, there have low velocity. And for \( \emptyset = 0.2 \) and 0.8, the flow have high velocity. But at the range \( 0.4 < \emptyset < 0.6 \) the flow behavior becoming like straight duct. Figure 10(d) shows that the velocity magnitude of the average surface on the cut plane increases due to decreasing of viscosity.
Figure 9(b). Aspect ratio effect on velocity contour. When $L = 40m$, $De = 180$, $t = 300^{th}$s and $\phi = 0.0$ at outer domain.
Figure 9(c). Aspect ratio effect on vector plot of flow field. When $L = 40m$, $De = 180$, $t = 300^{th}s$ and $\emptyset = 0.0$ at outer domain.
5. Conclusions

A three-dimensional two-phase fluid flow passes through a curved duct with square and rectangular cross-sections was investigated numerically. The major findings of the current study are given as follows:

- Increasing the radius of curvature, the number of Contour and Dean vortex decreases, and Dean flow becomes chaotic to steady state. For $L \geq 100$, the flow become steady. The axial flow velocity becomes multi-periodic to periodic after increasing of radius of curvature.

- Small Dean number causes a large number of Contour and Dean vortex while increasing Dean number of Contour and Dean vortex decreases and Dean flow becomes chaotic to steady state.

- Changing the aspect ratio from (1:1) to (1:6), Contour and Dean vortex becomes large number to small number that indicates the dean flow become chaotic to steady state.

- Comparison of two-phase flow between different fluids shows that secondary flow is a steadier for the fluid with large viscosity than that of small viscosity.
• As the time increases, the volume fraction of fluid becomes steady and high-density flow stays at the lower portion and low-density flow stays on the upper portion.
• When the curvature length is low, the velocity profile is low and velocity profile is periodic, after increasing length of curvature, the velocity profile also increases. Also, for a large curvature length, the velocity behavior is like a straight channel.
• Due to increasing of the dean number the velocity is also increasing.
• High-velocity magnitude of the average surface of a cut plane is found for particle concentration at $\emptyset = 0.2$ and $0.8$ and it becomes steady at the range $0.4 \leq \emptyset \leq 0.6$.
• The velocity magnitude of the average surface of the cut plane increases due to increasing viscosity.

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