Engineering Advances, 2023, 3(4), 321-327
https://www.hillpublisher.com/journals/ea/
ISSN Online: 2768-7961

Global Endeavor to Read the Black Hole—The Working Mechanisms of Event Horizon Telescope

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How to cite this paper: Zirui Yu. (2023) Global Endeavor to Read the Black Hole—The Working Mechanisms of Event Horizon Telescope. *Engineering Advances*, 3(4), 321-327.
DOI: 10.26855/ea.2023.08.009

Received: July 12, 2023
Accepted: August 9, 2023
Published: September 8, 2023

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Abstract

For centuries, human beings cooperated to make endeavors to reach for the full image of the galaxy. This dream has finally come real in the past decades—by constructing the Event Horizon Telescope. On May 12th, 2022, the image of Sagittarius A* is first observed and reveals the existence of a super massive black hole. This is the “masterpiece” of EHT. The most unique characteristic of this momentous engineering is the span of the project: over 8 countries engaged in data processing and image capturing jobs, just for obtaining valuable information about black holes from outer space. To uncover the secret of high-resolution picture-capturing ability and telecommunicating technology, getting familiar with optical interferometry and information processing is indispensable. From the basic concept of resolving images—Rayleigh Criterion—to the Fourier transform of electromagnetic wave, immense knowledge is involved in taking a picture of the black hole. This is why low-pass filters can sift out the higher frequencies out of the band.

Keywords

Black holes, Event Horizon Telescope, Interferometry, Angular Resolution Estimation, Rayleigh Criterion, Aperture Synthesis, Fourier Transform, Wavelength, Visibility, Noise, Wave Filter, Heaviside step-function

1. Brief Introduction to EHT (Event Horizon Telescope)

Event Horizon Telescope is a significant engineering that involved globe-spanning cooperation, from Hawaii to Spain and Arizona to Chile. The project of EHT consists of gathering the radio telescopes world-round by Very Long Baseline Interference (i.e. VLBI) technology. Allowing close coordination between telescopes, the separation of every radio telescope reaches one hundred thousand kilometers in attempt to accurately record data gained from detecting the galaxy. To be one major aim of EHT, it is an efficient tool for astronomers to delve into the operating pattern within the galaxy and for theoretical physicists to learn and verify the general relativity proposed by Albert. Einstein. In order to reach these objectives by collecting highly precise data, engineers and theoretical physicists in EHT Collaboration incessantly endeavor to increase the angular resolution ratio of EHT to a level that allows scientists to "get closer" to the effect of the event horizon [1].

EHT has gained many feats in assisting scientists to make breakthrough for astrophysics and theoretical physics. For instance, on April 10th 2019, the first black hole captured by EHT in this manner is a super massive one at the heart of a nearby galaxy, called Messier 87. This is an invaluable break to understand the principles of general relativity due to the astonishing capability of Messier 87 to distort the nearby gravitational fields [2].

2. Angular Resolution Estimation & Data Derivation

Taking a clear photo of M87, a black hole, with high-resolution level is very difficult due to the spinning dusts and celestial bodies around it [3]. Consequently, the traditional optical telescopes are "abandoned" in the project of EHT to capture the image of M87 because of their lack of capability to capture enough light from the radiation source. Instead, capturing the radiation with relatively higher frequency and transforming into electrical signal are still possible because of closeness of the wavelength and the mean separation of dusts particles [4].

There is a strong "ruler" for scientists to confirm whether the image or spectrum line in the case of M87 black hole can be resolved clearly—Rayleigh criterion. It brings a kind of quantitative standards to determine the ability of angular resolution of telescopes. Rayleigh criterion states that: if the angular separation of two images or spectrum lines equals to 1.22 times of the ratio of received wavelength and the diameter of the object lens of a telescope, the two can be just resolved. In a quantitative way, Rayleigh criterion can be expressed as:

$$\mathrm{\delta\theta} = 1.22\frac{\lambda}{\mathrm{D}}$$

When the angular separation, in degree, is smaller than the standard separation given by Rayleigh criterion, the two images or spectrum lines cannot be resolved. Vice versa.

Back to the case of EHT, M87 has a distance of approximately 5,5000 thousand ly (light year) from the Earth, namely 5.203×10^{23}m. Meanwhile, Sgr A* has a distance of approximately 25 thousand ly from the Earth, namely 2.365×10^{20}m. To calculate the visual angle observed from the Earth by EHT, gaining an approximate value of the black holes radius is significant.

$$R_S = \frac{2GM}{c^2}$$

This is the simplest form of Schwarzschild radius. Once the radius of a planet is smaller than this radius, no object can escape from the surface of the planet, thus forming a black hole.

After gaining the masses of Sgr A* and M87 black holes through planet dynamics, astrophysicists predicted the angular sizes (diameters) of the event horizon of a black holes. By calculating the Schwarzschild radius of each, it can be calculated that the angular size is 20 mas for Sgr A* and 14 mas for M87 by constructing sectors and doing approximation.

Since the visual angles are confirmed if the two black holes don't move, what scientists can do to make the images of two images easy to resolve is to increase the diameter of EHT and decrease the received wavelength according to Rayleigh criterion. It is known that 1°=3.6×10^9mas. So 20 mas equals to approximately $\frac{20mas}{3.6\times10^9\mathrm{mas}}$=$5.6 \times 10^{-9}$degree and 14 mas equals to $\frac{14mas}{3.6\times10^9\mathrm{mas}}$=$3.9 \times 10^{-9}$degree. These are extremely small. In order to make the images easy to resolve, the standard angular separations determined by Rayleigh criterion need to be even smaller. Scientists chose to filter out all other wavelengths contained in the radiation from these black holes but remain the one with 1.3mm (227-229 GHz) and construct radio telescopes worldwide. The whole EHT is equivalent to a very huge telescope with the diameter of 10 thousand kilometers. According to Rayleigh criterion, the standard angular separation is:

$$\mathrm{\delta\theta} = 1.22\frac{\lambda}{\mathrm{D}} = 1.22\frac{1.3 \times 10^{-3}}{10000^3} = 1.6 \times 10^{-10\circ}$$

This value is smaller than the visual angle calculated above. So EHT is capable enough to resolve the images of two black holes.

3. Interferometry in EHT & Aperture Synthesis

The basic form of EHT is actually interferometer with the function of delaying the arriving wave front of radio wave in the spectrum of radiation from black holes. Being different from the traditional interferometer scientists use to observe the patterns of light strips, the basic idea of interferometry here is to gather the signal collected directly from the radiation source in every radio telescope and form constructive interference as much as possible via optical fiber or waveguide. Currently, the technique of interferometry is applied in astronomy to allow astronomers and astrophysicists to pick up details about celestial bodies. This technique can be an alternative choice to solve the problem that the angular resolution ability is not strong enough to distinguish images or spectrum lines because it can increase the intensity of signals by many times of constructive interference.

Before discussing the case of EHT, modeling a two-element interferometer is helpful for learning more about multi-element system.

It is assumed that the radiation source only conveys signal with single frequency and the difference in phase of every signal is unchanged with respect to time. According to image 5, two radio telescopes are connected with a radio correlator, which can be considered as a device that filters out the uncorrelated signals by identifying their frequencies. Due to the presence of phase difference τ_g, the amplitudes of two signals differ by $V_{R2} - V_{R1}$. Now just identify V_{R2} and

V_{R1} as uncorrelated quantities. So when two signals are combined in the cross-correlator, the collected amplitude can be expressed as [5]:

$$\langle V_1 \cdot V_2 \rangle = \langle (V_{R1}V_S + V_{R1}V_{R2} + V_S V_{R2} + V_S^2) \rangle = \langle V_S^2 \rangle$$

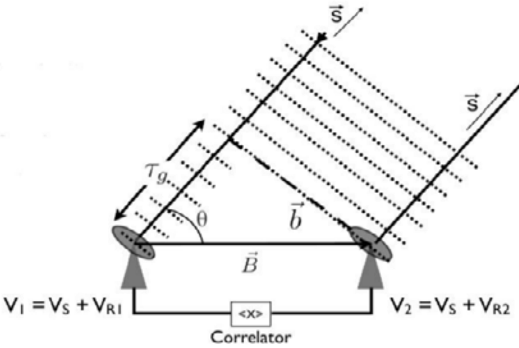

Figure 1. A simplified graph of equivalence of an ideal two-element interferometer.

There is fluctuation in the amplitudes of signals collected as the amplitude of signal is directly proportional to the electric field generated from each telescope.

$$V(t) \propto E \sin(\omega t) + N(t)$$

That is, the effective receiving area of a two-element system equals to that of one. If an interferometer consists of an array of N identical telescopes, then there are N (N-1)/2 possible independent two-element interferometers within the array [6].

$$A_{effective}^{interferometer} = \sqrt{A_1 A_2}$$

The information provided by a two-element interferometer system too limited. So we extend the range of the question to a N-element interferometer system, which is just the case of EHT. There are N (N-1)/2 combinations of two-element interferometer systems. Applying VLBI directly is a fast and convenient way to gather the most information. EHT telescopes outputs can be combined to yield N (N-1)/2 unique baselines, and N (N-1) correlators can be combined to filter out noises.

4. Correlator Outputs & Interferometric Visibility & Complex Visibility

As introduced in the former section, correlators connect radio telescopes to combine their collection of useful signal from noise and jamming signals.

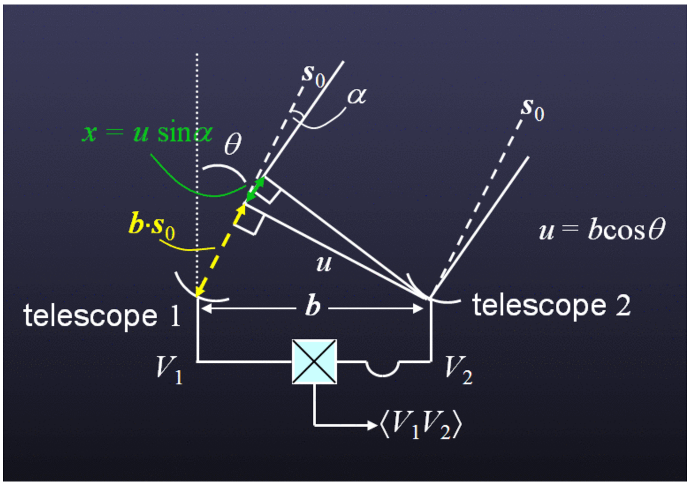

Figure 2. A two-element interferometer system using correlator to integrate the voltage signal.

Here, S_0 represents the direction of antennas. The radio wave is radiated at an angle of α with the extension line of antenna. According to the geometric relationship, the vertical distance between two antennas is expressed as u=b·cos (θ), where b is the distance between 2 telescopes. So it is easy to deduce that the vertical distance between two rays is x=u·sin (α). In order to simplify the expressions later, designate l=sin (α). Expand to 2D by introducing β orthogonal to α, m = sin (β), and v orthogonal to u, so that in this direction the extra path y = vm.

In this way, V_1 and V_2 can be connected by exponential function (or by Eulers expansion):

$$V_2 = V_1 e^{-2\pi i(ul+vm)}$$

where (ul+vm) equals to the total path difference.

The output of the correlator is defined as the multiplication of two fluctuating voltage signals and taking time average. V_1 and V_2 are respectively the function of l and m [7]. Thus,

$$C = \langle V_1 V_2 \rangle = \left\langle \iint V_1(l,m)\,dl\,dm \iint V_2(l,m)\,dl\,dm \right\rangle$$

Equivalently, V_1 and V_2 are sinusoidal functions. Once they are multiplied and took average, the terms with different m and different l will be eliminated. Finally, there will only be one term left with exactly same l and m. Since it is the square of amplitude of voltage, it is proportional to the intensity of the voltage signal. So the following substitution is deduced [8]:

$$\mathrm{C} = \iint \mathrm{I(l,m)e^{-2\pi i(vm+ul)}dl\,dm}$$

This is the actual output of one set of correlator. In actual place, there will be a set of logic gates to restrict the output, which is not included in the discussion.

As introduced before, the amplitude of signal is directly proportional to the electric field generated from each telescope. So:

$$\mathrm{V_1 = E\cos[\omega(t-t_g)]}$$

$$\mathrm{V_2 = E\cos[\omega t]}$$

As these voltage signals enter the correlator, they will first be multiplied and second be averaged. It turns out to have the form of $R_c = P\cos(\omega t_g) = P\cos\left(2\pi \frac{\boldsymbol{b}\cdot\boldsymbol{s}}{\lambda}\right)$, where P is called power coefficient.

Back to the case of EHT, one main job of it is to provide enough information to determine the visibility of the signal diagrams and images. For correlators between EHT telescopes, they are called complex if they produce sinusoidal fringes. Still, complex visibility can be defined from two separated correlators with output of R_1 and R_2. So complex visibility has the form of subtraction:

$$\tilde{v} = R_1 - iR_2$$

Define two more physical quantities: phase error (ϕ) and amplitude error (A).

$$\mathrm{A} = \sqrt{\mathrm{R_1^2 + R_2^2}}$$

$$\phi = \tan^{-1}\left(\frac{\mathrm{R_2}}{\mathrm{R_1}}\right)$$

It is shown that the correlator can both measure the real and imaginary parts of the complex visibility. On a complex plane, the phase error and amplitude error can be both shown [9].

As same as the interferometric visibility, the complex visibility is related to the intensity of radiation sources as well. The differences exist in the specific relationship. According to van Cittert-Zernike theorem, the relationship between complex visibility and the light source can be obtained from doing Fourier transform because these two are Fourier transform pair. To be more specific:

$$\mathcal{V}(u,v) = \iint I(l,m)e^{-2\pi i(ul+vm)}dl\,dm$$

$$I(l,m) = \iint \mathcal{V}(u,v)e^{2\pi i(ul+vm)}du\,dv$$

Being similar to the content in Fourier Optics, the mechanisms of correlator to process signals and connect complex visibility and intensity together can both be explained by Fourier transform. Intensity of the radiation and the complex visibility can both be abstracted to be two separate but conjugate planes [10].

5. Signal Analysis & Wave Filters in Radio telescopes

Signals collected by different telescopes of EHT need to be analyzed together. The general form of transmission of informational signals is by varying current. Consider an electrical conductor along which is sent a varying current. The potential difference between two terminals of terminating impedance of one ohm is a time-depending function with sinusoidal fluctuation. The time average of the voltage difference is:

$$\left\langle \mathrm{V(t)} \right\rangle = \frac{1}{2\mathrm{T}}\int_{-\mathrm{T}}^{\mathrm{T}} \mathrm{V(t)dt}$$

The mean value of the voltage difference is taken in two complete periods. As introduced before, the power delivered by the voltage signal has similar form [11].

To process a broad range of random signals to gain useful data, a standard of measuring the signals power content versus frequency is needed. Power Spectral Density can be used to reach this. PSD is a useful tool to reflect the characteristics of a set of broad random signals and to digitalize the signals. PSD can be defined to be:

$$\frac{|\mathrm{C(f)}|^2}{2\mathrm{T}} = \mathrm{G(f)}$$

where C (f) is the Fourier transform of V(t). By applying Wiener-Khinchine theorem it can be proved that the function of PSD is also the Fourier transform of the autocorrelation of a signal [12].

During the transmission of signals between radio telescopes, noises will be produced all the time. As defined, the autocorrelation of every signal in an array is zero for noise since the randomness of signals that convey no resultant information. In practice, noise can be processed by wave filter with setting of low-pass or high-pass.

Wave filters always compose of capacitors, inductances and resistors. They do only two things: shift the phase of input signals and attenuates the amplitudes. Wave filters work by the selective mechanism of inductances and capacitors: the restriction of alternating current signals and high-frequency signals. The frequency-dependence of the impedance of the wave filter can be expressed by filter function Z (f) [13].

$$\mathrm{Z(f)} = \frac{\mathrm{V_o}}{\mathrm{V_i}} = \mathrm{A(f)e^{i\Phi(f)}}$$

where V_o and V_i are analytic representations of input and output of voltage signals, so they contain the information of change in both amplitudes and phase shifts.

This filter function can be used to determine the best ratio of transmitted noise and signals. After multiplying the filter function into the PSD function with the frequency spectrum, the PSD function with filter mechanism becomes:

$$\frac{|\mathrm{C(f)Z(f)}|^2}{2\mathrm{T}} = \mathrm{G(f)}$$

Suppose that noise is mixed in an array of transmitted signals. The PSD for noise is:

$$\frac{|\mathrm{N(f)}|^2}{2\mathrm{T}}$$

If the noise is white noise, the frequency spectrum N (f) is constant for every wave band. So PSD for noise is constant, say equal to A. So the transmitted noise power is:

$$\frac{\mathrm{A}}{2\mathrm{T}}\int_{-\infty}^{\infty} |\mathrm{Z(f)}|^2 \mathrm{df}$$

Consequently, the ratio of power of signal to that of the white noise is:

$$\frac{\mathrm{S}}{\mathrm{N}} = \frac{\int_{-\infty}^{\infty} |\mathrm{C(f)Z(f)}|^2\, \mathrm{df}}{\mathrm{A}\int_{-\infty}^{\infty} |\mathrm{Z(f)}|^2 \mathrm{df}}$$

According to Schwartz’s inequality:

$$|f(x)g(x)|^2 \leq |f(x)|^2|g(x)|^2$$

It can be obtained that:

$$\left[\int_{-\infty}^{\infty} |\mathrm{C(f)Z(f)}|^2\, \mathrm{df}\right]^2 \leq \int_{-\infty}^{\infty} |\mathrm{Z(f)}|^2 \mathrm{df} \int_{-\infty}^{\infty} |\mathrm{C(f)}|^2 \mathrm{df}$$

So the conclusion for the greatest ratio of S and N power can be generalized if the filter function Z (f) has the same shape as the frequency content of the signal to be received.

Using low pass filter to process the voltage signals is a commonly used way to accomplish wave band selecting. The passage of some signals through the transmission tunnel once the antenna collects the radiation ray is virtually instantaneous. This is the quantitative representation of heaviside step-function, H (t). With a similar definition as δ function, H(t)=0 when t<0 and H(t)=1 when t>0.

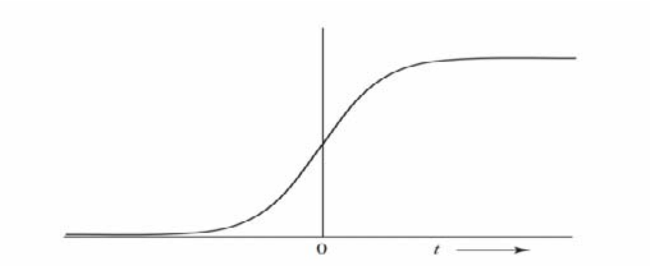

Figure 3. Normal heaviside step-function.

This is the simplified diagram for the normal heaviside step-function. It has the expression as: $\mathrm{H(t)} = \lim_{a\to 0}\left[\frac{1}{1+e^{-2t/a}}\right]$. It has no Fourier transform pair formed because it doesn't meet Dirichlet condition of square-integrable. Instead, constructing a substitution function with same effect is possible; it is called sgn (t):

$$\operatorname{sgn}(t) = \begin{cases} -1, & -\infty < t < 0, \\ +1, & 0 < t < \infty \end{cases}$$

To be more specific, another trial for getting a square-integrable function succeeds:

$$f(t) = \begin{cases} \lim\limits_{a\to 0} \dfrac{-(at+1)}{2}, & -1/a < x < 0, \\ \lim\limits_{a\to 0} \dfrac{(1-at)}{2}, & 0 < x < 1/a \end{cases}$$

This heaviside step-function has the graph like this:

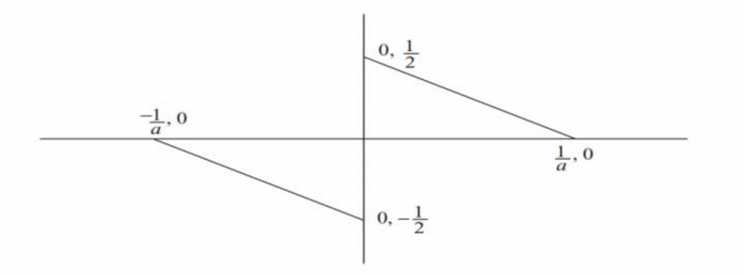

Figure 4. Heaviside step-function.

We can get the same result using a pair of exponentials:

$$H(t) = \begin{cases} \dfrac{1}{2} + \lim\limits_{a\to 0} \dfrac{1}{2}\left(e^{at} - 1\right), & -\infty < t < 0, \\ \dfrac{1}{2} + \lim\limits_{a\to 0} \dfrac{1}{2}\left(1 + e^{-at}\right), & 0 < t < \infty. \end{cases}$$

The reason for explaining much about heaviside step-function is that the "barrier" of the voltage in the low-pass filter is a heaviside step-function with respect to time. This is why low-pass filters can sift out the higher frequencies out of the band [15].

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