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A New Grey Prediction IANGM (1,1,k,k^2) Model

Xinyi Dong

School of Mathematics and Information, China West Normal University, Nanchong, Sichuan, China.

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*Corresponding author: Xinyi Dong, School of Mathematics and Information, China West Normal University, Nanchong, Sichuan, China.

Abstract

In this paper, a new grey prediction IANGM (1,1,k,k^2) model with quadratic time-varying function is constructed, which is suitable for approximate homogeneous exponential type, approximate non-homogeneous exponential type, exponential linear combination type, and exponential parabolic combination type characteristic sequence. The undetermined coefficient method is used to determine the shadow equation matching the model, and the time response formula of the IANGM (1,1, k,k^2) model is derived based on the constant variation method. Through numerical simulation and double modeling analysis of China's domestic heat consumption and soft soil foundation settlement data from 2011 to 2020, the results show that the IANGM (1,1,k,k^2) model has higher simulation and prediction accuracy.

Keywords

IANGM (1,1,k,k^2) model, parameter estimation, mean relative deviation, prediction

1. Introduction

Since Mr Deng Julong put forward the grey theory in 1982, the grey prediction model has been actively considered by many scholars and has been widely used in many fields [1-5]. The traditional grey modeling method is generally based on one-time accumulation (1-AGO) generation to model [6-8]. Due to the existence of different characteristics of data in real life, most of the models constructed by such methods cannot achieve better modeling results. Therefore, scholars have studied the optimization methods of the model from different angles and integrated the research methods: First, the improvement of the model, such as optimizing the grey derivative [9, 10], the grey action [11-13], etc; the second is the improvement of the sequence generation method, such as reverse accumulation [14-19]and one-time accumulation [20, 21]. These new optimization methods have achieved certain practical application results and further broadened the application scope of the grey model.

In [22], Xu Huafeng pointed out that the amount of grey action changes with time, however, the role of the time power term is to make the amount of grey action dynamic, that changes the traditional amount of grey action is an invariant constant. by introducing a correction term, to better reflect the objective law of data changes. Therefore, scholars have studied the addition of time power terms to effectively improve the accuracy of simulation and prediction. However, the modeling process is always based on the generation of 1-AGO, and the simulation and prediction of the data must be realized through the reduction reduction. This method is relatively complex. Therefore, a new grey modeling method appears, that is one-time accumulation generation (1-IAGO).

In [20], Wang Jian first proposed the definition of 1-IAGO, and established a new model based on 1-IAGO. The experimental results show that the model can achieve better prediction results. What's more, In [20], Chen Jing optimized this model, and proposes a new IANGM (1,1,k,k^2) model with a higher matching degree between the grey differential equation and shadow equation, which is suitable for approximate non-homogeneous exponential data, and enhances the adaptability of the model to data. It shows that 1-IAGO on modeling data will make the model simpler, and no longer through the cumulative reduction. It directly participates in the simulation and prediction of data.

Based on the above theoretical analysis, this paper proposes a new method to establish a grey model with a quadratic time power term by using 1-IAGO. The shadow equation matching the grey differential equation of the model is determined by the method of undetermined coefficient, and the time response equation of the model is obtained by derivative reduction solution. The properties and application scope of the model are explored. Finally, the new IANGM $(1,1,k,k^2)$ model is verified by numerical simulation and example application.

2. IANGM $(1,1,k)$ model

Definition 1 [21]. Assume that $X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n)\}$ is the original series of data; $X^{(-1)}$ is a decreasing sequence of $X^{(0)}$, $X^{(-1)} = \{x^{(-1)}(2), x^{(-1)}(3), \cdots, x^{(-1)}(n)\}$, where, $x^{(-1)}(k) = x^{(0)}(k) - x^{(0)}(k-1)$; $Z^{(0)} = \{z^{(0)}(2), z^{(0)}(3), \cdots, z^{(0)}(n)\}$, where, $z^{(0)}(k) = 0.5(x^{(0)}(k-1) + x^{(0)}(k))$, $k = 2, 3, \cdots, n$. the equation

$$x^{(-1)}(k) + az^{(0)}(k) = bk + c \tag{1}$$

is the basic form of the IANGM $(1,1,k)$ model.

Definition 2 [21]. The first-order differential equation

$$\frac{dx^{(0)}(t)}{dt} + ax^{(0)}(t) = b(t + \frac{1}{2}) + c \tag{2}$$

is defined as the whitening differential of the IANGM $(1,1,k)$ model.

Theorem 1 [21]. Let $\partial=[a,b,c]^T$ be the parameter column of the IANGM $(1,1,k)$ model, and

$$B = \begin{bmatrix} -z^{(0)}(2) & 2 & 1 \\ -z^{(0)}(3) & 3 & 1 \\ \vdots & \vdots & \vdots \\ -z^{(0)}(n) & n & 1 \end{bmatrix}, \quad Y = \begin{bmatrix} x^{(-1)}(2) \\ x^{(-1)}(3) \\ \vdots \\ x^{(-1)}(n) \end{bmatrix}$$

Then according to the theory of matrix and the least squares method, we have $\partial = (B^T B)^{-1} B^T Y$.

Theorem 2 [21]. The equation

$$\hat{x}^{(0)}(k) = (x^{(0)}(1) - \frac{3b + 2c}{2a} + \frac{b}{a^2})e^{-a(k-1)} + \frac{b}{a}k + \frac{b + 2c}{2a} - \frac{b}{a^2} \tag{3}$$

is called the time response function of the IANGM$(1,1,k)$ model.

3. A new grey prediction model IANGM $(1,1,k,k^2)$

3.1 IANGM $(1,1,k,k^2)$ model

Definition 3. $x^{(-1)}(k)$ and $z^{(0)}(k)$ are shown in Definition 1, and the equation

$$x^{(-1)}(k) + az^{(0)}(k) = bk^2 + ck + d \tag{4}$$

is the basic form of the IANGM $(1,1,k,k^2)$ model.

Theorem 3. The first-order differential equation

$$\frac{dx^{(0)}(t)}{dt} + ax^{(0)}(t) = b(t^2 + t + \frac{1}{6}) + c(t + \frac{1}{2}) + d \tag{5}$$

is defined as the whitening differential of the IANGM $(1,1,k,k^2)$ model.

Proof. Using the undetermined coefficient method to prove.

Assume the equation is

$$\frac{dx^{(0)}(t)}{dt} + ax^{(0)}(t) = mt^2 + nt + p \tag{6}$$

Where, m and n and p are undetermined coefficient.

For integrating Eq.(6) on the interval $[k-1,k]$, we can get

$$\int_{k-1}^{k}\frac{dx^{(0)}(t)}{dt}dt + a\int_{k-1}^{k}x^{(0)}(t)dt = m\int_{k-1}^{k}t^{2}dt + n\int_{k-1}^{k}tdt + p\int_{k-1}^{k}dt$$

Then

$$x^{(-1)}(k) + a\int_{k-1}^{k}x^{(0)}(t)dt = mk^{2} + (n-m)k + \frac{m}{3} - \frac{n}{2} + p \tag{7}$$

Comparing Eq.(4) and Eq.(7), we can get

$$\begin{cases} m = b \\ n - m = c \\ \dfrac{m}{3} - \dfrac{n}{2} + p = d \end{cases} \Rightarrow \begin{cases} m = b \\ n = b + c \\ p = \dfrac{b}{6} + \dfrac{c}{2} + d \end{cases} \tag{8}$$

We substitute Eq.(8) into Eq.(6), and finally get Eq.(5).

3.2 Parameter estimation of IANGM (1,1, k , k^2) model

Theorem 4. Let $\partial = [a, b, c, d]^{T}$ be the parameter column of IANGM(1,1, k , k^2) model, and

$$B = \begin{bmatrix} -z^{(0)}(2) & 4 & 2 & 1 \\ -z^{(0)}(3) & 9 & 3 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ -z^{(0)}(n) & n^{2} & n & 1 \end{bmatrix}, \quad Y = \begin{bmatrix} x^{(-1)}(2) \\ x^{(-1)}(3) \\ \vdots \\ x^{(-1)}(n) \end{bmatrix} \tag{9}$$

Then according to the theory of matrix and the least squares method, we have $\partial = \left(B^{T}B\right)^{-1} B^{T}Y$.

Proof. Substituting the values of k into Eq.(1), we can get

$$\begin{cases} x^{(-1)}(2) + az^{(0)}(2) = b\cdot 2^{2} + c\cdot 2 + d \\ x^{(-1)}(3) + az^{(0)}(3) = b\cdot 3^{2} + c\cdot 3 + d \\ x^{(-1)}(4) + az^{(0)}(4) = b\cdot 4^{2} + c\cdot 4 + d \\ \vdots \\ x^{(-1)}(n) + az^{(0)}(n) = b\cdot n^{2} + c\cdot n + d \end{cases}$$

Constructing the error square sum function: $f(a,b,c,d) = \sum_{k=2}^{n}(x^{(-1)}(k) + az^{(0)}(k) - bk^{2} - ck - d)^{2}$.

In order to minimize the value of this function, where, a and b and c and d must be established:

$$\begin{cases} \dfrac{\partial}{\partial a} f(a,b,c,d) = 2\sum_{k=2}^{n}[x^{(-1)}(k) + az^{(0)}(k) - bk^{2} - ck - d]z^{(0)}(k) = 0 \\ \dfrac{\partial}{\partial b} f(a,b,c,d) = -2\sum_{k=2}^{n}[x^{(-1)}(k) + az^{(0)}(k) - bk^{2} - ck - d]k^{2} = 0 \\ \dfrac{\partial}{\partial c} f(a,b,c,d) = -2\sum_{k=2}^{n}[x^{(-1)}(k) + az^{(0)}(k) - bk^{2} - ck - d]k = 0 \\ \dfrac{\partial}{\partial d} f(a,b,c,d) = -2\sum_{k=2}^{n}[x^{(-1)}(k) + az^{(0)}(k) - bk^{2} - ck - d] = 0 \end{cases}$$

Then

$$\begin{cases} -\sum_{k=2}^{n} x^{(-1)}(k)z^{(0)}(k) = a\sum_{k=2}^{n} [z^{(0)}(k)]^2 - b\sum_{k=2}^{n} k^2 z^{(0)}(k) - c\sum_{k=2}^{n} kz^{(0)}(k) - d\sum_{k=2}^{n} z^{(0)}(k) \\ \sum_{k=2}^{n} x^{(-1)}(k)k^2 = -a\sum_{k=2}^{n} z^{(0)}(k)k^2 + b\sum_{k=2}^{n} k^4 + c\sum_{k=2}^{n} k^3 + d\sum_{k=2}^{n} k^2 \\ \sum_{k=2}^{n} x^{(-1)}(k)k = -a\sum_{k=2}^{n} z^{(0)}(k)k + b\sum_{k=2}^{n} k^3 + c\sum_{k=2}^{n} k^2 + d\sum_{k=2}^{n} k \\ \sum_{k=2}^{n} x^{(-1)}(k) = -a\sum_{k=2}^{n} z^{(0)}(k) + b\sum_{k=2}^{n} k^2 + c\sum_{k=2}^{n} k + d(n-1) \end{cases}$$

The above equation can be obtained:

$$\begin{bmatrix} \sum_{k=2}^{n} (z^{(0)}(k))^2 & -\sum_{k=2}^{n} k^2 z^{(0)}(k) & -\sum_{k=2}^{n} kz^{(0)}(k) & -\sum_{k=2}^{n} z^{(0)}(k) \\ -\sum_{k=2}^{n} k^2 z^{(0)}(k) & \sum_{k=2}^{n} k^4 & \sum_{k=2}^{n} k^3 & \sum_{k=2}^{n} k^2 \\ -\sum_{k=2}^{n} kz^{(0)}(k) & \sum_{k=2}^{n} k^3 & \sum_{k=2}^{n} k^2 & \sum_{k=2}^{n} k \\ -\sum_{k=2}^{n} z^{(0)}(k) & \sum_{k=2}^{n} k^2 & \sum_{k=2}^{n} k & n-1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} -\sum_{k=2}^{n} x^{(-1)}(k)z^{(0)}(k) \\ \sum_{k=2}^{n} x^{(-1)}(k)k^2 \\ \sum_{k=2}^{n} x^{(-1)}(k)k \\ \sum_{k=2}^{n} x^{(-1)}(k) \end{bmatrix}$$

Therefore,

$$B^T Y = \begin{bmatrix} -z^{(0)}(2) & 2^2 & 2 & 1 \\ -z^{(0)}(3) & 3^2 & 3 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ -z^{(0)}(n) & n^2 & n & 1 \end{bmatrix}^T \begin{bmatrix} x^{(-1)}(2) \\ x^{(-1)}(3) \\ \vdots \\ x^{(-1)}(n) \end{bmatrix} = \begin{bmatrix} -\sum_{k=2}^{n} x^{(-1)}(k)z^{(0)}(k) \\ \sum_{k=2}^{n} x^{(-1)}(k)k^2 \\ \sum_{k=2}^{n} x^{(-1)}(k)k \\ \sum_{k=2}^{n} x^{(-1)}(k) \end{bmatrix}$$

$$B^T Y = \begin{bmatrix} -z^{(0)}(2) & 2^2 & 2 & 1 \\ -z^{(0)}(3) & 3^2 & 3 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ -z^{(0)}(n) & n^2 & n & 1 \end{bmatrix}^T \begin{bmatrix} x^{(-1)}(2) \\ x^{(-1)}(3) \\ \vdots \\ x^{(-1)}(n) \end{bmatrix} = \begin{bmatrix} -\sum_{k=2}^{n} x^{(-1)}(k)z^{(0)}(k) \\ \sum_{k=2}^{n} x^{(-1)}(k)k^2 \\ \sum_{k=2}^{n} x^{(-1)}(k)k \\ \sum_{k=2}^{n} x^{(-1)}(k) \end{bmatrix}$$

In summary, we can obtain: $\partial = [a, b, c, d]^T = (B^T B)^{-1} B^T Y$.

3.3 The time response function of IANGM (1,1,k,k^2) model

Theorem 5. Under the initial condition of $\hat{x}^{(0)}(1) = x^{(0)}(1)$, the time response function of IANGM (1,1,k,k^2) model is

$$\hat{x}^{(0)}(k) = C^* e^{-ak} + \frac{b}{a}k^2 + (\frac{b+c}{a} - \frac{2b}{a^2})k + \frac{b+3c+6d}{6a} - \frac{b+c}{a^2} + \frac{2b}{a^3} \tag{10}$$

Where, $C^* = e^a(x^{(0)}(1) - \frac{13b+9c+6d}{6a} + \frac{3b+c}{a^2} - \frac{2b}{a^3})$.

Proof. Using the constant variation method to prove.

Step 1: First, the general solution of the homogeneous equation corresponding to Eq.(5) is obtained.

According to Eq.(5), we can obtain a homogeneous equation: $\frac{dx^{(0)}(t)}{dt} + ax^{(0)}(t) = 0$, and Integrate both sides of it. Finally, We obtain the general solution of the homogeneous equation:

$$x^{(0)}(t) = c_1 e^{-at} \tag{11}$$

Step 2: We change the constant c_1 to $\eta(t)$, then compute $\eta(t)$.

Let $\eta(t) = c_1$, and Eq.(11) is transformed into $x^{(0)}(t) = \eta(t)e^{-at}$, we can get

$$\frac{dx^{(0)}(t)}{dt} = \eta'(t)e^{-at} - a\eta(t)e^{-at} \tag{12}$$

Then simultaneous Eq.(5) and Eq.(11)and Eq.(12):

$$\eta'(t)e^{-at} - a\eta(t)e^{-at} = b(t^2 + t + \frac{1}{6}) + c(t + \frac{1}{2}) + d - a\eta(t)e^{-at} \tag{13}$$

Multiply both sides of Eq.(13) by e^{at}:

$$\eta'(t) = [b(t^2 + t + \frac{1}{6}) + c(t + \frac{1}{2}) + d]e^{at} \tag{14}$$

Integral on both sides of Eq.(14):

$$\eta(t) = e^{at}[\frac{b}{a}t^2 + (\frac{b+c}{a} - \frac{2b}{a^2})t + \frac{b+3c+6d}{6a} - \frac{b+c}{a^2} + \frac{2b}{a^3})] + C_1 \tag{15}$$

Where, C_1 is a parameter to be estimated.

Step 3: The time response $x^{(0)}(t)$ of the IANGM(1,1,k,k^2) model under the initial condition $x^{(0)}(1) = \hat{x}^{(0)}(1)$ is solved.

Substituting Eq.(15) into Eq.(11):

$$\hat{x}^{(0)}(t) = C_1 e^{-at} + \frac{b}{a}t^2 + (\frac{b+c}{a} - \frac{2b}{a^2})t + \frac{b+3c+6d}{6a} - \frac{b+c}{a^2} + \frac{2b}{a^3} \tag{16}$$

Let $t = 1$, then simplify and organize to get:

$$C_1 = (x^{(0)}(1) - \frac{13b+9c+6d}{6a} + \frac{3b+c}{a^2} - \frac{2b}{a^3})\ e^a \tag{17}$$

Substituting Eq.(17) into Eq.(16):

$$\hat{x}^{(0)}(t) = (x^{(0)}(1) - \frac{13b + 9c + 6d}{6a} + \frac{3b + c}{a^2} - \frac{2b}{a^3})e^{-a(t-1)} + \frac{b}{a}t^2 + (\frac{b + c}{a} - \frac{2b}{a^2})t \rightarrow$$
$$\leftarrow + \frac{b + 3c + 6d}{6a} - \frac{b + c}{a^2} + \frac{2b}{a^3} \quad (18)$$

3.4 Properties of IANGM(1,1, k, k^2) model

According to Theorem 5, the time response function of the IANGM (1,1, k, k^2) model is the form of $x^{(0)}(t) = Ae^{Bt} + Ct^2 + Dt + E$. Therefore, it has the following properties:

Property 1. The model of IANGM (1,1, k, k^2) is suitable for exponential parabolic combination data sequence modeling, which satisfies $x^{(0)}(t) = Ae^{Bt} + Ct^2 + Dt + E$.

Proof. It can be seen from Theorem 5 that it is obviously true, and the following proof is similar.

Property 2. When $b = 0$, $C = 0$, IANGM(1,1, k, k^2) model is suitable for the exponential linear combination data sequence modeling, which satisfies $x^{(0)}(t) = Ae^{Bt} + Dt + E$.

Property 3. When $c = 0$ or $d = 0$, IANGM(1,1, k, k^2) model is suitable for exponential parabolic combination data sequence modeling, which satisfies $x^{(0)}(t) = Ae^{Bt} + Ct^2 + Dt + E$.

Property 4. When $b = c = 0$, $C = D = 0$, IANGM(1,1, k, k^2) model is suitable for approximate non-homogeneous exponential data sequence modeling, which satisfies $x^{(0)}(t) = Ae^{Bt} + E$.

Property 5. When $b = c = d = 0$, $C = D = E = 0$, IANGM(1,1, k, k^2) model is suitable for approximate homogeneous exponential data sequence modeling, which satisfies $x^{(0)}(t) = Ae^{Bt}$.

It should be noted that the introduction of the quadratic time power term in the model broadens the scope of application of the model. Therefore, based on 1-IAGO, a new grey prediction model of IANGM(1,1, k, k^2) with quadratic time power term is constructed, which is an effective promotion of the IANGM(1,1, k) model, moreover, it is suitable for the simulation and prediction of approximate homogeneous exponential type and approximate non-homogeneous exponential type and exponential linear combination type and exponential parabolic combination type data.

4. Numerical Simulation

In order to verify the simulation effect of the new model of IANGM(1,1, k, k^2) and test its application scope, the numerical simulation is analyzed. The low growth sequence ($X_1^{(0)}, X_2^{(0)}, X_3^{(0)}$) and the high growth sequence ($X_4^{(0)}$) are selected for the simulation test, including homogeneous exponential type $X_1^{(0)}$, non-homogeneous exponential type $X_2^{(0)}$, exponential linear combination type $X_3^{(0)}$ and exponential parabolic combination type $X_4^{(0)}$. The four sets of data are as follows: (All results retain four decimal places)

$$X_1^{(0)} = 2e^{0.2k} = \{2.4428, 2.9836, 3.6442, 4.4511, 5.4366, 6.6402\}$$

$$X_2^{(0)} = 2e^{0.2k} + 6 = \{8.4428, 8.9836, 9.6442, 10.4511, 11.4366, 12.6402\}$$

$$X_3^{(0)} = 2e^{0.2k} + 2k + 6 = \{10.4428, 12.9836, 15.6442, 18.4511, 21.4366, 24.6402\}$$

$$X_4^{(0)} = 2e^{0.2k} + 1700k^2 + 2k + 6$$
$$= \{1710.4428, 6812.9836, 15315.6442, 27218.4511, 42521.4366, 61224.6402\}$$

The traditional GM(1,1) model, NGM(1,1, k) model [11], NGM(1,1, k, c) model [12], GM(1,1, k, k^2) model [13] and the new model of IANGM(1,1, k, k^2) are used to model the above different characteristic data sequences, and the fitting values of the five models are compared. The results are listed in Table 1.

From Table 1, IANGM(1,1, k, k^2) model can effectively simulate the homogeneous exponential, non-homogeneous exponential, exponential linear combination, and exponential parabolic combination characteristic data sequences, and the fitting results are optimal, which is consistent with the above properties.

Table 1. Comparison of modeling effects of five types of grey models under four data sequences.

Sequence	Model	Simulative value	MAPE(%)
$X_1^{(0)}$	GM(1,1)	2.4428,2.9727,3.6285,4.4289,5.4059,6.5984	0.4983
	NGM(1,1, k)	2.4428,1.8789,3.0942,4.1742,5.1339,5.9866	14.7501
	NGM(1,1, k , c)	2.4428,2.9726,3.6284,4.4288,5.4057,6.5981	0.5016
	GM(1,1, k , k^2)	2.4428,2.9727,3.6285,4.4290,5.4060,6.5985	0.4972
	IANGM(1,1, k , k^2)	2.4428,2.9816,3.6394,4.4422,5.4222,6.6182	0.1987
$X_2^{(0)}$	GM(1,1)	8.4428,8.8687,9.6695,10.5427,11.4947,12.5327	0.7554
	NGM(1,1, k)	8.4428,5.2383,8.6891,10.3810,11.2105,11.6172	12.4670
	NGM(1,1, k , c)	8.4428,9.6342,10.4359,11.4144,12.6088,14.0665	9.2404
	GM(1,1, k , k^2)	8.4428,8.9727,9.6285,10.4290,11.4060,12.5985	0.2188
	IANGM(1,1, k , k^2)	8.4428,8.9816,9.6394,10.4422,11.4222,12.6182	0.0913
$X_3^{(0)}$	GM(1,1)	10.4428,13.2841,15.5260,18.1463,21.2087,24.7880	1.2770
	NGM(1,1, k)	10.4428,8.3251,13.4468,17.3578,20.3441,22.6245	13.8256
	NGM(1,1, k , c)	10.4428,12.0696.14.6554,17.4073,20.3358,23.4523	5.7948
	GM(1,1, k , k^2)	10.4428,12.9360,15.5837,18.3743,21.3393,24.5171	0.4246
	IANGM(1,1, k , k^2)	10.4428,12.9816,15.6394,18.4422,21.4222,24.6182	0.0501
$X_4^{(0)}$	GM(1,1)	1710.4428,10783.6192,16865.5185,26377.5742,41254.3746,64521.6052	15.9708
	NGM(1,1, k)	1710.4428,6677.1849,13401.2528,22793.5111,35912.7298,54237.8111	11.5429
	NGM(1,1, k , c)	1710.4428,2831.8566,10779.2606,20965.4526,34021.1004,50754.5311	29.6238
	GM(1,1, k , k^2)	1710.4428,6993.5572,15872.0000,26624.0000,42496.0000,60928.0000	1.8023
	IANGM(1,1, k , k^2)	1710.4428,6755.7140,15196.9216,27037.3200,42280.8774,60932.4324	0.6648

5. Application

5.1 Data analysis of China's domestic heat consumption from 2011 to 2020

This paper takes 10 data on China's domestic heat consumption from 2011 to 2020 as the research object [22], and establishes the traditional GM(1,1) model, NGM(1,1, k , c) model [12], GM(1,1, k , k^2) model [13], IANGM(1,1, k) model [21] and the new model of IANGM(1,1, k , k^2). The simulation results of the five models from 2011 to 2019 and the prediction results in 2020 are listed in Table 2.

From Table 3, The simulation prediction accuracy of the IANGM(1,1, k , k^2) model is higher than that of the traditional GM(1,1) model, NGM(1,1, k , c) model, GM(1,1, k , k^2) model and IANGM(1,1, k) model, which shows that the new model of IANGM(1,1, k , k^2) is better, and its prediction results are basically consistent with the trend of China 's living heat consumption from 2011 to 2020. On the other side, it is also shown that adding a quadratic time power term to the IANGM(1,1, k) model can improve the adaptability of the model to the modeling data and achieve better modeling results.

Table 2. China 's domestic heat consumption in 2011-2020 (ten thousand / million kilojoules)

Year	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
consumption	70044	77608	81472	86482	93841	98623	106330	121684	128832	141349

Table 3. Comparison of modeling effects of five models

Year	Actual value	GM(1,1)	NGM(1,1, k , c)	GM(1,1, k , k^2)	IANGM(1,1, k)	IANGM(1,1, k , k^2)
2011	70044	70044.0000	70044.0000	70044.0000	70044.0000	70044.0000
2012	77608	74834.8350	82391.5167	77715.0676	75101.8489	76676.1095
2013	81472	80766.6893	87571.7144	80957.2804	80639.1998	81922.0060
2014	86482	87168.7377	93715.8718	85311.8414	86723.9215	87119.4413
2015	93841	94078.2506	101003.3677	90857.0549	93433.4888	93009.0041
2016	98623	101535.4526	109646.9609	97636.7838	100856.3421	100000.6891
2017	106330	109583.7567	119899.0015	105696.5068	109093.4401	108321.4724
2018	121684	118270.0174	132058.7980	115083.3779	118260.0306	118097.0095
2019	128832	127644.8028	146481.3563	125846.2883	128487.6741	129396.8640
MAPE (%)		1.9033	9.4765	1.8358	1.6137	1.2541
Year	Actual value	predicted value				
2020	141349	137762.6894	163587.7435	138035.9303	139926.5518	142259.5470
APE (%)		17.4803	15.7332	2.3175	1.0063	0.6442

5.2 Analysis of settlement data of soft soil foundation

Eight data in [24] are selected. The first seven data are used as modeling data, and the eighth data is used as data to test the prediction effect. GM(1,1) model (Model I), GM(1,1, k , k^2) model optimized by background value in [13] (Model II), GM(1,1, k , k^2) model optimized by background value and time response function (Model III) and IANGM(1,1, k , k^2) model (Model IV) are established respectively. The modeling results of the four models are compared. The results are listed in Table 4.

Table 4. Comparison of modeling effects of four models

Actual value	Model I	APE(%)	Model II	APE(%)	Model III	APE(%)	Model IV	APE/(%)
70.87	70.8700	0.0000	70.8700	0.0000	70.8700	0.0000	70.8700	0.0000
83.71	87.1695	4.1327	83.8505	0.3970	83.8372	0.1519	83.4500	0.3106
92.91	92.1287	0.8410	92.5411	0.0192	92.5290	0.4100	92.5727	0.3630
99.73	97.3700	2.3664	99.7491	0.5080	99.7382	0.0082	99.6488	0.0814
105.08	102.9094	2.0656	105.6138	0.4841	105.6038	0.4985	105.1716	0.0871
109.73	108.7640	0.8803	110.2612	1.4401	110.2522	0.4759	109.3133	0.3797
112.19	114.9517	2.4616	113.8056	2.5568	113.7975	1.4328	112.1344	0.0496
MAPE(%)	2.1246		0.5030		0.4962		0.2119	
Actual value	predicted value							
113.45	121.4914		116.3507		116.3433		113.6559	
APE(%)	7.0881		2.5568		2.5503		0.1185	

Note: The simulated predicted values and relative prediction errors of Model I, Model II, and Model III in Table 4 are all from reference [13].

From Table 4, The MAPE of the four models is 2.1246%, 0.5030%, 0.4962% and 0.2119%. Among them, the IANGM(1,1, k , k^2) model has the best simulation effect. For the predicted value APE, the new model of IANGM(1,1, k , k^2) is 0.1185%. Therefore, not only the simulated value MAPE, but also the predicted value APE are significantly lower than model I, model II and model III, indicating that the IANGM(1,1, k , k^2) model is better.

6. Conclusion

In this paper, a new grey prediction model IANGM(1,1, k , k^2) is derived and constructed based on the first-order cumulative reduction (1-IAGO). It is a kind of extension of the IANGM(1,1, k) model. A quadratic time power term is added on the basis of the IANGM(1,1, k) model. The shadow equation matching the grey differential equation of the model is determined by the undetermined coefficient method, and then the derivative reduction or constant variation method is used to obtain the time response of the model. It shows that the new model is suitable for modeling the characteristic sequence of approximate homogeneous exponential type, approximate non-homogeneous exponential type, exponential linear combination type and exponential parabolic combination type. To a large extent, it broadens the scope of application of the model. The numerical analysis results show that the new model IANGM(1,1, k , k^2) has better simulation performance. The new model is used to simulate and predict the data of China's domestic thermal consumption and soft soil settlement. The comparison results of multiple models, show that the IANGM(1,1, k , k^2) model has certain feasibility and better modeling effect. This model can be used to accurately predict China's domestic thermal consumption and soft soil settlement in the next five years. If the idea of function transformation is introduced on the basis of this model, several traditional optimization model methods and a more general new model with sub-time power will be the further expansion and supplement of the existing grey model.

References

[1] Liu S F, Dang Y G, & Fang Z G. Grey system theory and application [M]. Beijing: Science Press, 2018.

[2] Long Z, Wei Y, & Long X. A new method of background value optimization in grey prediction model [J]. Journal of Grey System, 2015, 18(01): 41-46.

[3] Xiang X W, Zhang P, & Yu L. Forecasting oil consumption with novel fractional grey prediction model based on Simpson formula [J]. Asia J Math, 2019, 15(02): 1-27.

[4] Cheng ML, Shi G J, & Xiang M Y. On the improvement of the parameter estimation of the grey model GM(1,1) and model application [J]. Communications in Statistics-Simulation and Computation, 2020, 49(05): 1367-1384.

[5] Dai Y Q, Yan Y L, & Liu Z. Prediction and analysis of the total health expenditure and its composition trend in Beijing based on GM (1,1) model [J]. Modern Preventive Medicine, 2021, 48(11): 1996-2000.

[6] Tang L W & Lu Y Y. An Improved Non-equal Interval GM(1,1) Model Based on Grey Derivative and Accumulation [J]. Journal of Grey System, 2020, 32(02): 77-88.

[7] Wu Z T & Zhang T. The improvement and application of the GM(1,1) model [J]. Statistics And Decision, 2019, 35(09): 15-18.

[8] Yin K D, Geng Y, & Li X M. Improved grey prediction model based on exponential grey action quantity [J]. Journal of Systems Engineering and Electronics, 2018, 29(03): 560-570.

[9] Xu J L & Chen Y J. DGM(2,1) model based on grey action and grey derivative optimization [J]. Journal of Luoyang Normal University, 2023, 42(05): 1-8.

[10] Zhang Y. Optimize the MGM(1,m) model of grey derivative [J]. The Practice And Understanding of Mathematics, 2022, 52(10): 136-141.

[11] Cui J, Dang Y G, & Liu S F. A new grey prediction model and its modeling mechanism [J]. Control and Decision, 2009, 24(11): 1702-1706.

[12] Cui X K & Lu X Y. Prediction method of agricultural product yield based on NGM(1,1,k) model [J]. Microelectronics & Computer, 2011, 28(08): 201-203+207.

[13] Kong X H, Chen J J, & Zhao Y. Optimization of background value and time response function of grey GM(1,1,k,k^2) model [J]. Operations Research, and Management Science, 2022, 31(07): 109-113.

[14] Liu J F, Liu S F, & Fang Z G. Fractional-order Reverse Accumulation Generation GM(1,1) Model and its Applications [J]. Journal of Grey System, 2015, 27(4): 52-62.

[15] Zhang K, Wang C Y, & He L J. An improved GOM(1,1) model and its application [J]. Journal of Engineering Mathematics, 2021, 38(01): 38-48.

[16] Zeng L & Jiang A P. Four basic forms of reverse accumulation GOM(1,1) model and their comparison [J]. Practice and Cognition of Mathematics, 2020, 50(06): 219-228.

[17] Ding S, Dang Y G, & Xu N. Construction and optimization of NGOM(1,1) model with approximate non-homogeneous exponential

decreasing sequence [J]. Control and Decision, 2017, 32(08): 1457-1464.

[18] Yang Z, Ren P, & Dang Y G. Reverse cumulative generation and optimization of grey GOM(1,1) model [J]. Theory and Practice of Systems Engineering, 2009, 29(08): 160-164.

[19] Lian J W, Dang Y G, & Wang Z X. The characteristics of reverse accumulation generation and the optimization of GOM(1,1) model [J]. Theory and Practice of Systems Engineering, 2013, 33(09): 2306-2312.

[20] Wang J. Optimization and application of a new grey prediction model [J]. Statistics and Decision, 2015(17): 86-88.

[21] Chen J. & Chen Y J. Construction of IANGM(1,1,k) model for approximate non-homogeneous exponential increasing sequence [J]. Statistics and Decision, 2020, 36(20): 24-27.

[22] Xu H F, Liu S F, & Fang Z G. Optimization of grey action of GM(1,1) model [J]. Practice and Cognition of Mathematics, 2010, 40(02): 26-32.

[23] National Bureau of Statistics of China. China statistical yearbook [M]. Beijing: China Statistical Publishing House, 2022.

[24] Niu X X. Analysis and improvement of GM (2,1) model for settlement prediction of soft soil foundation [J]. Safety and Environmental Engineering, 2014, 21(02): 139-142.