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Reachable Set Estimation for Neutral Markovian Jump Systems with Time-varying Delay and Distributed Delay

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Abstract

Neutral Markov jump systems are a special class of switched systems with time-varying neutral delays. The estimation of the boundary of reachable sets for such systems is one of the important properties of dynamic theory and is currently a research hotspot in dynamic properties. The main focus of this paper is to investigate the estimation of reachable set boundaries for neutral Markov jump systems with time-varying delays, distributed delays, and bounded disturbances. This paper primarily explores and analyzes the boundary problem of reachable sets by constructing appropriate Lyapunov functions, utilizing linear matrix inequality analysis techniques, and combining the approach of free-weight matrices. The objective is to find an ellipsoid set as small as possible to bound the reachable set defined in this paper. By doing so, we aim to derive a less conservative boundary condition for the reachable set. Subsequently, numerical examples are employed to demonstrate the effectiveness of the obtained results, thus confirming the correctness and validity of the findings presented in this paper.

Keywords

Reachable set, Neutral Markovian jump system, Lyapunov function, Linear matrix inequalities

1. Introduction

In practice and engineering applications, due to stochastic failures or repairs of the components, many dynamical systems may cause abrupt variations in their structure, changes in the interconnections of subsystems, sudden environment changes, and so on. Markov jump system is a multi-modal random switching system, and the random switching between each mode is described by a set of Markov chains. Because the Markov jump system has many switching modes, the switching probability between modes in the system will greatly affect the behavior and performance of the system. In the past few decades, the Markov jump system has been widely studied, see [1-4] and the references therein.

A reachable set was proposed in the 1960s, which is a hot issue in the control theory of dynamic systems. It refers to all the state sets that can be reached from a certain initial condition under bounded disturbance. Reachable set estimation is not only an important issue in the control theory [5-8], which plays an important role in solving the problem of state estimation and parameter estimation, but also in practical engineering when safe operation is required through synthesizing controllers to avoid undesirable (or unsafe) regions in the state space. For linear systems with bounded input perturbations, based on the augmented Lyapunov-Krasovskii functional, the ellipsoidal boundary of the reachable set of the system is obtained in the form of linear matrix inequalities in [7]. The authors in [8] investigate the reachable set estimation problem for Markov jump systems with time-varying delays and bounded disturbance. Literature [9] studied the reachable set boundary problem of the neutral system by constructing augmented Lyapunov functional and combining matrix inequality technique, and obtained the smallest reachable set boundary as possible. In [10], a suitable Lyapunov-Krasovskii function is constructed, and then the reachable set ellipsoid description of the neural network system is derived by using Wirtinger-based integral inequality and augmented reciprocating convex matrix inequality. A class of time-delay neural networks with Markov jump parameters and bounded disturbance are studied in [11]. With suitably constructing Lyapunov-Krasovskii functionals, a less conservative delay-dependent condition of finding an ellipsoid-like set to contain all state trajectories that start from the origin is derived in terms of linear matrix inequalities. Then the matrix inequality technique is used to further reduce the conservatism of some integral terms, the proposed reachable set estimation approach is extended to the case that transition probabilities are partially known.

However, the bound of reachable sets for neutral Markovian jump systems with bounded peak disturbances and distributed delays has not been investigated, which motivates this paper. In this paper, combining the technique of linear matrix inequality and the idea of free weight matrix by constructing the Lyapunov function, we study the boundary problem of reachable set for neutral Markov jump systems with time-varying delay and distributed delay and obtain a method to find the boundary of reachable set. Therefore, in this paper, the research on the reachable set boundary of the neutral Markov jump system has great theoretical significance and practical application value.

Notations: The notations used throughout the paper are fairly standard. The superscript T stands for matrix transposition; $\Re^n$ denotes the n-dimensional Euclidean space; the notation $P_{1i} > 0$ means that P_{1i} is a positive definite matrix. In symmetric block matrices, we use an asterisk * to represent a term which is induced by symmetry. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. Problem Statement

Consider the following Markov jump systems with disturbances

$$\begin{cases} \dot{x}(t) - C_{(t,r_t)}\dot{x}(t-\tau(t)) = A_{(t,r_t)}x(t) + B_{(t,r_t)}x(t-h(t)) + B_{1(t,r_t)}\int_{t-r}^{t} x(s)ds + D_{(t,r_t)}\omega(t), \\ x(t_0+\theta) = 0, \forall\theta \in [-\rho, 0], \end{cases} \tag{1}$$

where $x(t) \in \Re^n$ is the state vector, $\int_{t-r}^{t} x(s)ds$ is the distributed delay, $\omega(t) \in \Re^m$ is the disturbance, and satisfy

$$\omega^T(t)\omega(t) \le \omega_m^2, \tag{2}$$

the discrete time-varying delay $ h (t) > 0 $ and distributed delay $\tau(t) > 0$ satisfy

$$0 \le h(t) \le h, 0 \le \dot{h}(t) \le \mu_1 < 1, 0 \le \tau(t) \le \tau, 0 \le \dot{\tau}(t) \le \mu_2 < 1, \tag{3}$$

where h, τ, μ_1 and μ_2 are constants, $A_{(t,r_t)}$, $B_{(t,r_t)}$, $C_{(t,r_t)}$, $B_{1(t,r_t)}$ and $D_{(t,r_t)}$ are known constant matrices of the Markovian process, $\{r_t, t \ge 0\}$ is a Markovian process taking values on the probability space in a finite state $\wp = \{1, 2, \cdots, N\}$ with a generator $\Lambda = \{\lambda_{ij}\}(i, j \in \wp)$ given by

$$\Pr\{r_{t+\Delta} = j | r_t = i\} = \begin{cases} \lambda_{ij}\Delta + o(\Delta), & j \ne i, \\ 1 + \lambda_{ij}\Delta + o(\Delta), & j = i, \end{cases} \tag{4}$$

where $\Delta > 0$, $\lim_{\Delta\to 0^+} \frac{o(\Delta)}{\Delta} = 0$, $\lambda_{ij} \ge 0$, for $j \ne i$ is the transition probability from mode i at time t to mode j at time $t+\Delta$, $\lambda_{ii} = -\sum_{j=1, j\ne i}^{N} \lambda_{ij}$. For notational simplicity, when $(t, r_t) = i, i \in \wp$, $A_{(t,r_t)}$, $B_{(t,r_t)}$, $C_{(t,r_t)}$, $B_{1(t,r_t)}$ and $D_{(t,r_t)}$ are denoted as A_i, B_i, C_i, B_{1i} and D_i respectively.

Since the state transition probability of the Markovian jump process is considered in this paper is partially known, the transition probability matrix of the Markovian jumping process Λ is defined as

$$\Lambda = \begin{pmatrix} \lambda_{11} & ? & \cdots & \lambda_{1N} \\ ? & \lambda_{22} & \cdots & \lambda_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{N1} & ? & \cdots & \lambda_{NN}, \end{pmatrix} \tag{5}$$

where ? represents the unknown transition rate. For notational clarity, $\forall i \in \wp$, the set $U^i = U_k^i \bigcup U_{uk}^i$ with

$$U_k^i \underline{\underline{\Delta}} \{ j : \lambda_{ij} \ is \ known \ for \ j \in \wp \},$$
$$U_{uk}^i \underline{\underline{\Delta}} \{ j : \lambda_{ij} \ is \ unknown \ for \ j \in \wp \}.$$

Moreover, if $U_k^i \neq \varnothing$, it is further described as $U_k^i = \{ k_1^i, k_2^i, \cdots, k_m^i \}$, where m is a non-negation integer with $1 \le m \le N$ and $k_j^i \in Z^+$, $1 \le k_j^i \le N, j = 1, 2, \cdots, m$ represent the known element of the ith row and jth column in the state transition probability matrix Λ.

For the sake of brevity, $x(t)$ is used to represent the solution of the system under initial conditions $x(t, t_0, r_0)$ and $\{x(t), t\}$ satisfies the initial condition $\{x(0), r_0\}$. And its weak infinitesimal generator, acting on function V is defined in [12].

$$LV(x(t), t, i) = \lim_{\Delta \to 0^+} \frac{1}{\Delta} \left[\varepsilon \left(V \left(x(t+\Delta), t+\Delta, r_{t+\Delta} \right) \middle| x(t), r_t = i \right) - V(x(t), t, i) \right]. \tag{6}$$

A reachable set that bounds the state of system (1) is defined by

$$\Re_x = \left\{ x(t) \in \Re^n \middle| x(t), \omega(t) \ satisfy \ (1)(2)(3) \right\}. \tag{7}$$

Based on the ideas proposed in [13], this reachable set estimation problem can be transformed into the problem of finding an ellipsoid to bound the $\Re_x$. We will bound $\Re_x$ by an ellipsoid of the form

$$\Im(P_{1i}, 1) \triangleq \left\{ x(t) \in \Re^n : x^T(t) P_{1i} x(t) \le 1; P_{1i} > 0 \right\}. \tag{8}$$

In this paper, the following Lemma are needed.

Lemma 1 [13]. Let $V(t, x(0)) = 0$ and $\omega^T(t)\omega(t) \le \omega_m^2$, if

$$LV(t, x_t) + \alpha V(t, x_t) - \beta \omega^T(t)\omega(t) \le 0, \alpha > 0, \beta > 0,$$

then we have $V(t, x_t) \le \frac{\beta}{\alpha} \omega_m^2$ for $\forall t \ge 0$.

Lemma 2 [14]. Suppose $h \in R^n$ and $x(t) \in R^n$, for any positive definite matrix W the following inequality holds

$$-h \int_{t-h}^{t} \dot{x}^T(s) W \dot{x}(s) ds \le \begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix}^T \begin{bmatrix} -W & W \\ W & -W \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix}.$$

Lemma 3 [15]. For any constant matrix $M = M^T > 0$ and a scalar $h > 0$ such that the following integrations are well defined, then

$$-\int_{t-h}^{t} \omega^T(s) M \omega(s) ds \le -\frac{1}{h} \left(\int_{t-h}^{t} \omega(s) ds \right)^T M \left(\int_{t-h}^{t} \omega(s) ds \right),$$
$$-\int_{-h}^{0} \int_{t+\theta}^{t} \omega^T(s) M \omega(s) ds d\theta \le -\frac{2}{h^2} \left(\int_{-h}^{0} \int_{t+\theta}^{t} \omega(s) ds d\theta \right)^T M \left(\int_{-h}^{0} \int_{t+\theta}^{t} \omega(s) ds d\theta \right).$$

3. Main Results

Our aim is to find an ellipsoid set as small as possible to bound the reachable set defined in (7). In this section, based on an appropriate Lyapunov functional and matrix inequality techniques, following Theorems are derived.

Theorem 1. Consider the neutral Markov system (1) with constraints (2) (3), if there exist real matrices $P_{2i}, P_{3i}, P_{1i} > 0, W_i > 0 (i = 1, \cdots, N), Q_1 > 0, Q_2 > 0, S > 0, W > 0, R_1 > 0, R_2 > 0$, any matrices M_1, M_2, M_3 with appropriate dimension and a scalar $\alpha > 0$ satisfying the following matrix inequalities:

$$\Phi_i = \begin{bmatrix} \varphi_{i11} & \varphi_{i12} & \varphi_{i13} & e^{-\alpha\tau}W & \varphi_{i15} & \varphi_{16} & \varphi_{17} & \varphi_{i18} & \varphi_{i19} \\ * & \varphi_{i22} & \varphi_{i23} & 0 & \varphi_{i25} & 0 & 0 & \varphi_{i28} & \varphi_{i29} \\ * & * & \varphi_{33} & 0 & B_i^T M_3^T & \varphi_{36} & 0 & 0 & 0 \\ * & * & * & -e^{-\alpha\tau}W & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \varphi_{i55} & 0 & 0 & M_3 B_{1i} & M_3 D_i \\ * & * & * & * & * & \varphi_{66} & 0 & 0 & 0 \\ * & * & * & * & * & * & \varphi_{77} & 0 & 0 \\ * & * & * & * & * & * & * & \varphi_{88} & 0 \\ * & * & * & * & * & * & * & * & -\dfrac{\alpha}{\omega_m^2} I \end{bmatrix} < 0, \tag{9}$$

$$P_{1j} - W_i \le 0,\ j \in U_{uk}^i, i \ne j, \tag{10}$$

$$P_{1j} - W_i \ge 0,\ j \in U_{uk}^i, i = j, \tag{11}$$

where

$$\begin{aligned}
\varphi_{i11} &= \alpha P_{1i} + P_{2i}^T A_i + A_i^T P_{2i} + Q_1 + h^2 S_1 - (1-\mu_1) e^{-\alpha h} S_3 + r R_1 - e^{-\alpha\tau} W \\
&\quad - h^2 e^{-\alpha h} R_2 + M_1 A_i + A_i^T M_1^T + \sum_{j \in U_k^i} \lambda_{ij} (P_{1j} - W_i), \\
\varphi_{i12} &= P_{1i} - P_{2i}^T + A_i^T P_{3i} + h^2 S_2 - M_1 + A_i^T M_2^T,\ \varphi_{17} = h e^{-\alpha h} R_2, \\
\varphi_{i13} &= P_{2i}^T B_i + (1-\mu_1) e^{-\alpha h} S_3 + M_1 B_i,\ \varphi_{16} = -(1-\mu_1) e^{-\alpha h} S_2^T, \\
\varphi_{i15} &= P_{2i}^T C_i + M_1 C_i + A_i^T M_3^T,\ \varphi_{i18} = P_{2i}^T B_{1i} + M_1 B_{1i},\ \varphi_{i19} = P_{2i}^T D_i + M_1 D_i, \\
\varphi_{i22} &= -P_{3i}^T - P_{3i} + Q_2 + \tau^2 W + h^2 S_3 + \frac{h^4}{4} R_2 - M_2 - M_2^T, \\
\varphi_{i23} &= P_{3i}^T B_i + M_2 B_i,\ \varphi_{i25} = P_{3i}^T C_i + M_2 C_i - M_3^T,\ \varphi_{i28} = P_{3i}^T B_{1i} + M_2 B_{1i}, \\
\varphi_{i29} &= P_{3i}^T D_i + M_2 D_i,\ \varphi_{33} = -(1-\mu_1) e^{-\alpha h} Q_1 - (1-\mu_1) e^{-\alpha h} S_3, \\
\varphi_{36} &= (1-\mu_1) e^{-\alpha h} S_2^T,\ \varphi_{55} = -(1-\mu_2) e^{-\alpha\tau} Q_2 + M_3 C_i + C_i^T M_3^T, \\
\varphi_{66} &= -(1-\mu_1) e^{-\alpha h} S_1,\ \varphi_{77} = -e^{-\alpha h} R_2,\ \varphi_{88} = -\frac{1}{r} e^{-\alpha r} R_1.
\end{aligned}$$

Then, the reachable sets of the system (1) having the constraints (2)(3) is bounded by a boundary $\bigcap_{i \in \wp} \Im(P_{1i}, 1)$, which $\Im(P_{1i})(i \in \wp)$ is defined in (8).

Proof. We choose the following Lyapunov-Krasovskii functional candidate as follows:

$$V(t, x_t, i) = \sum_{i=1}^{6} V_i(t, x_t, i), \tag{12}$$

where

$$\begin{aligned}
V_1(t, x_t, i) &= x^T(t) P_{1rt} x(t) = \begin{bmatrix} x^T(t) & \dot{x}^T(t) \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} P_{1rt} & 0 \\ P_{2rt} & P_{3rt} \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}, \\
V_2(t, x_t, i) &= \int_{t-h(t)}^{t} e^{\alpha(s-t)} x^T(s) Q_1 x(s) ds + \int_{t-\tau(t)}^{t} e^{\alpha(s-t)} \dot{x}^T(s) Q_2 \dot{x}(s) ds, \\
V_3(t, x_t, i) &= \int_{t-h(t)}^{t} e^{\alpha(s-t)} (h(t) - t + s) g^T(s) S g(s) ds, \\
V_4(t, x_t, i) &= \tau \int_{-\tau(t)}^{0} \int_{t+\theta}^{t} e^{\alpha(s-t)} \dot{x}^T(s) W \dot{x}(s) ds d\theta,
\end{aligned}$$

$$V_5(t,x_t,i)=\int_{-r}^{0}\int_{t+\theta}^{t}e^{\alpha(s-t)}x^T(s)R_1x(s)dsd\theta,$$

$$V_6(t,x_t,i)=\frac{h^2}{2}\int_{-h}^{0}\int_{v}^{0}\int_{t+\theta}^{t}e^{\alpha(s-t)}x^T(s)R_2x(s)dsd\theta dv,$$

where $g^T(s)=\left[x^T(s)\quad \dot{x}^T(s)\right]$.

When $r_t=i$, then $P_{2i},P_{3i},P_{1i}>0(i=1,2,\cdots,N),Q_1>0,Q_2>0,S>0,W>0,R_1>0,R_2>0$ and $\alpha>0$ are solutions of (9).

First, we show that $V(t,x_t,i)$ in (12) is a good $L-K$ functional candidate. For $t-h\le s\le t$, so we have $0<e^{-\alpha h}\le e^{-\alpha h(t)}\le e^{\alpha(s-t)}\le 1$. Furthermore, for $t-\tau\le s\le t$, we have $0<e^{-\alpha\tau}\le e^{-\alpha\tau(t)}\le e^{\alpha(s-t)}$. Then $\sum_{j=2}^{6}V_j(t,x_t,i)\ge 0$.

Therefore, we get

$$\begin{cases}V(t,x_t,i)=\sum_{j=1}^{6}V_j(t,x_t,i)\ge V_1(t,x_t,i)=x^T(t)P_{1r_t}x(t),\\ V(t,x_t)=0,\ when\ x(\theta)=0,\ \theta\in[t-\rho,t].\end{cases}\tag{13}$$

Taking derivative of $V(t,x_t)$ along the trajectories of system (1), we can obtain the following

$$LV=LV_1+LV_2+LV_3+LV_4+LV_5+LV_6,\tag{14}$$

where

$$\begin{aligned}LV_1(t,x_t,i)&=2\left[x^T(t)\quad \dot{x}^T(t)\right]\begin{bmatrix}P_{1i}&P_{2i}^T\\0&P_{3i}^T\end{bmatrix}\begin{bmatrix}\dot{x}(t)\\0\end{bmatrix}+x^T(t)\left(\sum_{j=1}^{N}\lambda_{ij}P_{1j}\right)x(t)\\
&=2\begin{bmatrix}x(t)\\\dot{x}(t)\end{bmatrix}^T\begin{bmatrix}P_{1i}&P_{2i}^T\\0&P_{3i}^T\end{bmatrix}\begin{bmatrix}\dot{x}(t)\\ \begin{Bmatrix}A_ix(t)+B_ix(t-h(t))+C_i\dot{x}(t-\tau(t))\\-\dot{x}(t)+B_{1i}\int_{t-r}^{t}x(s)ds+D_i\omega(t)\end{Bmatrix}\end{bmatrix}+x^T(t)\left(\sum_{j=1}^{N}\lambda_{ij}P_{1j}\right)x(t)\\
&=x^T(t)\left[P_{2i}^TA_i+A_i^TP_{2i}\right]x(t)+2x^T(t)\left[P_{1i}+P_{2i}^T+A_i^TP_{3i}\right]\dot{x}(t)+2x^T(t)P_{2i}^TB_ix(t-h(t))\\
&\quad+2x^T(t)P_{2i}^TC_i\dot{x}(t-\tau(t))+2x^T(t)P_{2i}^TB_{1i}\cdot\left(\int_{t-r}^{t}x(s)ds\right)+2x^T(t)P_{2i}^TD_i\omega(t)\\
&\quad-\dot{x}^T(t)\left[P_{3i}^T+P_{3i}\right]\dot{x}(t)+2\dot{x}^T(t)P_{3i}^TB_ix(t-h(t))+2\dot{x}^T(t)P_{3i}^TC_i\dot{x}(t-\tau(t))\\
&\quad+2\dot{x}^T(t)P_{3i}^TB_{1i}\cdot\left(\int_{t-r}^{t}x(s)ds\right)+2\dot{x}^T(t)P_{3i}^TD_i\omega(t)+x^T(t)\left(\sum_{j=1}^{N}\lambda_{ij}P_{1j}\right)x(t).\end{aligned}$$

Taking into account the situation that the information of transition probabilities is not accessible completely, due to $\sum_{j=1}^{N}\lambda_{ij}=0$, the follows zero equations hold for arbitrary matrices $W_i=W_i^T$ is satisfied

$$-x^T(t)\left(\sum_{j=1}^{N}\lambda_{ij}W_i\right)x(t)=0.\tag{15}$$

Hence,

$$\begin{aligned}LV_1(t,x_t,i)&=x^T(t)\left[P_{2i}^TA_i+A_i^TP_{2i}\right]x(t)+2x^T(t)\left[P_{1i}+P_{2i}^T+A_i^TP_{3i}\right]\dot{x}(t)+2x^T(t)P_{2i}^TB_i\cdot x(t-h(t))\\
&\quad+2x^T(t)P_{2i}^TC_i\dot{x}(t-\tau(t))+2x^T(t)P_{2i}^TB_{1i}\left(\int_{t-r}^{t}x(s)ds\right)+2x^T(t)P_{2i}^TD_i\omega(t)\\
&\quad-\dot{x}^T(t)\left[P_{3i}^T+P_{3i}\right]\dot{x}(t)+2\dot{x}^T(t)P_{3i}^TC_i\dot{x}(t-\tau(t))+2\dot{x}^T(t)P_{3i}^TB_ix(t-h(t))\\
&\quad+2\dot{x}^T(t)P_{3i}^TB_{1i}\left(\int_{t-r}^{t}x(s)ds\right)+2\dot{x}^T(t)P_{3i}^TD_i\omega(t)+x^T(t)\left(\sum_{j\in U_k^i}\lambda_{ij}(P_{1j}-W_i)\right)x(t)+x^T(t)\left(\sum_{j\in U_{uk}^i}\lambda_{ij}(P_{1j}-W_i)\right)x(t),\end{aligned}\tag{16}$$

$$\begin{aligned} LV_2(t,x_t,i) \le{}& x^T(t)Q_1x(t) + \dot{x}^T(t)Q_2\dot{x}(t) - (1-\mu_1)e^{-\alpha h}x^T(t-h(t))Q_1 \\ &\cdot x(t-h(t)) - (1-\mu_2)e^{-\alpha\tau}\dot{x}^T(t-\tau(t))Q_2\dot{x}(t-\tau(t)) - \alpha V_2, \end{aligned} \tag{17}$$

Based on Lemma 3, so, $LV_3(t,x_t)$ can be rewritten as:

$$\begin{aligned} LV_3(t,x_t,i) \le{}& h^2g^T(t)Sg(t) - (1-\mu_1)e^{-\alpha h}\int_{t-h}^{t} g^T(s)Sg(s)ds - \alpha V_3 \\ \le{}& x^T(t)\left[h^2S_1 - (1-\mu_1)e^{-\alpha h}S_3\right]x(t) + 2x^T(t)\left[h^2S_1\right]\dot{x}(t) \\ &+ \dot{x}^T(t)\left[h^2S_3\right]\dot{x}(t) + 2x^T(t)\left[(1-\mu_1)e^{-\alpha h}S_3\right]x(t-h(t)) + 2x^T(t) \\ &\cdot\left[-(1-\mu_1)e^{-\alpha h}S_2^T\right]\left(\int_{t-h(t)}^{t} x(s)ds\right) + 2x^T(t-h(t))\left[(1-\mu_1)e^{-\alpha h}S_2^T\right] \\ &\cdot\left(\int_{t-h(t)}^{t} x(s)ds\right) + x^T(t-h(t))\left[-(1-\mu_1)e^{-\alpha h}S_3\right]x(t-h(t)) \\ &+ \left(\int_{t-h(t)}^{t} x(s)ds\right)^T\left[-(1-\mu_1)e^{-\alpha h}S_1\right]\left(\int_{t-h(t)}^{t} x(s)ds\right) - \alpha V_3, \end{aligned} \tag{18}$$

So, according to Lemma 2, we have

$$\begin{aligned} LV_4(t,x_t,i) \le{}& \dot{x}^T(t)\left(\tau^2W\right)\dot{x}(t) + x^T(t)(-e^{-\alpha\tau}W)x(t) + 2x^T(t)\left(e^{-\alpha\tau}W\right) \\ &\cdot x\left(t-\tau(t)\right) + x^T\left(t-\tau(t)\right)\left(-e^{-\alpha\tau}W\right)x\left(t-\tau(t)\right) - \alpha V_4, \end{aligned} \tag{19}$$

Using Lemma 3, we further have

$$\begin{aligned} LV_5(t,x_t,i) ={}& x^T(t)\left(rR_1\right)x(t) - \int_{t-r}^{t} e^{\alpha(s-t)}x(s)R_1x(s)ds - \alpha V_5 \\ \le{}& x^T(t)\left(rR_1\right)x(t) - \frac{e^{-\alpha r}}{r}\left(\int_{t-r}^{t} x(s)ds\right)^T R_1\left(\int_{t-r}^{t} x(s)ds\right) - \alpha V_5, \end{aligned} \tag{20}$$

$$\begin{aligned} LV_6(t,x_t,i) \le{}& \frac{h^4}{4}\dot{x}^T(t)R_2\dot{x}(t) - \frac{h^2}{2}e^{-\alpha h}\int_{-h}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)R_2\dot{x}(s)dsd\theta - \alpha V_5 \\ \le{}& \dot{x}^T(t)\left(\frac{h^4}{4}R_2\right)\dot{x}(t) - e^{-\alpha h}\left(\int_{-h}^{0}\int_{t+\theta}^{t}\dot{x}(s)dsd\theta\right)^T R_2\left(\int_{-h}^{0}\int_{t+\theta}^{t}\dot{x}(s)dsd\theta\right) - \alpha V_5 \\ ={}& \dot{x}^T(t)\left(\frac{h^4}{4}R_2\right)\dot{x}(t) + x^T(t)\left(-e^{-\alpha h}h^2R_2\right)x(t) + 2x^T(t)\left(e^{-\alpha h}R_2\right)\left(\int_{t-h}^{t} x(s)ds\right) \\ &+ \left(\int_{t-h}^{t} x(s)ds\right)^T\left(-e^{-\alpha h}R_2\right)\left(\int_{t-h}^{t} x(s)ds\right) - \alpha V_6, \end{aligned} \tag{21}$$

Meanwhile, for any matrices M_1, M_2 and M_3 with appropriate dimension, the following equation is true:

$$\begin{aligned} &2\left[x^T(t)M_1 + \dot{x}^T(t)M_2 + \dot{x}^T(t-\tau(t))M_3\right] \\ &\left[A_ix(t) - \dot{x}(t) + B_ix(t-h(t)) + C_i\dot{x}^T(t-\tau(t)) + B_{1i}\int_{t-r}^{t} x(s)ds + D_i\omega(t)\right] = 0 \end{aligned} \tag{22}$$

Combining Eqs. (13)-(22), we can obtain

$$LV(t,x_t,i) + \alpha V(t,x_t,i) - \frac{\alpha}{\omega_m^2}\omega^T(t)\omega(t) \le \xi^T(t)\Phi_i\xi(t) + x^T(t)\left(\sum_{j\in U_{uk}^i}\lambda_{ij}\left(P_{1j} - W_i\right)\right)x(t) \tag{23}$$

where Φ_i is the same as defined in the Theorem 1 and

$$\xi^T(t) = \left[x^T(t) \quad \dot{x}^T(t) \quad x^T(t-h(t)) \quad x^T(t-\tau(t)) \quad \dot{x}^T(t-\tau(t))\int_{t-h(t)}^{t} x^T(s)ds \quad \int_{t-h}^{t} x^T(s)ds \quad \int_{t-r}^{t} x^T(s)ds \quad \omega^T(t)\right].$$

Thus, from matrix inequalities (9)-(11), we get

$$LV(t,x_t,i)+\alpha V(t,x_t,i)-\frac{\alpha}{\omega_m^2}\omega^T(t)\omega(t)\le 0.$$

Therefore, we can obtain the inequality $V(t,x_t,i)\le x^T(t)P_{1i}x(t)\le 1$. Hence, $V(t,x_t,i)\le 1$ is true by using Lemma 1. This completes the proof.

Corollary 1. Consider the neutral Markov system (1) with constraints (2) (3) when the transition rate matrix Λ is completely known, if there exist real matrices $P_{2i},P_{3i},P_{1i}>0,W_i>0(i=1,\cdots,N),Q_1>0,Q_2>0,S>0,W>0,R_1>0,R_2>0$, any matrices M_1,M_2,M_3 with appropriate dimension and a scalar $\alpha>0$ satisfying the following matrix inequalities:

$$\Phi_i=\begin{bmatrix}\varphi_{i11} & \varphi_{i12} & \varphi_{i13} & e^{-\alpha\tau}W & \varphi_{i15} & \varphi_{16} & \varphi_{17} & \varphi_{i18} & \varphi_{i19}\\ * & \varphi_{i22} & \varphi_{i23} & 0 & \varphi_{i25} & 0 & 0 & \varphi_{i28} & \varphi_{i29}\\ * & * & \varphi_{33} & 0 & B_i^TM_3^T & \varphi_{36} & 0 & 0 & 0\\ * & * & * & -e^{-\alpha\tau}W & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & \varphi_{i55} & 0 & 0 & M_3B_{1i} & M_3D_i\\ * & * & * & * & * & \varphi_{66} & 0 & 0 & 0\\ * & * & * & * & * & * & \varphi_{77} & 0 & 0\\ * & * & * & * & * & * & * & \varphi_{88} & 0\\ * & * & * & * & * & * & * & * & -\frac{\alpha}{\omega_m^2}I\end{bmatrix}<0, \tag{24}$$

where

$$\varphi_{i11}=\alpha P_{1i}+P_{2i}^TA_i+A_i^TP_{2i}+Q_1+h^2S_1-(1-\mu_1)e^{-\alpha h}S_3+rR_1$$
$$-e^{-\alpha\tau}W-h^2e^{-\alpha h}R_2+M_1A_i+A_i^TM_1^T+\sum_{j\in U_k^i}\lambda_{ij}(P_{1j}-W_i),$$
$$\varphi_{i12}=P_{1i}-P_{2i}^T+A_i^TP_{3i}+h^2S_2-M_1+A_i^TM_2^T,\ \varphi_{17}=he^{-\alpha h}R_2,$$
$$\varphi_{i13}=P_{2i}^TB_i+(1-\mu_1)e^{-\alpha h}S_3+M_1B_i,\ \varphi_{16}=-(1-\mu_1)e^{-\alpha h}S_2^T,$$
$$\varphi_{i15}=P_{2i}^TC_i+M_1C_i+A_i^TM_3^T,\ \varphi_{i18}=P_{2i}^TB_{1i}+M_1B_{1i},\ \varphi_{i19}=P_{2i}^TD_i+M_1D_i,$$
$$\varphi_{i22}=-P_{3i}^T-P_{3i}+Q_2+\tau^2W+h^2S_3+\frac{h^4}{4}R_2-M_2-M_2^T,$$
$$\varphi_{i23}=P_{3i}^TB_i+M_2B_i,\ \varphi_{i25}=P_{3i}^TC_i+M_2C_i-M_3^T,\ \varphi_{i28}=P_{3i}^TB_{1i}+M_2B_{1i},$$
$$\varphi_{i29}=P_{3i}^TD_i+M_2D_i,\ \varphi_{33}=-(1-\mu_1)e^{-\alpha h}Q_1-(1-\mu_1)e^{-\alpha h}S_3,$$
$$\varphi_{36}=(1-\mu_1)e^{-\alpha h}S_2^T,\ \varphi_{55}=-(1-\mu_2)e^{-\alpha\tau}Q_2+M_3C_i+C_i^TM_3^T,$$
$$\varphi_{66}=-(1-\mu_1)e^{-\alpha h}S_1,\ \varphi_{77}=-e^{-\alpha h}R_2,\ \varphi_{88}=-\frac{1}{r}e^{-\alpha r}R_1.$$

Then, the reachable sets of the system (1) having the constraints (2) (3) is bounded by a boundary $\bigcap_{i\in\wp}\Im(P_{1i},1)$, which $\Im(P_{1i})(i\in\wp)$ is defined in (8).

Corollary 2. Consider the neutral Markov system (1) with constraints (2) (3) when the transition rate matrix Λ is completely unknown if there exist real matrices $P_{2i},P_{3i},P_{1i}>0,W_i>0(i=1,\cdots,N),Q_1>0,Q_2>0,S>0,W>0,R_1>0,R_2>0$, any matrices M_1,M_2,M_3 with appropriate dimension and a scalar $\alpha>0$ satisfying the following matrix inequalities:

$$\Phi_i = \begin{bmatrix} \varphi_{i11} & \varphi_{i12} & \varphi_{i13} & e^{-\alpha\tau}W & \varphi_{i15} & \varphi_{16} & \varphi_{17} & \varphi_{i18} & \varphi_{i19} \\ * & \varphi_{i22} & \varphi_{i23} & 0 & \varphi_{i25} & 0 & 0 & \varphi_{i28} & \varphi_{i29} \\ * & * & \varphi_{33} & 0 & B_i^T M_3^T & \varphi_{36} & 0 & 0 & 0 \\ * & * & * & -e^{-\alpha\tau}W & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \varphi_{i55} & 0 & 0 & M_3 B_{1i} & M_3 D_i \\ * & * & * & * & * & \varphi_{66} & 0 & 0 & 0 \\ * & * & * & * & * & * & \varphi_{77} & 0 & 0 \\ * & * & * & * & * & * & * & \varphi_{88} & 0 \\ * & * & * & * & * & * & * & * & -\frac{\alpha}{\omega_m^2} I \end{bmatrix} < 0, \tag{25}$$

where

$$\begin{aligned} \varphi_{i11} &= \alpha P_{1i} + P_{2i}^T A_i + A_i^T P_{2i} + Q_1 + h^2 S_1 - (1-\mu_1) e^{-\alpha h} S_3 + r R_1 \\ &\quad - e^{-\alpha\tau} W - h^2 e^{-\alpha h} R_2 + M_1 A_i + A_i^T M_1^T, \\ \varphi_{i12} &= P_{1i} - P_{2i}^T + A_i^T P_{3i} + h^2 S_2 - M_1 + A_i^T M_2^T,\ \varphi_{17} = h e^{-\alpha h} R_2, \\ \varphi_{i13} &= P_{2i}^T B_i + (1-\mu_1) e^{-\alpha h} S_3 + M_1 B_i,\ \varphi_{16} = -(1-\mu_1) e^{-\alpha h} S_2^T, \\ \varphi_{i15} &= P_{2i}^T C_i + M_1 C_i + A_i^T M_3^T,\ \varphi_{i18} = P_{2i}^T B_{1i} + M_1 B_{1i},\ \varphi_{i19} = P_{2i}^T D_i + M_1 D_i, \\ \varphi_{i22} &= -P_{3i}^T - P_{3i} + Q_2 + \tau^2 W + h^2 S_3 + \frac{h^4}{4} R_2 - M_2 - M_2^T, \\ \varphi_{i23} &= P_{3i}^T B_i + M_2 B_i,\ \varphi_{i25} = P_{3i}^T C_i + M_2 C_i - M_3^T,\ \varphi_{i28} = P_{3i}^T B_{1i} + M_2 B_{1i}, \\ \varphi_{i29} &= P_{3i}^T D_i + M_2 D_i,\ \varphi_{33} = -(1-\mu_1) e^{-\alpha h} Q_1 - (1-\mu_1) e^{-\alpha h} S_3, \\ \varphi_{36} &= (1-\mu_1) e^{-\alpha h} S_2^T,\ \varphi_{55} = -(1-\mu_2) e^{-\alpha\tau} Q_2 + M_3 C_i + C_i^T M_3^T, \\ \varphi_{66} &= -(1-\mu_1) e^{-\alpha h} S_1,\ \varphi_{77} = -e^{-\alpha h} R_2,\ \varphi_{88} = -\frac{1}{r} e^{-\alpha r} R_1. \end{aligned}$$

Then, the reachable sets of the system (1) having the constraints (2) (3) is bounded by a boundary $\bigcap_{i\in\wp} \Im(P_{1i},1)$, which $\Im(P_{1i})(i \in \wp)$ is defined in (8).

Remark. The solution for (9)-(11), (24) or (25), if it exists, need not be unique. It is well-known that the volume of $\Im(P_{1i})$ defined in (8) is proportional to $\det(P_{1i})^{\frac{1}{2}}$, so the minimization of $\det(P_{1i})^{-\frac{1}{2}}$ is the same as minimizing the volume of $\Im(P_{1i})$. That is, maximize $\det(P_{1i})^{\frac{1}{2}}$ subject to $\delta I \le P_{1i}$ which can be equivalent to the following optimization problem:

$$\min \left\{ \min_{j\in\wp} \det(P_{ij})^{-\frac{1}{2}} \right\}$$

$$s.t. \begin{cases} P_{1i} - \delta_i \ge 0, i \in \wp \\ (9)-(11), or (24), or (25). \end{cases}$$

4. Example

In this section, two examples are used to demonstrate that the effectiveness and correctness of the main results derived above.

Example 1. Consider the neutral Markov jump system (1) with three operation modes whose state matrices are listed as following:

$$A_1 = \begin{bmatrix} -2 & -1 \\ 0 & -2 \end{bmatrix}, B_1 = \begin{bmatrix} -1 & 0 \\ -1 & -2 \end{bmatrix}, B_{11} = \begin{bmatrix} -0.1 & -0.1 \\ 0 & 1.5 \end{bmatrix}, C_1 = \begin{bmatrix} -1 & 0 \\ -1 & -2 \end{bmatrix},$$

$$D_1 = \begin{bmatrix} -0.15 \\ 0.15 \end{bmatrix}, A_2 = \begin{bmatrix} -3 & 0 \\ 0 & -2 \end{bmatrix}, B_2 = \begin{bmatrix} -2 & 0 \\ -1 & -1 \end{bmatrix}, B_{12} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} -2 & 0 \\ -1 & -1 \end{bmatrix}, D_2 = \begin{bmatrix} -0.14 \\ 0.35 \end{bmatrix}, A_3 = \begin{bmatrix} -2 & 0 \\ -1 & -1.5 \end{bmatrix}, B_3 = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix},$$

$$B_{13} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, C_3 = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}, D_3 = \begin{bmatrix} -0.2 \\ 0.3 \end{bmatrix}.$$

The transition rate matrix Λ is considered as the following three case.

Case 1. The transition rate matrix Λ is completely known, which is considered as

$$\Lambda = \begin{bmatrix} -0.6 & 0.2 & 0.4 \\ 0.6 & -1 & 0.4 \\ 0.3 & 0.5 & -0.8 \end{bmatrix}.$$

Case 2. The transition rate matrix Λ is partly known, which is considered as

$$\Lambda = \begin{bmatrix} -0.6 & 0.2 & 0.4 \\ ? & -1 & ? \\ ? & ? & ? \end{bmatrix}.$$

Case 3. The transition rate matrix Λ is completely unknown, which is considered as

$$\Lambda = \begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix}.$$

By using corollary 1 and solving the optimization problem (1) in case 1, we can obtain minimization of $\det(P_{1i})^{-\frac{1}{2}}$ is 0.2308 when $\alpha = 0.1$, and the corresponding feasible matrices are given as $P_{11} = \begin{bmatrix} 6.2857 & 2.5749 \\ 2.5749 & 3.2765 \end{bmatrix}, P_{12} = \begin{bmatrix} 7.4468 & 2.5201 \\ 2.5201 & 2.5907 \end{bmatrix}, P_{13} = \begin{bmatrix} 10.5817 & 4.6112 \\ 4.6112 & 3.7835 \end{bmatrix}$. The reachable sets of the system (1) in case 1 is bounded by a intersection of three ellipsoids: $\bigcap_{i=1}^{3} \Im(\tilde{P}_{1,i})$, which is depicted in Figure 1.

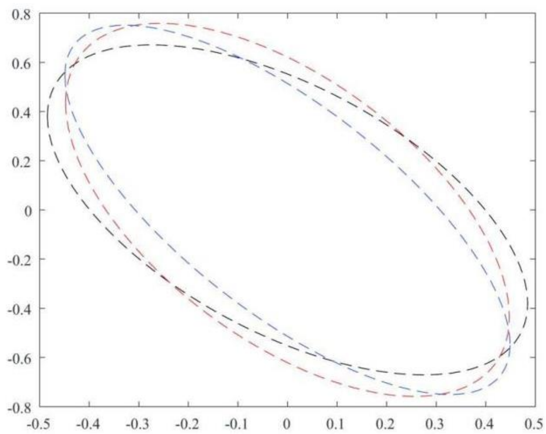

Figure 1. The ellipsoidal bound $\Im$ and state trajectory of system (27) for case 1.

By using theorem 1 and solving the optimization problem (1) in case 2, we can obtain a minimization of $\det(P_{1i})^{-\frac{1}{2}}$ is 0.3356 when $\alpha = 0.1$, and the corresponding feasible matrices are given as $P_{11} = \begin{bmatrix} 3.6977 & 1.2090 \\ 1.2090 & 2.7969 \end{bmatrix}, P_{12} = \begin{bmatrix} 4.8705 & 1.1716 \\ 1.1716 & 1.7549 \end{bmatrix}, P_{13} = \begin{bmatrix} 3.9786 & 1.5603 \\ 1.5603 & 1.7610 \end{bmatrix}$. The reachable sets of the system (1) in case 2 is bounded by a intersection of three ellipsoids: $\bigcap_{i=1}^{3} \Im\left(\tilde{P}_{1,i}\right)$ which is depicted in Figure 2.

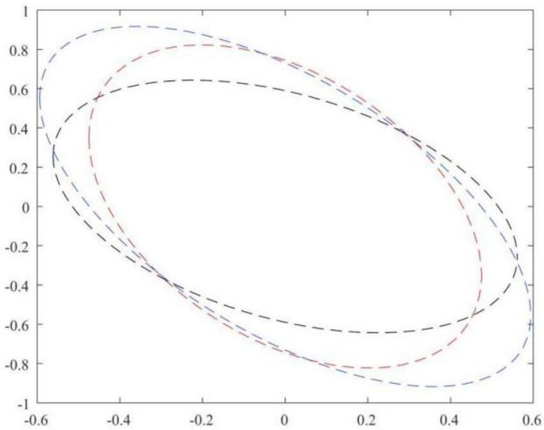

Figure 2. The ellipsoidal bound $\Im$ and state trajectory of system (27) for case 2.

By using corollary 2 and solving the optimization problem (1) in case 3, we can obtain minimization of $\det(P_{1i})^{-\frac{1}{2}}$ is 0.3311 when $\alpha = 0.1$, and the corresponding feasible matrices are given as $P_{11} = \begin{bmatrix} 3.7721 & 1.3125 \\ 1.3125 & 2.8745 \end{bmatrix}, P_{12} = \begin{bmatrix} 5.4273 & 1.3360 \\ 1.3360 & 1.8069 \end{bmatrix}, P_{13} = \begin{bmatrix} 4.4624 & 1.7442 \\ 1.7442 & 1.8708 \end{bmatrix}$. The reachable sets of the system (1) in case 3 is bounded by a intersection of three ellipsoids: $\bigcap_{i=1}^{3} \Im\left(\tilde{P}_{1,i}\right)$, which is depicted in Figure 3.

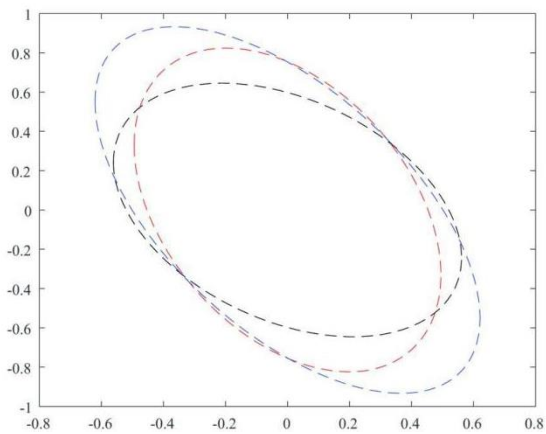

Figure 3. The ellipsoidal bound $\Im$ and state trajectory of system (27) for case 3.

Example 2. Consider the neutral Markov jump system and its state matrix is as follows

$$\begin{cases} \dot{x}(t) - C_{(t,r_t)}\dot{x}(t-0.1) = A_{(t,r_t)}x(t) + B_{(t,r_t)}x(t-0.5) + D_{(t,r_t)}\omega(t), \\ x(t_0+\theta) = 0, \forall \theta \in [-\rho, 0], \end{cases} \tag{27}$$

$$A_1=\begin{bmatrix}-2 & -1\\ 0 & -2\end{bmatrix}, B_1=\begin{bmatrix}-1 & 0\\ -1 & -2\end{bmatrix}, C_1=\begin{bmatrix}-1 & 0\\ -1 & -2\end{bmatrix}, D_1=\begin{bmatrix}-0.15\\ 0.15\end{bmatrix},$$

$$A_2=\begin{bmatrix}-3 & 0\\ 0 & -2\end{bmatrix}, B_2=\begin{bmatrix}-2 & 0\\ -1 & -1\end{bmatrix}, C_2=\begin{bmatrix}-2 & 0\\ -1 & -1\end{bmatrix}, D_2=\begin{bmatrix}-0.14\\ 0.35\end{bmatrix},$$

$$A_3=\begin{bmatrix}-2 & 0\\ -1 & -1.5\end{bmatrix}, B_3=\begin{bmatrix}-2 & 0\\ 0 & -2\end{bmatrix}, C_3=\begin{bmatrix}-2 & 0\\ 0 & -2\end{bmatrix}, D_3=\begin{bmatrix}-0.2\\ 0.3\end{bmatrix}.$$

Firstly, by giving the transition probabilities, Λ a possible mode evolution of the neutral Markov jump system (27) is derived as shown in Figure 4. Based on the mode evolution shown in Figure 4, and choosing disturbances $\omega(t)$ as the random signal satisfying $\omega^T(t)\omega(t)\le 1$, all the reachable states of neutral Markov jump system (27) starting from the origin are given in Figure 5.

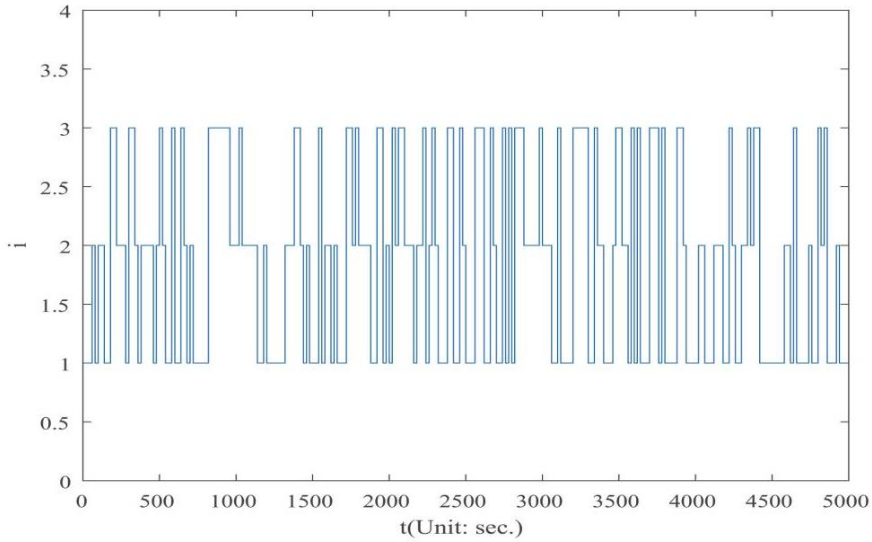

Figure 4. Random jumping mode $r(t)$ of neutral Markov jump system (27).

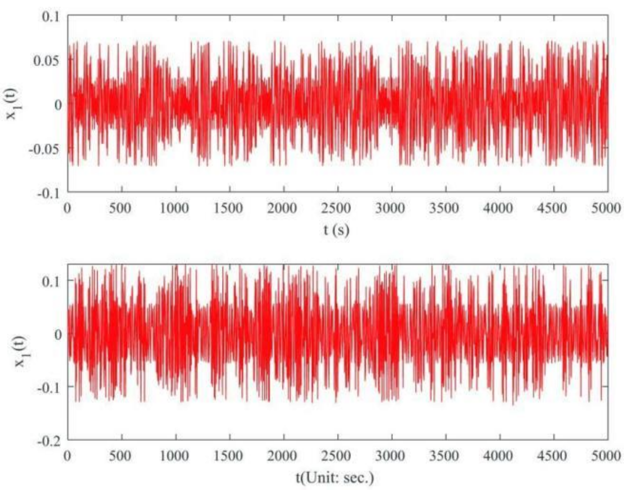

Figure 5. The time responses of state variable $x(t)$ of Markov jump system (27) for case 1.

By using corollary 1 and solving the optimization problem (27) in case 1, we can obtain minimization of $\det\left(P_{1i}\right)^{-\frac{1}{2}}$ is 0.4694 when $\alpha = 0.1$, and the corresponding feasible matrices are given as $P_{11} = \begin{bmatrix} 2.1532 & 0.1069 \\ 0.1069 & 2.1130 \end{bmatrix}, P_{12} = \begin{bmatrix} 1.9186 & 0.2970 \\ 0.2970 & 1.3932 \end{bmatrix}, P_{13} = \begin{bmatrix} 1.5613 & 0.3191 \\ 0.3191 & 1.1630 \end{bmatrix}$. The reachable sets of the system (27) in case 1 is bounded by a intersection of three ellipsoids: $\det\left(P_{1i}\right)^{-\frac{1}{2}}$, which is depicted in Figure 6.

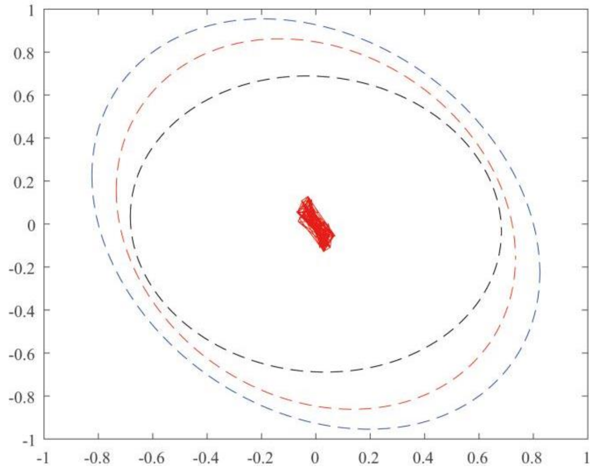

Figure 6. The ellipsoidal bound $\Im$ and state trajectory of system (27) for case 1.

By using theorem 1 and solving the optimization problem (27) in case 2, we can obtain minimization of $\det\left(P_{1i}\right)^{-\frac{1}{2}}$ is 0.4620 when $\alpha = 0.1$, and the corresponding feasible matrices are given as $P_{11} = \begin{bmatrix} 2.1557 & 0.1187 \\ 0.1187 & 2.1800 \end{bmatrix}, P_{12} = \begin{bmatrix} 1.8260 & 0.1895 \\ 0.1895 & 1.3434 \end{bmatrix}, P_{13} = \begin{bmatrix} 1.5160 & 0.3384 \\ 0.3384 & 1.0547 \end{bmatrix}$. The reachable sets of the system (27) in case 2 is bounded by a intersection of three ellipsoids: $\bigcap_{i=1}^{3} \Im\left(\tilde{P}_{1,i}\right)$, which is depicted in Figure 7.

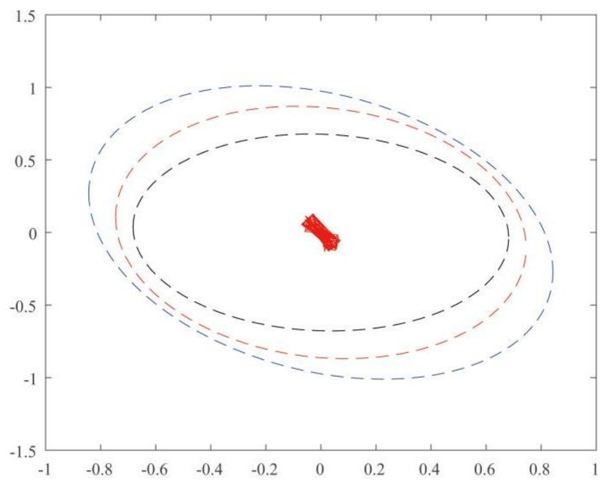

Figure 7. The ellipsoidal bound $\Im$ and state trajectory of system (27) for case 2.

By using corollary 2 and solving the optimization problem (27) in case 3, we can obtain minimization of $\det\left(P_{1i}\right)^{-\frac{1}{2}}$ is 0.4636 when $\alpha = 0.1$, and the corresponding feasible matrices are given as

$P_{11}=\begin{bmatrix}2.1518 & 0.1120\\ 0.1120 & 2.1676\end{bmatrix}, P_{12}=\begin{bmatrix}1.8648 & 0.1834\\ 0.1834 & 1.3468\end{bmatrix}, P_{13}=\begin{bmatrix}1.5554 & 0.3187\\ 0.3187 & 1.0551\end{bmatrix}$. The reachable sets of the system (27) in case 3 is bounded by a intersection of three ellipsoids: $\bigcap_{i=1}^{3}\Im\left(\tilde{P}_{1,i}\right)$, which is depicted in Figure 8.

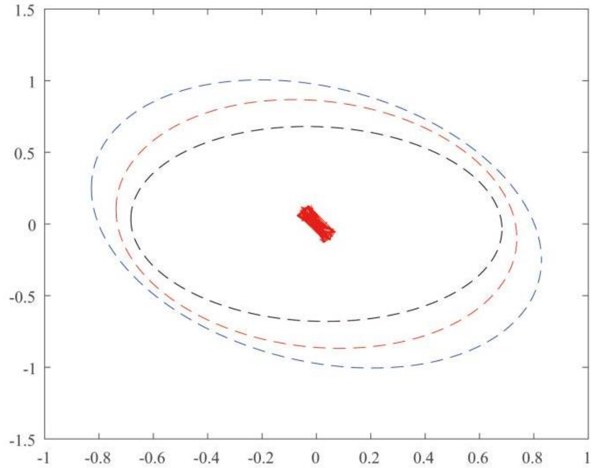

Figure 8. The ellipsoidal bound $\Im$ and state trajectory of system (27) for case 3.

5. Conclusion

In this paper, we study the reachable set boundary estimation for neutral Markov jump systems with time-varying and distributed delays. By choosing the appropriate Lyapunov function, using the infinitesimal operator and matrix inequality to analyze the constructed functional, and then using the definition of the reachable set and the necessary lemma conditions, the boundary conditions of the reachable set satisfying the definition and as small as possible are obtained. Finally, numerical examples are given to verify the effectiveness and correctness of the results.

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