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Using the Pythagorean Theorem to Make a Square Equal to the Area of a Circle

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Abstract

The drawing method of turning a circle into a square explored in this study does not need the circumference of a circle, nor does it need to draw a line segment equal to a circle or an arc. It only needs to draw 1800 arcs and diameters and draw a right triangle on a semicircle with the hypotenuse R as a right-angled side, and the square of the other right-angled side reaches 95.49% of the area of the circle. According to the Pythagorean theorem, the square of the other right-angled side can be increased. When the trial coefficient K = 0.9265, The square of the other right-angled side is equal to the area of the circle by 5 digits. In the process of continuing to try, a formula for calculating k is found, so that the diameter of the given circle is a right-angled triangle with the hypotenuse K*R as the right-angled side, and the square of the other right-angled side is equal to s =$\sqrt{\pi R^2}$.

Keywords

R is the radius; K is the coefficient; Pythagorean theorem; Area of circle = s = R2

1. Introduction

Archimedes, a Greek mathematician, discussed the study of turning a circle into a square: "When the radius of a circle is known, the circumference is 2 πr. From this, if a right-angled triangle can be made, and the lengths of the two sides between the right angles are the circumference of a known circle 2 and the radius R, then the area of this triangle is equal to half the area of its circle. It is not difficult to make a square with the same area as this triangle." However, how to make the side of this right triangle, that is, how to make a line segment equal to the circumference of a known circle, Archimedes can't solve this problem. Later, I realized that it is impossible to complete this problem only by using rulers and compasses. Since then, mathematicians have had to admit that the ruler can't make a square equal to the area of a circle and give up the research on this difficult problem. In fact, there are other methods to study turning a circle into a square, such as making full use of a ruler to draw irrational numbers, making transcendental numbers into integer sets and rational number fields, making square roots, and changing the proportion of two right-angled sides by using Pythagorean theorem [1-3]. Based on this, it is possible to make a square equal to the area of a circle. Drawing analysis: make a right triangle ZJC1 on the semicircle, ZJ = 2R is the hypotenuse, JC1 = R is the right-angled side, and the other right-angled side Z C1 = √(2R2-R2), and the square surface with Z C1 as the side length = $\sqrt{3}$ has a circular area of $\sqrt{3.141582653}$ 95.4929674% (See Figure 1).

According to the Pythagorean theorem, narrowing the right-angled side can increase the square of the other right-angled side and make it closer to 99.7934167%.

Increase the coefficient to 0.9265, JC2 = 0.9265R. The area of the square with the other right angle ZC2 as the side length is equal to the area of the square R2, and the number of digits reaches 5, as shown in Figure 1. If R=1.6, R _ 2 = 3.1415926 * 1.62 = 8.04247, ZC2 = ZJ - JC = 10.24 - 2.197509 = 8.04247, the area of a square line segment drawn in this drawing is equal to the area of a circle with 6 digits. The difference is 2.0 parts per million. More importantly, in the process of trying to get the exact coefficient, the formula for calculating the coefficient was found, that is, K = √[(2R2)- Σ R2] = √ 4-3.141592653 = √ 0.858407347 = 0.926502751. The coefficient obtained by this method can increase with the increase of the number of digits, which is the calculated coefficient, so the square area made in this drawing is equal to √ R2. Give examples to prove:

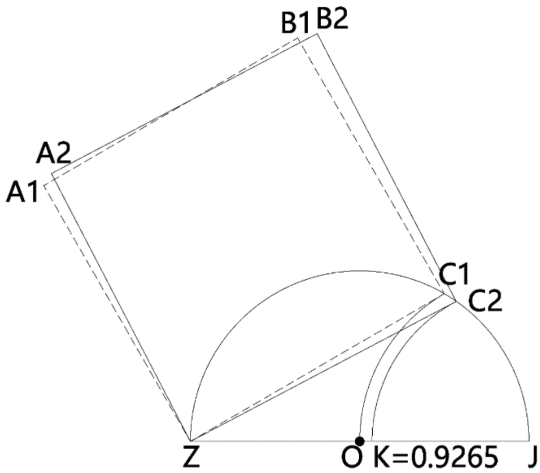

Figure 1. An attempt to turn a circle into a square.

2. Theoretical proof

Example 1

Known: r = 0.7, k = 0.952751,

Find the line segment ZC, so that ZC2 = UR2 = 3.14653 * 0.72.

Drawing: See Figure 2 for the proof of turning a circle into a square.

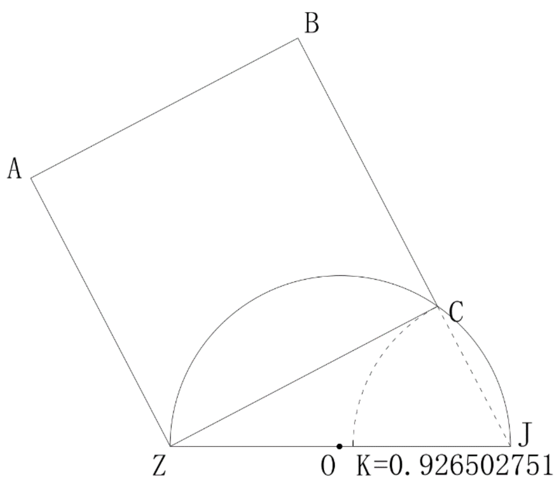

Figure 2. Proof diagram of rounding into a square.

1) Draw an arc on a straight line with a point as the center and a radius, and cross both ends of the straight line at point Z and point J respectively. $R = 0.7$
2) Make KJ on OJ line, so that KJ = 0.96502751R
3) Take point J as the center and KJ as the radius, and make an arc intersecting ZJ at point C,
4) Connect ZC, JC
5) Make a square AB with four sides = ZC. ZC

Verification: zc2 = √ 3.141592653 * 0.72 = √ 1.5804.

Prove:

$$\because \overrightarrow{OJ} = R,\ R = 0.7,\ K = 0.926502751,\ K \times R = 0.7 \times 0.926502751$$

$\therefore \sqrt{\overrightarrow{KJ}^2} = \sqrt{(0.926502751 \times 0.7)^2} = \sqrt{0.648551925^2} = \sqrt{0.4206196}$

$\because \overrightarrow{JK} = \overrightarrow{JC}$, $\overrightarrow{JK} = \sqrt{0.4206196} = 0.648551925$

$\therefore \sqrt{\overrightarrow{JC}^2} = \sqrt{0.4206196} = 0.648551925$

$\because \overrightarrow{OJ} = \overrightarrow{OC} = \overrightarrow{OZ} = \overrightarrow{ZJ}/2$

∴∠ZCJ=900, $\perp \overrightarrow{ZC}\overrightarrow{JC}$

∵∠ZCJ=900, $\perp \overrightarrow{ZC}\overrightarrow{JC}$

∴ ZJC=⊿

$\because \varDelta ZJC$,, ,$\sqrt{\overrightarrow{ZC}^2} = \sqrt{\overrightarrow{ZJ}^2 - \overrightarrow{JC}^2}\sqrt{\overrightarrow{JC}^2} = \sqrt{0.4206196}\sqrt{\overrightarrow{ZJ}^2} = \sqrt{1.4^2} = \sqrt{1.96}$,

$\therefore \sqrt{\overrightarrow{ZC}^2} = \sqrt{\overrightarrow{ZJ}^2 - \overrightarrow{JC}^2} = \sqrt{1.96 - 0.4206196} = \sqrt{1.5393804}$,

$\because \sqrt{\overrightarrow{ZC}^2} = \sqrt{\overrightarrow{ZJ}^2 - \overrightarrow{JC}^2} = \sqrt{1.96 - 0.4206196} = \sqrt{1.5393804}$

$\sqrt{\pi R^2} = \sqrt{3.141592653 \times 0.7^2} = \sqrt{1.5393804}$

$\therefore \sqrt{\overrightarrow{ZC}^2} = \sqrt{\pi R^2}$, which is the area of a square with side length equal to $\overrightarrow{ZC}\sqrt{\pi R^2}$.

Example 2

Known: r = 1.9, k = 0.952751,

Find the line segment ZC, so that ZC2 = UR2 = 3.14653 * 1.92.

Drawing: See Figure 2 for the proof of turning a circle into a square.

1) Draw an arc on a straight line with the point as the center and R = 1.9 as the radius, and cross the two ends of the straight line at point Z and point J respectively.O
2) make KJ on OJ line, so that KJ=0.96502751R
3) Take point J as the center and KJ as the radius, and make an arc intersecting ZJ at point C,
4) Connect ZC, JC
5) Make a square AB with four sides = ZC. ZC

Proof: zc2 = √ 3.14592653 * 1.92 = √ 11.40889.99989889895

Prove:

$\because \overrightarrow{OJ} = R, R = 1.9, K = 0.926502751, K \times R = 1.9 \times 0.926502751$

$\therefore \sqrt{\overrightarrow{KJ}^2} = \sqrt{(0.926502751 \times 1.9)^2} = \sqrt{1.760355227^2} = \sqrt{3.098850525}$

$\because \overrightarrow{JK} = \overrightarrow{JC}$, $\sqrt{\overrightarrow{JK}^2} = \sqrt{1.760355227^2} = \sqrt{3.098850525}$

$\therefore \sqrt{\overrightarrow{JC}^2} = \sqrt{1.760355227^2} = \sqrt{3.098850525}$

$\because \overrightarrow{OJ} = \overrightarrow{OC} = \overrightarrow{OZ} = \overrightarrow{ZJ}/2$

∴∠*ZCN=900*, $\perp \overrightarrow{ZC}\overrightarrow{NC}$

∵∠ZCN=900, $\perp \overrightarrow{ZC}\overrightarrow{NC}$

∴ ZJC=⊿

$\because \varDelta ZJC$, $\sqrt{\overrightarrow{ZC}^2} = \sqrt{\overrightarrow{ZJ}^2 - \overrightarrow{JC}^2}$, $\overrightarrow{CJ} = \sqrt{3.098850525}$ $\overrightarrow{ZJ} = \sqrt{3.8^2} = \sqrt{14.44}$

$\therefore \sqrt{\overrightarrow{ZC}^2} = \sqrt{\overrightarrow{ZJ}^2 - \overrightarrow{JC}^2} = \sqrt{14.44 - 3.098850525} = \sqrt{11.34114948},$

$\because \sqrt{\overrightarrow{ZC}^2} = \sqrt{11.34114948}, \sqrt{\pi R^2} = \sqrt{3.141592653 \times 1.9^{\ 2}} = 11.34114948$

$\therefore \sqrt{\overrightarrow{ZC}^2}$=, the area of a square with side length is equal to $\sqrt{\pi R^2}\overrightarrow{ZC}\sqrt{\pi R^2}$.

Example 3

Known: r = 0.876, k = 0.95751,

Find the line segment ZC, so that ZC2 = UR2 = 3.14653 * 0.8762.

Drawing: See Figure 2 for the proof of turning a circle into a square.

1) Draw an arc on a straight line with the point as the center and R = 1.9 as the radius, and cross the two ends of the straight line at point Z and point J respectively. O
2) make KJ on OJ line, so that KJ = 0.96502751R
3) Take point J as the center and KJ as the radius, and make an arc intersecting ZJ at point C,
4) Connect ZC, JC
5) Make a square AB with four sides = ZC. ZC

Verification: zc2 = √ 3.141592653 * 0.8762 = √ 2.40803.0000000006

Prove:

$\because \overrightarrow{OJ} = R, R = 0.876,\ K =\ 0.926502751, K \times R = 0.876 \times 0.926502751$

$\therefore \sqrt{\overrightarrow{KJ}^2} = \sqrt{(0.926502751 \times 0.876)^2} = \sqrt{0.811616409^2} = \sqrt{0.658721196}$

$\because \overrightarrow{JK} = \overrightarrow{JC},\ \ \sqrt{\overrightarrow{JK}^2} = \sqrt{0.811616409^2} = \sqrt{0.658721196}$

$\therefore \sqrt{\overrightarrow{JC}^2} = \sqrt{0.811616409^2} = \sqrt{0.658721196}$

$\because \overline{\mathrm{OJ}} = \overrightarrow{OC} = \overrightarrow{OZ} = \overline{\mathrm{ZJ}}/2$

∴∠ZCN=900, $\perp\overrightarrow{ZC}\overrightarrow{JC}$

$\because \angle ZCN = 900, \overrightarrow{ZC} \perp \overrightarrow{JC}$

∴ ZJC=⊿

$\because ⊿ZJC,\ \ \sqrt{\overrightarrow{ZC}^2} = \sqrt{\overrightarrow{ZJ}^2 - \overrightarrow{JC}^2},\ \ \ \overrightarrow{CJ}^2 = \sqrt{0.658721196}\ \overrightarrow{ZJ}^2 = \sqrt{1.752^2} = \sqrt{3.069504}$

$\therefore \sqrt{\overrightarrow{ZC}^2} = \sqrt{\overrightarrow{ZJ}^2 - \overrightarrow{JC}^2} = \sqrt{3.069504 - 0.658721196} = \sqrt{2.410782804}$

$\because \sqrt{\overrightarrow{ZC}^2} = \sqrt{3.069504 - 0.658721196} = \sqrt{2.410782804}$

$\sqrt{\pi R^2} = \sqrt{3.141592653 \times 0.876^2} = \sqrt{2.410782804}$

∴=$\sqrt{\overrightarrow{ZC}^2}\sqrt{\pi R^2}$ $\quad \overrightarrow{ZC}$ The area of a square with side length is equal to $\sqrt{\pi R^2}$**.**

Example 4

Known: r = 2.345, k = 0.95751,

Find the line segment ZC, so that ZC2 = Σ R2 = 3.14653 * 2.3452.

Drawing: See Figure 2 for the proof of turning a circle into a square.

1) Draw an arc on a straight line with the point as the center and R = 1.9 as the radius, and cross the two ends of the straight line at point Z and point J respectively. O
2) make KJ on OJ line, so that KJ = 0.96502751R
3) Take point J as the center and KJ as the radius, and make an arc intersecting ZJ at point C,
4) Connect ZC, JC

5) Make a square AB with four sides = ZC. ZC
Proof: zc2 = √ 3.14592653 * 2.3452 = √ 17.50089.00000000005
Prove:

$\because \overrightarrow{OJ} = R$， $R = 2.345$，$K = 0.926502751$，$K \times R = 0.926502751 \times 2.345$

$\therefore \sqrt{\overrightarrow{KJ}^2} = \sqrt{(0.926502751 \times 2.345)^2} = \sqrt{2.17264898851^2} = \sqrt{4.720403465}$

$\because \overrightarrow{JK} = \overrightarrow{JC}$, $\overrightarrow{JK} = \sqrt{22.28220887} = 4.720403465$

$\therefore \sqrt{\overrightarrow{JC}^2} = \sqrt{22.28220887} = 4.720403465$

$\because \overrightarrow{\mathrm{OJ}} = \overrightarrow{OC} = \overrightarrow{OZ} = \overrightarrow{\mathrm{ZJ}}/2$

∴∠ZCN=900, $\perp \overrightarrow{ZC}\overrightarrow{NC}$

∵∠ZCN=900, $\perp \overrightarrow{ZC}\overrightarrow{NC}$

$\therefore \sqrt{\overrightarrow{ZC}^2} = \sqrt{\overrightarrow{ZJ}^2 - \overrightarrow{JC}^2} = \sqrt{21.9961 - 4.720403465} = \sqrt{17.27569654},$

$\because \sqrt{\overrightarrow{ZC}^2} = \sqrt{17.27569654}$, $\sqrt{\pi R^2} = \sqrt{3.141592653 \times 2.345^2} = \sqrt{17.27569654}$

$\therefore \sqrt{\overrightarrow{ZC}^2}$=， $\sqrt{\pi R^2}\ \overrightarrow{ZC}$The area of a square with side length is equal to $\sqrt{\pi R^2}$.

3. Conclusion

Rulers can add, subtract, multiply, divide any line segment and make square roots to make irrational numbers into regular line segments, and turn irrational numbers into integer sets and rational number fields. It can also make known line segments as right-angled sides and hypotenuse as right-angled triangles. According to the Pythagorean theorem, the edge can be reduced to the required length of another right-angled side. By making full use of the advantages of ruler drawing, it can be reduced to the required length, which is the condition that the ruler can make the square area equal to the circle area. K = √(2R) 2- √ R2 = 0.926502751, so the square area is equal to √(2R) 2-KR2. Conclusion: The ruler can make a square equal to the area of a circle. The drawing method is that the area of a given circle is equal to the area of the circle with the diameter of the hypotenuse of a right triangle, K*R as the right-angle side and the other angle side as the square $R2RRR$.

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