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# The Solution Of 2D Hydrodynamic Equations In The Boussinesq Approximation: A Mechanism Of Hydrocarbons Transport To The Earth's Surface

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## Abstract

Thermomechanical model of the mantle wedge subducting under the lithospheric plate with a velocity $V$ at an angle $\beta$ is obtained for the infinite Prandtl number fluid as a solution of non-dimensional 2D hydrodynamic equations in the Boussinesq approximation. Numerical modeling of mantle thermal dissipation-driven convection accounting for phase transitions and temperature and pressure dependence of the "wet" olivine viscosity shows the characteristic convective rolls scale be approximately of same scale as the spatial wavelength of localization of the oil- and gas-bearing zones in the Timan-Pechora region. The scale of localization periodicity of the oil- and gas-bearing zones equals approximately to convection cell dimension this equality being an extra evidence for convective mechanism of non-organic hydrocarbons transport to the Earth's surface.

## Keywords

2D hydrodynamic equations, Boussinesq approximation, Runge-Kutta method, hydrocarbons transport, thermal convection, subduction angle and velocity

## 1. Introduction

Assuming mantle hydrocarbons were transported to the Timan-Pechora lithospheric plate surface by the mechanism of thermal convection of the form of variable thickness rolls aligned perpendicular to subduction the scale of convective rolls being the same as the scale of the relief and density variations wavelength, one can estimate the mean velocity of the former subduction of the East-European plate at closing Urals ocean in Paleozoic (~5–6 cm$\times$yr$^{-1}$). This can be done taking into account total horizontal extent of the Timan-Pechora oil- and gas-bearing province similarly to [1]. As this estimate is obtained on the assumption that the mantle wedge is a constant viscosity fluid, it seems important to calculate the scale and the intensity of convection accounting for the effects of phase transitions and more realistic mantle rheology with the temperature- and pressure-dependent viscosity. The latter in the mantle wedge is probably rather small, i.e. ~$10^{18}$ Pa$\times$s or less, what can be due to the presence of water upwelling to the mantle wedge from the subducting slab [2]. As is indicated by [3], additional $10^2$ –$10^3$ g of water in the tone of rock reduces viscosity by two orders of magnitude as compared to dry conditions. The model proposed accounts for the mentioned characteristics of the mantle wedge rheology, i.e. rather low pressure- and temperature dependent viscosity.

## 2. Statement of research

Thermomechanical model of the mantle wedge between the base of the Timan-Pechora lithospheric plate and the upper surface of the East-European lithospheric plate subducting under the Timan-Pechora one with a velocity $V$ at an angle $\beta$ is obtained for the infinite Prandtl number fluid as a solution of non-dimensional 2D hydrodynamic equations in the Boussinesq approximation [4]:

$$(\partial^2_{zz} - \partial^2_{xx})\eta(\partial^2_{zz} - \partial^2_{xx})\psi + 4\partial^2_{xz}\eta\partial^2_{xz}\psi = RaT_x - Ra^{(410)}\Gamma^{(410)}_x - Ra^{(660)}\Gamma^{(660)}_x, \quad (1)$$

$$\partial_t T = \Delta T - \psi_z T_x + \psi_x T_z + \frac{Di}{Ra}\times\frac{\tau^2_{ik}}{2\eta} + Q. \quad (2)$$

for the streamfunction $\psi$ and temperature $T$. Here $\eta$ is non-dimensional dynamic viscosity, $\partial$ and indices denote partial derivatives with respect to coordinates $x$ (horizontal), $z$ (vertical) and time $t$, $\Delta$ is the Laplace operator, $\Gamma^{(410)}$ and $\Gamma^{(660)}$ are volume ratios of the heavy phase at the 410 km and 660 km phase boundaries, the velocity components $V_x$ and $V_z$ are expressed through $\psi$ as

$$V_x = \psi_z, \quad V_z = -\psi_x \quad (3)$$

and non-dimensional Rayleigh number $Ra$, phase $Ra^{(410)}$, $Ra^{(660)}$ and dissipative number $Di$ are

$$Ra = \frac{\alpha\rho g d^3 T_1}{\bar{\eta}\chi} = 5.55\times10^8, \quad Ra^{(410)} = \frac{\delta\rho^{(410)} g d^3}{\bar{\eta}\chi} = 6.6\times10^8,$$

$$Ra^{(660)} = \frac{\delta\rho^{(660)} g d^3}{\bar{\eta}\chi} = 8.5\times10^8, \qquad Di = \frac{\alpha g d}{c_p} = 0.165, \quad (4)$$

where $a = 3\cdot10^{-5}$ K$^{-1}$ is thermal expansion coefficient, $\rho$ = 3.3 g×cm$^{-3}$ is the density, $g$ is gravity acceleration, $c_p = 1.2\times10^3$ J×kg$^{-1}$×K$^{-1}$ is heat capacity at constant pressure, $T_1$ = 1950 K is the temperature at the mantle transient zone (MTZ) base at 660 km depth, regarded the lower boundary of the modeled domain, $Q = 6.25\cdot10^{-4}$ μ×W×m$^{-3}$ is the volumetric radiogenic heat relies power in the crust, $\tau_{ik}$ is the viscous stress tensor, $d$ = 660 km is the vertical dimension of the modeled domain, $\bar{\eta} = 10^{18}$ Pa×s is the viscosity scaling factor, $\chi = 10^{-2}$ cm$^2$×s$^{-1}$ is thermal diffusivity, $\delta\rho^{(410)} = 0.07\rho$ and $\delta\rho^{(660)} = 0.09\rho$ are the density changes at phase transitions at 410 km и 660 km depths. In (1), (2) the scaling factors for time $t$, stresses $\tau_{ik}$ and stream-function $\psi$ are ($d^2 \times \chi^{-1}$), $\bar{\eta}\chi\cdot d^{-2}$ and $\chi$ respectively. Previously in [5] convection was modeled in the assumption of linear rheology for the diffusion creep mechanism, dominating in the mantle at depths over ~200 km [6], and temperature $T$ and lithostatic pressure $p$ viscosity $\eta$ dependence was taken as [2]

$$\eta = \frac{\mu}{2A}\left(\frac{h}{b^*}\right)^m \exp\frac{E^* + pV^*}{RT}, \quad (5)$$

where for wet olivine $A = 5.3\times10^{15}$ s$^{-1}$, $m$ = 2.5, the grain size $h = 10^{-2}$–1 cm, Burgers vector is $b^* = 5\times10^{-7}$ mm [7], activation energy is $E^*$=240 kJ×mol$^{-1}$, activation volume $V^*$ = 5 cm$^3$ ×mol$^{-1}$, $\mu$ = 300 GPa is normalizing factor of the shear modulus, $R$ is universal gas constant. Under grain size $h$ = 1.6 mm, $\bar{\eta} = 10^{18}$ Pa×s and abovementioned values of constants non-dimensional viscosity also denoted $\eta$ is

$$\eta = 5.0\times10^{-7}\exp\frac{14.8 + 6.72\times(1-z)}{T}, \quad (6)$$

where $T$ is non-dimensional temperature, and $z$, normalized by $d$, is vertical coordinate measured upwards from the MTZ base ($x$-axis is pointing along the MTZ base against subduction). The modeled domain aspect ration is 1:6, i.e. for diagonal subduction the angle of subduction is $\beta = 9°$, while the trial subduction velocity $V$ = 6 cm×yr$^{-1}$ scaled by ($d^{-1} \times \chi$) is $V = 1.25\times10^3$ its components in subducting slab being $V_x = -1.233\times10^3$ and $V_z = -0.164\times10^3$. Following [8] we assume the phase functions $\Gamma^{(l)}$ as

$$\Gamma^{(l)} = \frac{1}{2}\left(1 - th\frac{z - z^{(l)}(T)}{w^{(l)}}\right), \quad z^{(l)}(T) = z_0^{(l)} - \frac{\gamma^{(l)}}{\rho g}(T - T_0^{(l)}), \quad (7)$$

where the signs are changed as $z$-axis is pointing upwards, $z^{(l)}(T)$ is the depth of the $l$-th phase transition ($l$ = 410, 660), $z_0^{(l)}$ and $T_0^{(l)}$ are the averaged depth and temperature of the $l$-th phase transition, $\gamma^{(410)} = 3$ MPa$\times$K$^{-1}$ and $\gamma^{(660)} = -3$ MPa$\times$K$^{-1}$ are the slopes of the phase equilibrium curves, $w^{(l)}$ is the characteristic thickness of the $l$-th phase transition, $T_0^{(410)} = 1800^\circ$ K, $T_0^{(660)} = 1950^\circ$ K are the mean phase transition temperatures. The heats of phase transitions are neglected in (2) as in [8]. From (7) it follows

$$\Gamma_x^{(l)} = -(g^{(l)} / 2 \times r \times g \times w^{(l)}) \times T_x \times ch^{-2}\{[(z - z_o^{(l)} + g^{(l)} \times (T - T_o^{(l)})) / (r \times g)] / w^{(l)}\} \quad (8)$$

where from it is clear the phase transition with $\gamma^{(l)} > 0$ facilitates convection (at $l$ = 410), while the phase transition with $\gamma^{(l)} < 0$ hinders convection (at $l$ = 660). In non-dimensional form $z_0^{(410)} = 0.38$, $z_0^{(660)} = 0$, $w^{(l)} = 0.05$, $\gamma^{(410)} = 2.5\times10^9$, $\gamma^{(660)} = -2.5\times10^9$, $T_0^{(410)} = 0.92$, $T_0^{(660)} = 1$, and in (1)

$$\Gamma_x^{(l)} = -(\mathrm{d} \times r^{(l)} \times g^{(l)} / 2 \times r \times Ra^{(l)} \times w^{(l)}) \times T_x \times ch^{-2}\{[z - z_o^{(l)} - g^{(l)} \times (\mathrm{d}\, r^{(l)} / r \times Ra^{(l)}) (T - T_o^{(l)})] / w^{(l)}\} \quad (9)$$

Equations (1)–(2) are solved for the isothermal horizontal and insolated vertical boundaries regarded no-slip impenetrable ones except for the "windows" for in- and outgoing subducting plate, where the plate velocity is specified. Vertical boundary distant from subduction zone is assumed penetrable at right angle, the latter boundary condition appears not too imposing in the case of very flat subduction. $Q$ in (2) is non-zero in the continental and oceanic crust 40 and 7 km thick. Initial vertical boundaries temperature is calculated for the half-space cooling model for $10^9$ $a$ and $10^8$ $a$ for Timan-Pechora (continental) and East-European (oceanic) lithospheric plates respectively. It is convenient to express dimensionless $t^2_{ik}$ in (2) through the streamfunction $\psi$.

$$\tau_{ik}^2 = 4\eta^2[(\psi_{zz} - \psi_{xx})^2 / 2 + 2\psi_{xz}^2].$$

## 3. Results and Discussion

Consistent model of small-scale convection in the mantle wedge between the overriding Timan-Pechora lithospheric plate and subducting East-European lithospheric plate should be constructed, first, by specifying in (1)–(2) vanishing non-dimensional numbers $Ra \to 0$, $Di = 0$, i.e. ignoring convection and viscous dissipation. This approach is applied as convection with $Ra$ and $Di$ (4) is very vigorous and the time steps in integrating (1)–(2) should be chosen very small thus making it difficult to model the thermal structure of the plates. Solving (1)–(2) by the finite element method in space on the grid 104×104 and the 3-rd order Runge-Kutta method in time one obtains for $Ra \to 0$, $Di = 0$ non-dimensional quasi steady-state $\psi$ and $T$ shown in Figures 1, 2, where the streamlines in Figure 1 are depicted with the step 0.25 and the isotherms in Figure 2 with an interval 0.05. Subducting plate was considered rigid, while the viscosity at the zone of plates friction (at temperatures below $1200^\circ$ K) was reduced by 2 orders of magnitude as compared to (5). The latter viscosity reduction at the plates contact zone accounts for lubrication effected by deposits partially entrained by the subducting plate. Such a lubrication prevents the overriding plate from gluing to the subducting one. Assuming $Ra = 5.55 \times 10^8$ and $Di$ = 0.165, i.e. switching dissipation and convection on, and taking into account effects of phase transitions, we obtain from (1)–(2) the forced mantle flow in the mantle wedge (shown by negative streamlines in Figure 1) to be destroyed during the time interval ~ $1.5\times10^{-4}$ (in dimensional form ~1.3 Myr) by convective vortices shown in Figure 3 with the streamlines interval 0.25. The scale of convective vortices shown in Figure 3 is of the order of 250 – 300 km while the streamlines density in them corresponds to several tens of cm$\times$yr$^{-1}$. Figure 4 presents non-dimensional steady-state isotherms. It should be noted the perpendicular to subduction convective rolls in the mantle wedge do probably originate exclusively in the case of very flat subduction being absent already at subduction angle $\beta = 30$ °[5]. The presence of 2D small-scale convection in the mantle wedge appears to be due to greater stresses (and consequently greater dissipative heating) in the narrow mantle wedge as compared to the wider one. Arrows ($a$), ($b$), ($c$) and ($d$) above the boundaries of the oppositely revolving convective vortices in Figure 3 indicate possible directions of transport of non-organic mantle hydrocarbons. The characteristic scale of convective vortices is clearly of the order of spatial wavelength of oil- and gas-bearing zones location in the region under consideration.

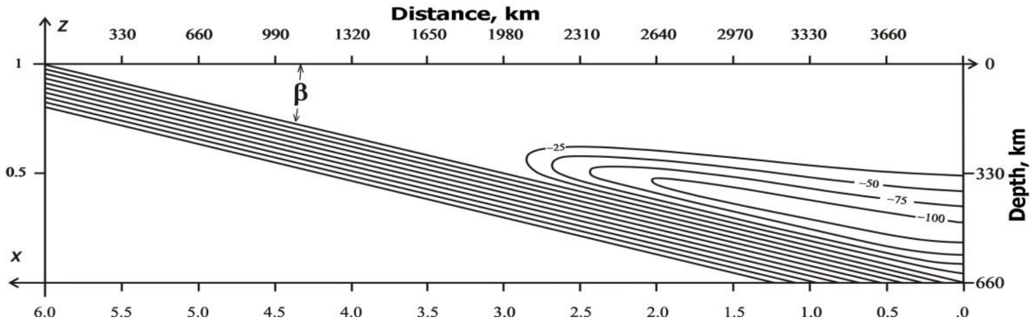


**Figure 1**. Quasi steady-state non-dimensional streamfunction in the zone of subduction of the East-European lithospheric plate under the Timan-Pechora lithospheric plate at Peleozoic with no effects of dissipative heating and small scale convection. Parallel equidistant streamlines represent subducting East-European lithospheric plate, streamlines with negative $\psi$ correspond to mantle flow induced by subduction.

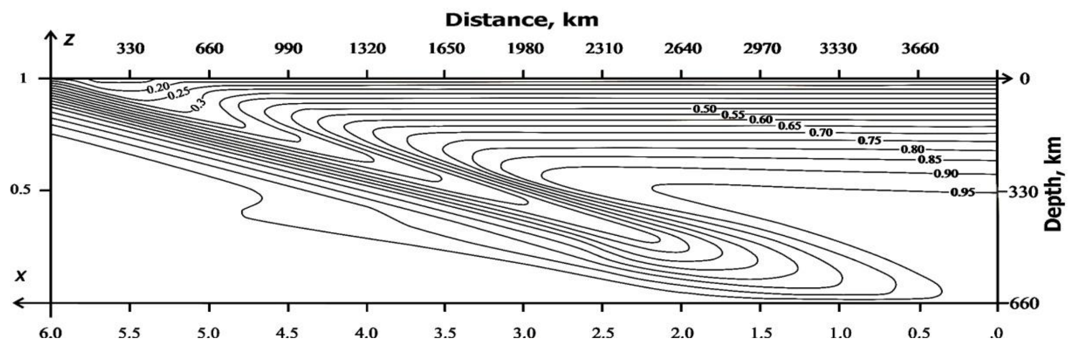


**Figure 2**. Quasi steady-state non-dimensional temperature in the mantle wedge with no effects of dissipative heating and small scale convective instability.

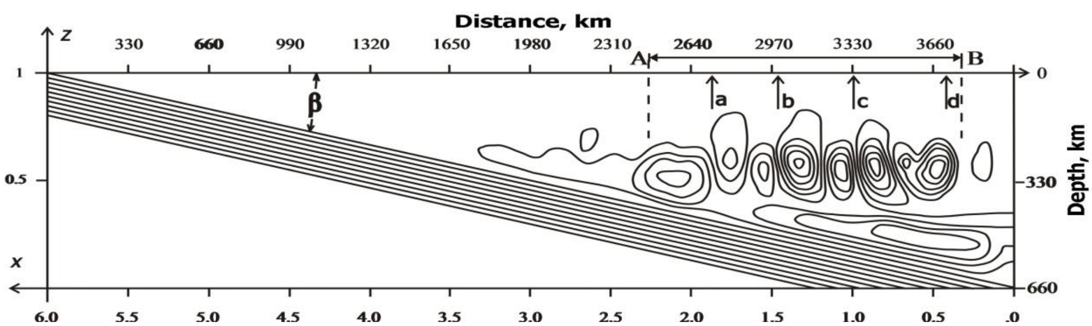


**Figure 3**. Quasi steady-state streamfunction above the lower boundary of mantle transition zone with dissipative heating, convective instability and phase transitions. Vortices correspond to convective flows possibly providing vertical transport of mantle hydrocarbons to the Earth's surface along the directions $a$, $b$, $c$ and $d$.

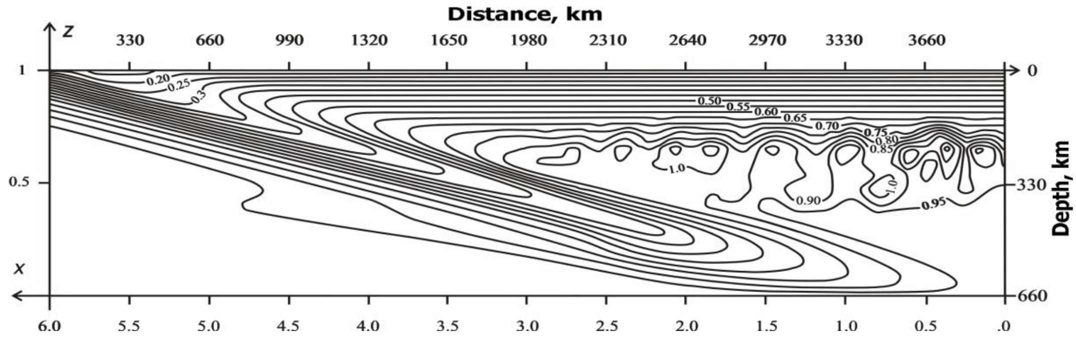


**Figure 4**. Quasi steady-state non-dimensional temperature above the lower boundary of mantle transition zone with dissipative heating, convective instability and phase transitions.

## 4. Conclusions

The scale of convection in the realistic rheology mantle wedge formed at Paleozoic in the course of closing of the former Urals ocean and subduction of the East-European lithospheric plate under the Timan-Pechora one equals to ~250–300 km, which is of the order of characteristic wavelength of location of oil- and gas-bearing stripped zones in this region. Velocities in convective whirls amount to tens of cm $\times yr^{-1}$, and that appears to be sufficient to effectively transport mantle hydrocarbons to the Earth's surface. The mantle wedge convection scale being of the order of the wavelength of the oil- and gas-bearing zones serves an indirect evidence for the estimate of velocity of subduction of the East-European lithospheric plate under the Timan-Pechora one (~ 6 cm $\times yr^{-1}$).


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## References:


[1]. Gavrilov S.V. Investigation of the mechanism of island arc formation and the back-arc spreading of the lithosphere. Geophysical Researches. 2014; 15: 35–43.
[2]. Zharkov V.N. Physics of the Earth's Interiors. Moscow, Science and Education, 2012, 384 p.
[3]. Fercshtatter G.B. Paleozoic intrusive magmatism at Urals – a key to understanding the orogenesis nature. The Lithosphere. 2012; 1: 3-13.
[4]. Schubert G., Turcotte D.L., Olson P. Mantle Convection in the Earth and Planets. New York, Cambridge University Press, 2001, 940 p.
[5]. Gavrilov S.V., Abbott D.H. Thermo-mechanical model of heat- and mass-transfer in the vicinity of subduction zone. Physics of the Earth. 1999; 35(12): 967–976.
[6]. Billen M., Hirth G. Newtonian versus non-Newtonian Upper Mantle Viscosity: Implications for Subduction Initiation. Geophys. Res. Lett. 2005; 32: (L19304), doi: 10.1029/2005GL023458.
[7]. Zharkov V.N. Geophysical Researches of the Planets and Satellites. Moscow, Joint Institute of Physics of the Earth RAS, 2003, 102 p.
[8]. Trubutsyn V.P., Trubitsyn A.P. Numerical model of formation of the set of lithospheric plates and their penetration through the 660 km boundary. Physics of the Earth. 2014; 6: 138-147.