

Numerical Investigation of Dynamical Response of Parametrically Excited System with Periodic and Chaotic Motions

Falade K. I.¹, Ayodele V. I.², Baoku I.G.³

¹Department of Mathematics, Faculty of Computing and Mathematical Sciences Kano University of Science and Technology, P.M.B 3244 Wudil Kano State, Nigeria

²Department of Computer Science and Mathematics, Faculty of Science, Nigeria Police Academy Wudil Kano State Nigeria

³Department of Mathematical Sciences, Federal University, Dutsin-Ma, P.M.B. 5001, DutsinMa, Katsina State, Nigeria

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*Corresponding author: Falade K. I., Department of Mathematics, Faculty of Computing and Mathematical Sciences Kano University of Science and Technology, P.M.B 3244 Wudil Kano State, Nigeria
Email: faladekazem2013@gmail.com

Abstract

This paper present numerical investigation of dynamics response of parametrically excited system with periodic and chaotic motions. The differential transformation method (DTM) was employed to obtained analytic-numerical solutions for both pe-riodic and chaotic motion of the system. Finally, we demonstrate the efficiency of the proposed method and the DTM solutions was compared with the analytical re-sults which showed a minimum relative error.

Keywords

Dynamic response, parametrically excited system, Differential transformation method (DTM), analytical solution, relative error.

1. Introduction

Parametrically excited systems are widely spread in many branches of physics and engineering, due to the many applica-tions of the Mathieu equation within these fields. In this paper we consider the second order non-linear homogenous parametrically excited system given of the form

(1)

Subject to initial conditions

(2)

where dots indicate differentiation with respect to the time, α is the parameter of nonlinearity, and $\beta \Omega \mu \in \mathbb{R}$.

Examples of this equation are found in many applications of mechanics, especially in problems of dynamic stability of elastic systems. In particular, the transverse vibration of abuckled column under the excitation of aperiodic end dis-placement is described by this equation. The system is called parametrically excited because the forcing term μc appears in the coefficient (parameter) of the equation.

In applied mathematics and engineering situations, the properties of parametric oscillations are widely used, for example, in the radio, the computer and laser engineering, in vibrant machines with special design [1], Paul trap mass spectrometers [2], as well as in a simulator for proving the equivalence of inertia and passive gravitational mass [3], numerical and analytical investigations into the chaotic behaviour of a parametrically excited pendulum system were carried out by [4-7].

In this paper, we present a reliable technique based on DTM to find numeric-analytic solution of the dynamics response of parametrically excited system with periodic and chaotic motions as the two cases of investigation.

2. Differential Transformation Method (DTM)

The concept of differential transformation method was first proposed by Zhou [8] and it was applied to solve linear and non-linear initial value problems in electric circuit analysis. This method constructs a semi analytical numerical technique that uses Taylor series for the solution of differential equations in the form of a polynomial. The differential transform is an iterative procedure for obtaining analytic Taylor series solutions of differential equations. The Differential transformation method is very effective and powerful for solving various kinds of differential equations. For example, it was applied to two-point boundary value problems [9], to differential-algebraic equations [10], to the KdV and mKdV equations [11], to the Schrodinger equations [12], to fractional differential equations [13] and to the Riccati differential equation [14] and just mention a few. The main advantage of this method is that it can be applied directly to linear and nonlinear ODEs without requiring linearization, discretization or perturbation. Another important advantage is that this method is capable of greatly reducing the size of computational work while still accurately providing the series solution with fast convergence rate.

2.1 Description Of Differential Transformation Method (DTM)

Consider an arbitrary function $x(t)$ which can be expanded in Taylor series about a point $t = 0$ as

$$x(t) = \sum_{k=0}^{\infty} \frac{t^k}{k!} \left[\frac{d^k x}{dt^k} \right]_{t=0} \tag{3}$$

The differential transformation of $x(t)$ is defined as

$$X(k) = \frac{1}{k!} \left[\frac{d^k x}{dt^k} \right]_{t=0} \tag{4}$$

Then the inverse differential transform is

$$x(t) = \sum_{k=0}^{\infty} t^k X(k) \tag{5}$$

The fundamental mathematical operations performed by differential transform method are listed in Table 1.

Table 1. One Dimensional Differential Transformation

Functional Form	Transformed Form
$x(t) = w(t) \pm v(t)$	$X(k) = W(k) \pm V(k)$
$x(t) = \eta v(t)$	$X(k) = \eta V(k)$, η is a constant
$x(t) = \frac{dx(t)}{dt}$	$X(k) = (k+1) X(k+1)$
$x(t) = \frac{d^m x(t)}{dt^m}$	$X(k) = (k + 1) \dots (k + m) X(k + m)$
$x(t) = x^m$	$X(k) = \delta(k-m) = \begin{cases} 1, & k = m \\ 0, & \text{otherwise} \end{cases}$
$x(t) = w(t) v(t)$	$X(k) = \sum_{r=0}^{\infty} W(r) V(k - r)$
$x(t) = x_1(t)x_2(t)x_3(t), \dots x_m(t)$	$X(k) = \sum_{k_{m-1}=0}^k \dots \sum_{k_1=0}^{k_2} X_1(k_1) X_2(k_2 - k_1) \dots X_m(k - k_{m-1})$

3. Relative error

The relative error used in this paper can be defined as

$$E_t = \left| \frac{\text{Analytical solution} - \text{Numerical solution}}{\text{Analytical solution}} \right| \times 100$$

4. Numerical Experiment

To illustrate the ability and reliability of the method for the numerical solution of equation (1), we investigate and present numerical solution at periodic motion ($\mu=0.3$) and chaotic motion ($\mu=0.4$) subject to initial condition $x(0) = 0$ and $\dot{x}(0) = 0.1$ [15]. The results reveal that the method is very effective and simple.

Case 1 Periodic Motion ($\mu=0.3$)

Consider equation (1) couple with the following parameters alpha (α) =1.0, beta (β) =0.2, Omega (Ω) =1.0, mu (μ)=0.3. We have

$$\ddot{x}(t) + 0.2\dot{x}(t) - (1 + 0.3\cos 1.0t)x + 1.0x^3 = 0 \quad t \in [0,1] \tag{6}$$

Subject to initial conditions

$$x(0) = 0, \dot{x}(0) = 0.1 \tag{7}$$

Taking the differential transform of (6), leads

$$(k + 1)(k + 2)X(k + 2) + 0.2(k + 1)X(k + 1) - (1 + 0.3\cos 1.0t)X(k) + 1.0[X_1(k_1)X_2(k_2 - k_1) + X_2(k_2)X_3(k_3 - k_2)] = 0$$

Taking $X(k + 2)$ as subject of relation

$$X(k + 2) = \frac{-0.2(k+1)X(k+1)+(1+0.3\cos 1.0t)X(k)+1.0[X_1(k_1)X_2(k_2-k_1)+X_2(k_2)X_3(k_3-k_2)]}{(k+1)(k+2)} \tag{8}$$

From the initial condition given by equation (8) we have

$$X(0) = 0 \tag{9}$$

$$X(1) = 0.1 \tag{10}$$

Substituting equations (9) and (10) into equation (8) and by recursive method when $k = 0,1,2,3,4 \dots \dots \dots 10$, the results are listed as follow:

$$\begin{aligned} X(0) = 0, X(1) = \frac{1}{10}, X(2) = -\frac{1}{100}, X(3) = \frac{67}{3000} \\ X(4) = -\frac{11}{5000}, X(5) = \frac{2219}{3000000}, X(6) = -\frac{5399}{90000000}, X(7) = -\frac{264967}{6300000000} \\ X(8) = \frac{24181}{3500000000}, X(9) = -\frac{41234519}{22680000000000}, X(10) = \frac{620677199}{11340000000000000} \\ X(11) = -\frac{24081766133}{1247400000000000000}, X(12) = \frac{9385753613}{20790000000000000000} \end{aligned} \tag{11}$$

Therefore, the closed form of the solution can be easily written as

$$\begin{aligned} x(t) = \sum_{k=0}^{12} X(k)t^k \approx \frac{1}{10}t - \frac{1}{100}t^2 + \frac{67}{3000}t^3 - \frac{11}{5000}t^4 + \frac{2219}{3000000}t^5 \\ - \frac{5399}{90000000}t^6 - \frac{264967}{6300000000}t^7 + \frac{24181}{3500000000}t^8 - \frac{41234519}{22680000000000}t^9 \\ + \frac{620677199}{11340000000000000}t^{10} - \frac{24081766133}{1247400000000000000}t^{11} \\ + \frac{9385753613}{20790000000000000000}t^{12} \end{aligned} \tag{12}$$

Case 2 Chaotic Motion ($\mu=0.4$)

Consider equation (1) couple with the following parameters alpha (α) =1.0, beta (β) =0.2, Omega (Ω) =1.0, mu (μ)=0.4. We have

$$\ddot{x}(t) + 0.2\dot{x}(t) - (1 + 0.4\cos 1.0t)x + 1.0x^3 = 0 \quad t \in [0,1] \tag{13}$$

Subject to initial conditions

$$x(0) = 0, \dot{x}(0) = 0.1 \tag{14}$$

Taking the differential transform of (13), leads

$$(k + 1)(k + 2)X(k + 2) + 0.2(k + 1)X(k + 1) - (1 + 0.4\cos 1.0t)X(k) + 1.0[X_1(k_1)X_2(k_2 - k_1) + X_2(k_2)X_3(k_3 - k_2)] = 0$$

Taking $X(k + 2)$ as subject of relation

$$X(k + 2) = \frac{-0.2(k+1)X(k+1)+(1+0.4\cos 1.0t)X(k)+1.0[X_1(k_1)X_2(k_2-k_1)+X_2(k_2)X_3(k_3-k_2)]}{(k+1)(k+2)} \tag{15}$$

From the initial condition given by equation (7) we have

$$X(0) = 0 \tag{16}$$

$$X(1) = 0.1 \tag{17}$$

Substituting equations (16) and (17) into equation (15) and by recursive method when $k = 0,1,2,3,4 \dots \dots \dots 10$, the results are listed as follow:

$$\begin{aligned} X(0) = 0, X(1) = \frac{1}{10}, X(2) = -\frac{1}{100}, X(3) = \frac{3}{125} \\ X(4) = -\frac{71}{30000}, X(5) = \frac{1087}{1500000}, X(6) = -\frac{869}{15000000}, X(7) = -\frac{34991}{525000000} \\ X(8) = \frac{1567}{164062500}, X(9) = -\frac{9845341}{5670000000000}, X(10) = \frac{54968567}{94500000000000}, \\ X(11) = -\frac{40843799}{243632812500000}, X(12) = \frac{42654977171}{93555000000000000} \end{aligned} \tag{18}$$

Therefore, the closed form of the solution can be easily written as

$$\begin{aligned} x(t) = \sum_{k=0}^{12} X(k)t^k \approx \frac{1}{10}t - \frac{1}{100}t^2 + \frac{3}{125}t^3 - \frac{71}{30000}t^4 + \frac{1087}{1500000}t^5 \\ - \frac{869}{15000000}t^6 - \frac{34991}{525000000}t^7 + \frac{1567}{164062500}t^8 - \frac{9845341}{5670000000000}t^9 \\ + \frac{54968567}{94500000000000}t^{10} - \frac{40843799}{243632812500000}t^{11} + \frac{42654977171}{93555000000000000}t^{12} \end{aligned} \tag{19}$$

Table 2. Case 1 Numerical solution of Periodic Motion ($\mu=0.3$)

t	Analytical Solution	DTM N= Computation length 12	Relative error = E_t (%)
0.0	0.000000000000	0.000000000000	0.000000000000
0.1	0.009922128761	0.009922120666	0.000081585314
0.2	0.019775381017	0.019775379000	0.000010214721
0.3	0.029686936442	0.029686924870	0.000038973371
0.4	0.039780280096	0.039780277000	0.000007792806
0.5	0.050176048332	0.050176039230	0.000018136143
0.6	0.060992527515	0.060992520710	0.000011165302
0.7	0.072346248264	0.072346244970	0.000004547575
0.8	0.084352341271	0.084352335030	0.000007397542
0.9	0.097124754737	0.097124761560	0.000007021897
1.0	0.110776434149	0.110776442900	0.000007943928

Table 3. Case 2 Numerical solution of Chaotic Motion ($\mu=0.4$)

t	Analytical Solution	DTM N= Computation length 12	Relative error = E_t (%)
0.0	0.000000000000	0.000000000000	0.000000000000
0.1	0.009923778963	0.009923770515	0.000085128861
0.2	0.019788442283	0.019788440680	0.000008489804
0.3	0.029730546141	0.029730534730	0.000038378037
0.4	0.039882498143	0.039882493280	0.000012185796
0.5	0.050373344303	0.050373337670	0.000013161722
0.6	0.061329217059	0.061329207310	0.000015897806
0.7	0.072873746115	0.072873747680	0.000002154411
0.8	0.085128337442	0.085128328370	0.000010654501
0.9	0.098212059686	0.098212073880	0.000014448327
1.0	0.112241684176	0.112241691700	0.000006682009

Figure 1 Analytical solution $x(t)/t$ Periodic Motion ($\mu=0.3$)

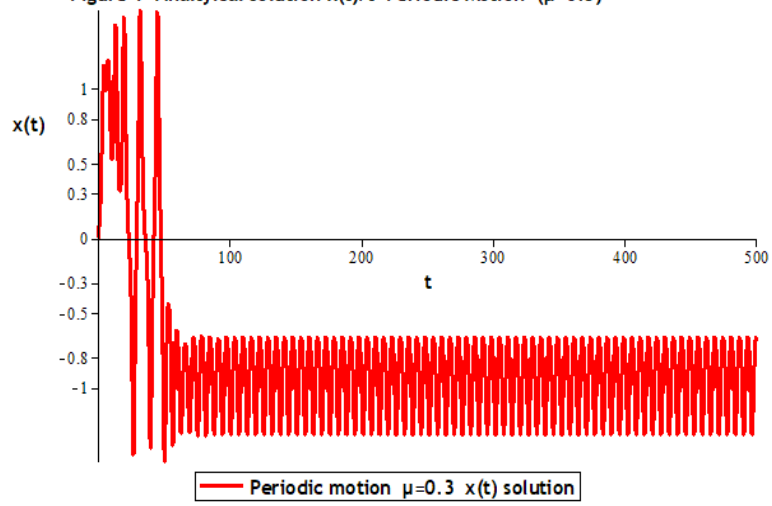


Figure 2 Analytical solution $x(t)/t$ Chaotic Motion ($\mu=0.4$)

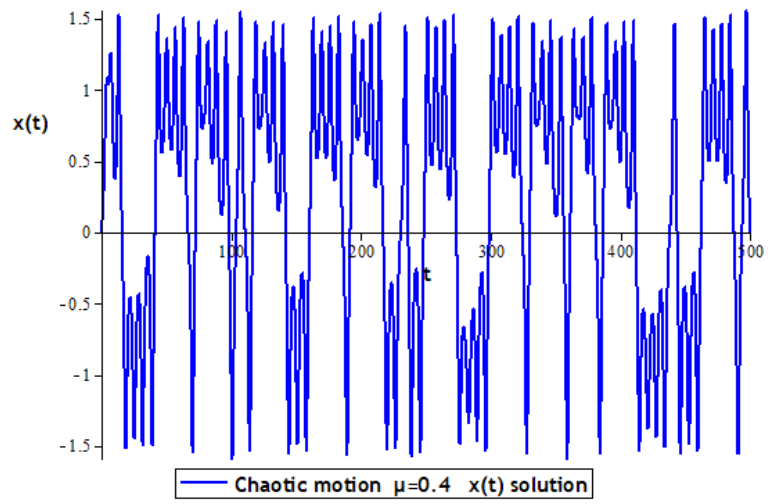


Figure 3 Analytical solution $Dx(t)/t$ Periodic Motion ($\mu=0.3$)

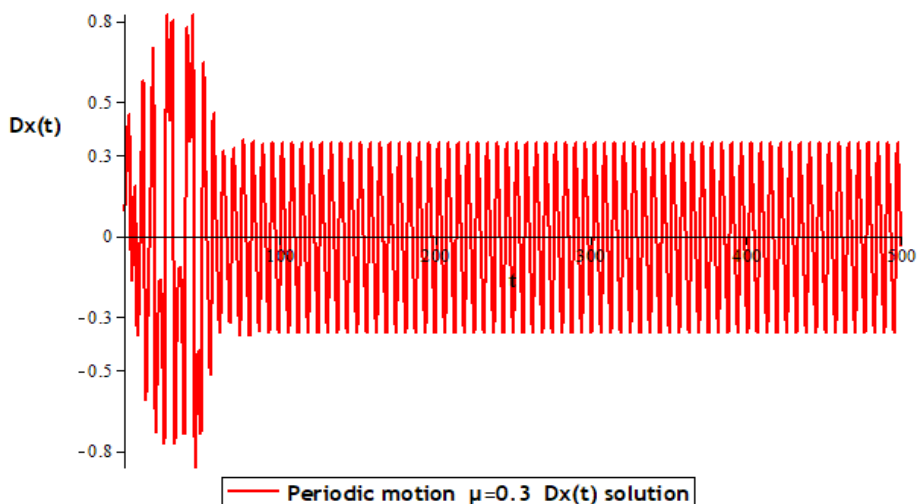


Figure 4 Analytical solution $Dx(t)/t$ Chaotic Motion ($\mu=0.4$)

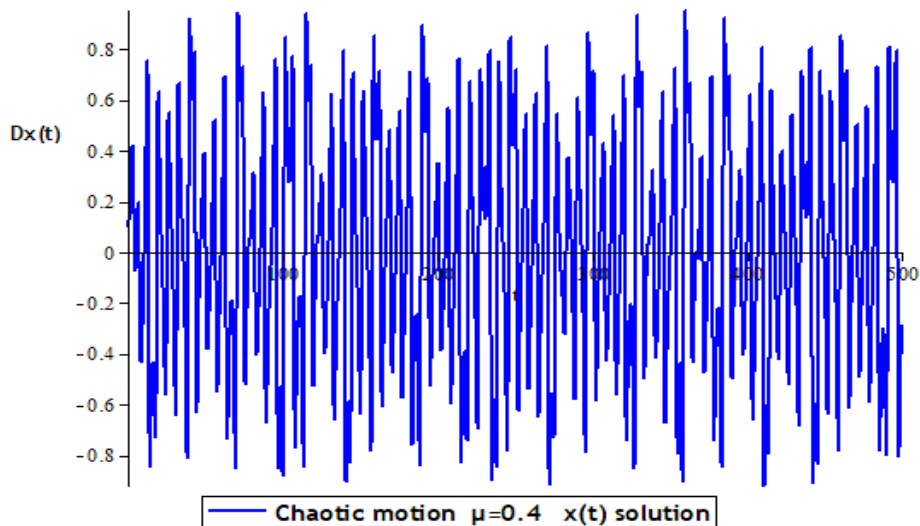
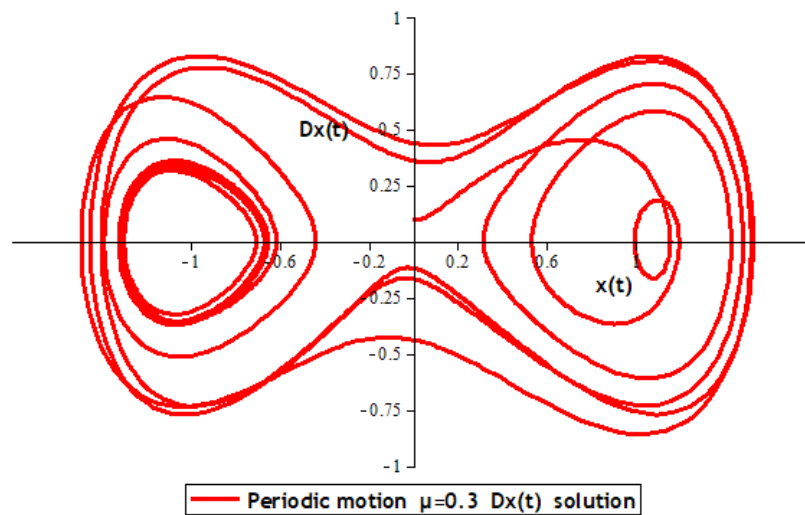
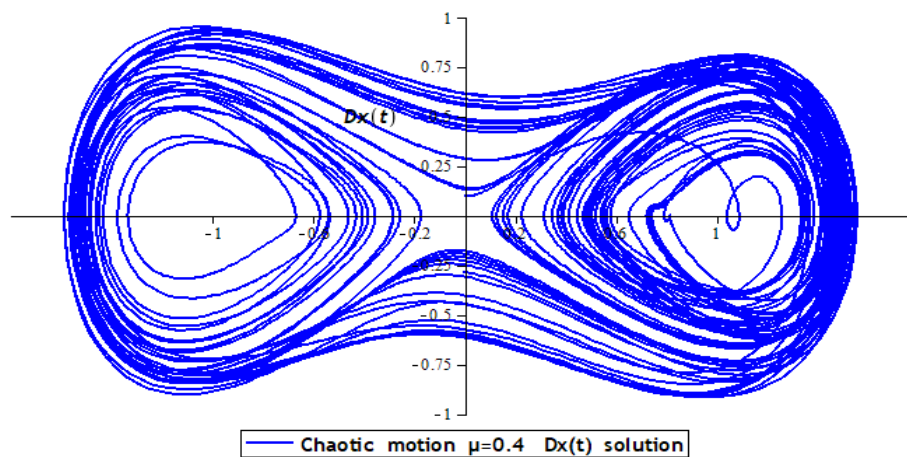


Figure 5 Analytical solution $Dx(t)/x(t)$ Periodic Motion ($\mu=0.3$)Figure 6 Analytical solution $Dx(t)/x(t)$ Chaotic Motion ($\mu=0.4$)

5. Conclusion

In this paper, we presented numerical technique (DTM) to solve the nonlinear parametrically excited equation. Solution of the equation (1) shows that the motions profile is affected by the variation or increase of the parameters $\mu=0.3$ to $\mu=0.4$ see figures (1,2,3,4,5,6). In other words, increasing of the μ yield higher numerical results obtained see tables (2,3). The DTM was used in a direct way without using linearization and we obtained numerical solutions for both periodic and chaotic motions. This method provides approximate closed-form solution at $N=12$. We conclude that the DTM is a promising tool for solving nonlinear of ordinary differential equations.

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