



Mathematical Modelling of Biogas Production from Animal Waste Via Anaerobic Biodegradation

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Abstract

The need to provide a scientific basis for biogas technology planning, control and management cannot be overemphasized in the search for sustainable renewable energy options. Hence, a proactive study to develop a predictive tool that would assist in the design of bioreactor was undertaken. This was achieved by investigating the kinetics of anaerobic digestion of substrates for biogas production under laboratory conditions using an experimental setup of a 10 litre bioreactor. Chicken droppings and cow dung were used differently as test samples, while biogas production was monitored and collected by water displacement method. 58.5 and 12 litres of biogas were obtained from poultry droppings and cow dung substrates, respectively within optimum pH of 6.5-7.3. Kinetic parameters for biodegradation of substrates were determined by conducting biochemical tests of dissolved oxygen (DO), biochemical oxygen demand (BOD), and total suspended solids (TSS). Biodegradation growth kinetics were derived, and kinetic constants were obtained from test results. This aided the formulation of mathematical expressions for biogas production estimate. The mathematical models obtained were checked for homogeneity using a system of dimensional analysis. The formulae were used to predict daily biogas yield, which was compared with the experimental biogas production. The volume of biogas produced was affected by the yield coefficient (y), substrate mass (M), retention time (θ), rate of biogas production (R), and the biogas potential of substrate (B_g). Overall, the mathematical model developed could be a veritable tool for the design, assessment and prediction of biogas production of any substrate.

Keywords

Chicken droppings; cow dung; bioreactor; anaerobic digestion; biogas

Nomenclature

X	Cell concentrations (g/l)
S_0	Influent substrate concentration ($\text{kg}_{\text{COD}}/\text{m}^3$)
R_{rs}	Volumetric substrate removal rate ($\text{kg}_{\text{COD}}/\text{m}^3\text{d}$)
K	First-order kinetic constant (1/d)
S_e	Effluent substrate concentration ($\text{kg}_{\text{COD}}/\text{M}^3$)
V	Bioreactor volume (m^3)
Q	Flow rate (m^3/d)
S	Substrate (l)
K_0	Saturation constant

K_m	Maximum rate of substrate utilization (g/l)
K_d	Decay rate coefficient
K_s	Half-velocity constant
K_I	Inhibition constant
V_g	Volume of biogas (m^3)
M	Mass (kg)
L	Length (m),
T	Time (s)
B_g	Biogas potential of substrate (kg/m^3)
R	Rate of biogas Production ($kg/s.m^3$)

Abbreviation

UNFCCC	United Nation Framework Convention on Climate Change
AD	Anaerobic Digestion
CO_2	Carbon Dioxide
TVS	Total Volatile Solids
TS	Total Solids
BOD	Biological Oxygen Demand
COD	Chemical Oxygen Demand
OLR	Organic Loading Rate
HRT	Hydraulic retention times
PLC	Percentage Lignin Content
DO	Dissolved Oxygen
SS	Suspended Solids or Non-filterable Solids
DS	Dissolved Solids or Filterable Solids

Greek Symbols

λ	Rate parameter or proportionality constant
μ	Specific growth rate (l/h)
τ	Hydraulic retention time (s)
θ	Temperature ($^{\circ}C$)
r_x	Rate of biomass formation(m^3/s)
r_s	Rate of substrate reaction (m^3/s)

Subscripts

g	biogas
d	decay
e	effluent
o	influent
m	maximum
o	saturation
x	biomass
s	substrate
i	inhibition

1. Introduction

The negotiations surrounding the United Nation Framework Convention on Climate Change (UNFCCC) and the growing awareness of the limits of traditional approaches to energy have spurred urgent need to further develop and adopt more climate-friendly energy technologies. The employment of Anaerobic Digestion (AD) of organic waste for biogas production is an established environmentally attractive technology, which has many environmental benefits with regards to waste treatment, pollution reduction, and production of CO_2 neutral renewable energy and the enhancement of agricultural practices by the use of the external economics of substrates of AD, i.e., the digestate, aside biogas. Renewable energy, such as biogas technology, is receiving heightened attention [1][2]. Biogas is leading

the pack in the new transition to a low-carbon energy future that guarantees energy security and environmental sustainability. Several studies on sustainable energy have all suggested that renewable energy, including biogas technology, might contribute to global energy supplies in the next century to a degree comparable to the use of fossil fuels today [3][4].

Global warming by greenhouse gases and climate change are the major negative impacts of the industrial revolution era. Developing countries of China, India, and Tanzania have developed biogas programmes in response to improving access to energy and mitigating the effects of climate change. In rural China, there are about 26.5 million biogas plants [5][6]. About 20 million biogas plants in India and Tanzania are currently producing 100 biogas plants monthly [7]. Before attaining this phenomenal growth, many types of biodigesters were popularized without much attention to research, training, and the quality of plants being constructed. This resulted in many digesters failing after a short period of operation. Pioneer biogas plants in Nigeria are the 10m³ biogas plant constructed in 1995 by Sokoto Energy Research Centre (SERC) in Zaria, 18m³ biogas plant constructed in 1996 at Ojokoro Ifelodun piggery farm Lagos by the Federal Institute of Industrial Research Oshodi (FIIRO). Finally, the 11 m³ biodigester constructed in 2010 by the National Centre for Energy Research and Development, University of Nigeria, Nsukka [8][9]. Work on biogas started in Nigeria after the establishment of two Renewable Energy Centres at Usman Danfodio University, Sokoto and University of Nigeria, Nsukka. These two centres along with many tertiary institutions have been involved in biogas research activities. Their activities in terms of biogas technology have remained at majorly investigating the biogas potential of different substrates [10][11]. Several other researchers have also designed and constructed biodigesters. These digesters have been constructed without a foreknowledge of the expected volume of biogas, and as such when disseminated to rural farmers and particular biodigester condition(s) altered, it will lead to biodigester failure [12][13]. Considering the very high investment costs and the regular personnel cost, the detrimental effect of methane, a chief component of biogas, on the environment has a global warming potential of 25 (time horizon 100 years); meanwhile, 25 times higher contribution to the greenhouse effect than CO₂ [14][15]. It should be discouraged to embark on construction of biodigesters without a rational or scientific basis for such designs. This was the case for many developing countries that have embarked on a biogas programme and encountered initial failures [16]. There is a need to learn from past experiences, adapt the biogas technology from Europe and Asia for local circumstances, keeping in mind that the results of the research should be applicable on a nation-wide scale and constitute a part of the country's biogas development plan [17][18]. Abdulahi *et al* [19] agreed that biogas production depends on substrate composition, type of substrate, retention time, and biodigester conditions of temperature and pH. Further, various studies contend that the sizing of biodigester, and invariably biogas production, depend on the mass of organic matter and retention time, without considering the kinetics of biodegradation in the digestion of organic matter. Aworanti *et al* [20] noted that the digestible substrate, which is difficult to measure but measurable in the nearest biochemical parameters of Total Volatile Solids (TVS), Total Solids (TS); Biological Oxygen Demand (BOD); and Chemical Oxygen Demand (COD). They are essentially used to determine the volume of the biogas biodigester, the biogas produced, and the gas production rate, whilst still maintaining optimum biodigester conditions of substrate, optimum temperature, pH, adequate water medium (good mixing ratio), frequent agitation and other factors vital for the maximum activity of the anaerobic bacteria responsible for biodegradation of substrates.

Nigeria's population is increasing geometrically and depends on non-renewable fossil fuels for its energy needs, making the contributions of global greenhouse gases significant. Being a responsible government, it is imperative that Nigeria should embark on a National Biogas Programme in order to improve access to energy and mitigate the effects of global warming as a contribution to providing a solution for the climate change menace. Nigeria's investment in biogas technology has been directed largely towards investigations on the biogas potential of available substrates. Numerous studies have already been conducted in that regard.

This present study seeks to further direct interest in obtaining relevant kinetic parameters of animal waste biodegradation, such as poultry droppings and cow dung, for biogas potential and productivity. It provides a major understanding of the design concept and mathematical model applied to biogas technology. Furthermore, it addresses the challenge of producing biogas from animal waste for optimal efficiency and low carbon generation. This contributes increasingly to a scientific basis for biogas technology planning, management, and controlled measures for anaerobic biodegradation of organic materials.

2. Methodology

In this study, the above-stated parameters used in describing the health of a digester were measured in a laboratory-scale anaerobic digestion using poultry droppings and cow dung to produce biogas. Relevant kinetic parameters were also determined and used to develop a mathematical estimate for the volume of biogas produced by simple

dimensional analysis. Thereby, setting a basis for informed design of biogas digesters, the kinetic parameters under various operating conditions are modeled and discussed for optimal performance.

2.1 Process parameters

Various parameters are influencing the anaerobic digestion of biogas production, such as Temperature, PH, Hydraulic retention time (HRT), the organic loading rate (OLR), etc. There are three different technical temperature ranges for methane formation in anaerobic digestion, and they include Psychrophilic temperature (or cryophilic) from 10° to 25°C; Mesophilic temperature from 25° to 35°C, and Thermophilic temperature from 49° to 60°C. In order to maintain a desired range and sustain gas production, the temperature is the single parameter to keep the process steady. The optimum temperature of digestion varies depending on feedstock composition and the type of digester [21]. A potential of Hydrogen pH is the most important parameter to know the health status of the digester. This is attributed to the change in time response to the biological conversion during anaerobic digestion process. Gas production is the only parameter that shows digester instability faster than pH. The range of acceptable pH for the bacteria participating in digestion is from 5.5 to 8.5, though the closer to neutral, the greater the chance that the methanogenic bacteria operate [22][23].

Hydraulic Retention Time (HRT) refers to the period required to achieve complete degradation of organic matter within a digester. Alternatively, it can be defined as the average duration that the substrate remains in the digestive system. HRT is influenced by several factors, notably the quality and composition of the substrate being processed, as well as the rate of microbial degradation. Lower degradation rates and slower bacterial doubling times typically necessitate a longer HRT to ensure efficient digestion.

For agricultural waste, the hydrolysis phase often serves as the rate-limiting step, as it involves breaking down complex organic compounds into simpler soluble molecules that can be further metabolized.

$$HRT = \frac{Liquid\ volume}{Daily\ flow} (days) \tag{1}$$

The organic loading rate (OLR) describes the amount of organic material or Volatile Solids (VS), which is fed daily per meter cube of digester volume.

For agricultural digesters, ORL is usually defined as,

$$OLR = \frac{Daily\ flow}{HRT} (days) \tag{2}$$

2.2 Stoichiometry and kinetics of biodegradation (Methanogenic processes)

Crampin et al [24] showed that mathematical models consist of chemical reaction pathways, which are made up of a number of elementary reactions, each of which can be represented by,



In which the chemical species *A* and *B* react to form species *C* and *D* in the proportions given by the integers a,b,c,d.

The molecularity of the reaction is determined by *a* and *b*, representing the number of molecules of *A* and *B*, occurring in the reaction, respectively. The unimolecular reactions refer to a single molecule transformation into one or more product molecules, while the biomolecular reactions involve the collision of two reactant molecules. Therefore, according to the mass action law of chemical kinetics, the reaction velocity *V(t)* is proportional to the product concentrations of the reactants.

$$V = \lambda X_A^a X_B^b \tag{4}$$

Where *X_A* and *X_B* represent the concentrations of *A* and *B*, while the rate parameter *λ* is the constant of proportionality.

The production rates of *C* and *D*, as well as the removal of *A* and *B*, obtained from the overall reaction velocity, are given by,

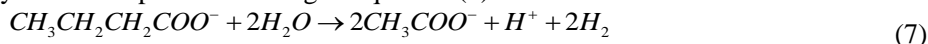
$$V(t) = -\frac{1}{a} \frac{dxA}{dt} = -\frac{1}{b} \frac{dxB}{dt} = \frac{1}{c} \frac{dxC}{dt} = \frac{1}{d} \frac{dxD}{dt} \tag{5}$$

Here, *C* is produced in the reaction at c-times this reaction velocity while imposing a structural constraint on the elementary kinetics reactions.

Delorme and Kapuscinski [25] presented the equation of propionate degradation as follows:

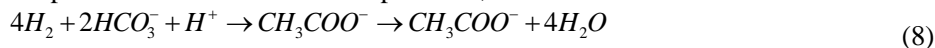


It has also been shown that butyrate decomposes according to equation (7)



In both cases, hydrogen molecule is a product, as well as an accumulation, at which the decomposition has the potential of inhibiting further fermentative acetogens.

However, hydrogen acetogenic respiration exhibits two sinks depicted as,



with the methanogenesis equation computed as,



and a stoichiometric equation often used in acetoclastic methanogenesis is given by,



The equations enumerated above describe major reactions, accounting for the breakdown of volatile fatty acids by acidogenic bacteria, the oxidation of molecular hydrogen, and the microbial formation of methane.

2.3 Modeling kinetic processes

A process that has constant substrate flow and gas production is referred as stationary or steady state process [26]. The substrate balance of a continuous or a discontinuous process is computed as,

$$\frac{ds}{dt} = DS_0 - DS + \left(\frac{ds}{dt}\right)_r \quad (11)$$

where the dilution rate D is the flow per fermenter volume in liter per hour, and S is the substrate concentration.

The reaction rate is the ratio between the product formation and cell concentration. The kinetics of bacterial growth provides the basic degradation process and is strongly dependent on the growth requirement and the medium.

The balance of bacteria cells is determined by,

$$\frac{dx}{dt} = DX_o - DX + \mu X + K_d X \quad (12)$$

where the cell concentration is X in grams per mole, while μ is the specific growth rate in liters per hour.

The bacterial growth depends on the specific growth rate, which cannot be infinite due to the limited availability of nutrients (substrate concentrations) and other ambient conditions, such as inhibitors.

The kinetics of bacterial growth are calculated as

$$\mu = \mu_{\max} \cdot \frac{S}{K + S} \quad (13)$$

According to this model, the specific growth rate μ increases strongly for low substrate concentration and slowly for high substrate concentration, until a saturation of bacteria is reached.

In Graef and Andrew model [27], the substrate inhibition on microbial growth is limited to the following formula,

$$\mu = \frac{\mu_{\max}}{1 + \frac{K_s}{S} + \frac{I}{K_I}} \quad (14)$$

where K_s is the half-velocity constant, K_I is the inhibition constant, and I is the inhibitor concentration, usually equal to S .

2.4 Dimensional analysis

Dimensions are made up of derived and fundamental quantities, usually adopted to check and determine key relationships between the valid analyses and derived engineering equations. For instance, in the design of a grandfather clock, it would be very useful to know the length of period taken to swing back and forth; then, the quantities on which this period might depend.

Recall weight depends on the object's mass and the acceleration due to gravity; one can therefore write,

$$P\alpha m^a g^b l^c \quad (15)$$

where P is the period, l is the length, g is the acceleration due to gravity, m is the mass, and a, b, c are unknown

constants.

The above equation can be determined as

$$P = km^a g^b l^c \tag{16}$$

where k , is an unknown constant of proportionality.

Equation (16) is found to be homogeneous; therefore, the dimensional analysis is described using equation (17).

$$T = [m]^a \left(\frac{[L]}{[T]^2} \right)^a L^c \tag{17}$$

The exponents are found by equating indices for each dimension,

$$[M]0 = a + 0 + 0 \Rightarrow a = 0 \tag{18}$$

$$[T]1 = 0 - 2b + 0 \Rightarrow b = -\frac{1}{2} \tag{19}$$

$$[L]0 = 0 + b + c \Rightarrow c = +\frac{1}{2} \tag{20}$$

The relationship is now obtained as

$$P \alpha m^a g^{-\frac{1}{2}} l^{\frac{1}{2}} = k g^{-\frac{1}{2}} l^{-\frac{1}{2}} = k \sqrt{\frac{l}{g}} \tag{21}$$

A constant may or may not have dimensions; therefore, the period formula is given by,

$$P = 2\pi \sqrt{\frac{l}{g}} \tag{22}$$

2.5 Experimental procedure

The process flow scheme of the experimental setup for the biodegradation of animal wastes is as shown in Fig. 1. The bioreactor is a 10 litre amber coloured bottle filled with the respective sample to two-thirds (2/3) volume of the bottle. Magnetic rods were placed inside the bioreactor to ensure continuous stirring of the sample by the action of the magnetic stirrer. Gas produced in the bioreactor was collected by water displacement into a graduated cylinder through a second amber bottle containing only water. The bottles are securely connected to form a unit using glass hollow rods. The amount/volume of water displaced is equal to the amount of gas produced by the bioreactor.

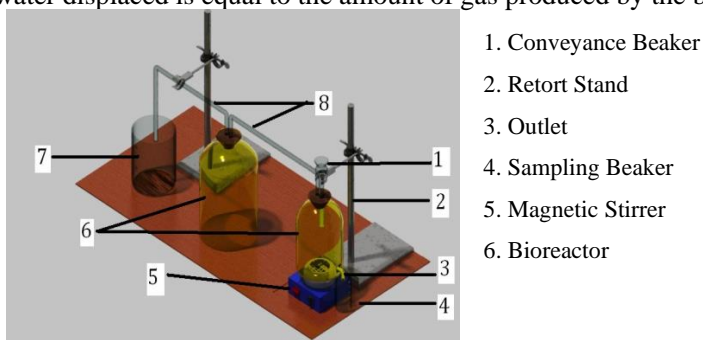


Figure 1. Schematic Diagram of the Experimental Setup.

2.5.1 Feedstock

The most important initial issue when considering the application of anaerobic digestion systems is the feedstock for the process. Feedstock is any substrate that can be converted to methane by the action of anaerobic bacteria (fermentation) in a bioreactor [28]. This can range from readily degradable organic waste to complex high-solid waste. Anaerobic bioreactors typically can accept any biodegradable material, but the level of biodegradability is the key factor for their successful application. In this study, cow dung and poultry droppings are used as substrates for digestion, as shown in Figs. 2 (a) and (b), respectively. Chicken droppings were collected from one of the private commercial farms at Olivet Hill junction, along the road leading to Ibagwa village of Igboeze South Local Government area of Enugu State. The sample collected was pounded and then sieved using the quarter grade British standard sieve (BSS

¼) and was later digested at a ratio of (1:2) volume of sample to volume of water accordingly. Cow dung was collected from the Nsukka abattoir in the new community market, Ikpa, Nsukka, Enugu State, South Eastern Nigeria, with geographical coordinates of 6°52'0" North and 7°23'0" East.



a) Sample of Chicken Droppings b) Freshly Collected Cow Dung

Figure 2. Feedstock samples.

2.5.2 Derivation of mathematical modeling equations

An association of methanogenic bacteria accomplishes the fermentative production of methane from organic compounds. These bacteria often compromise biomass quality and characteristics. Hence, making them susceptible to alterations in both space and time because of operational and environmental conditions. Kinetic study of anaerobic processes is an interesting exercise for design, prediction, and control purposes. Usually, the Monod model is efficiently applied for the description of organic matter removal during anaerobic digestion. Under these conditions, the first-order kinetics, usually at low effluent substrate concentration, is correlated to high reactor performance [29].

The first-order kinetics is expressed as follows:

$$R_{rs} = \frac{ds}{dt} = KS_e \quad (23)$$

Where R_{rs} is the volumetric substrate removal rate (kg_{COD}/m^3d), K is the first-order kinetic constant ($1/d$), and S_e is the effluent substrate concentration (kg_{COD}/m^3).

From the foregoing, it is possible to determine the kinetic constant K using equation (24).

$$K = \frac{R_{rs}}{S_e} = \frac{S_o - S_e}{\tau \cdot S_e} \quad (24)$$

$$\tau = \frac{V}{Q} \quad (25)$$

Where S_o is the influent substrate concentration (kg_{COD}/m^3), τ is the hydraulic retention time (d), V is the bioreactor volume (m^3), and Q is the flow rate (m^3/d).

Therefore, the rate of biomass formation (r_x) is found as

$$(r_x) = \frac{dX}{dt} = \frac{K_o SX}{K_m + S} - K_d X \quad (26)$$

While the rate of substrate reaction (r_s) is obtained by,

$$(r_s) = \frac{dS}{dt} = \frac{K_o SX}{K_m + S} \quad (27)$$

where S is the substrate, X is the biomass, K_o is the saturation constant, K_m is the maximum rate of substrate utilization per unit mass of cells produced, and K_d is Decay rate coefficient (lysis constant).

Writing the mass balance equation for biomass formation, accumulation is equal to inflow minus outflow plus Decay.

$$\frac{VdC}{dt} = QC_o - QC + V(-KC) \quad (28)$$

Where V is the reactor volume, C_o is the influent concentration, C is the effluent concentration, Q is the flow rate, and K is the reaction coefficient.

By substituting r_s and r_x respectively, S and X depend on the useful substrate reaction or biomass formation reaction.

$$QS_o - QS - \frac{K_o SXV}{y(K_m + S)} = 0 \tag{29}$$

$$QX_o - QX + \frac{K_o SXV}{K_m + S} - K_d XV = 0 \tag{30}$$

Since X_o is neglected, the equation becomes,

$$-QX + \frac{K_o SXV}{K_m + S} - K_d XV = 0 \tag{31}$$

By dividing through by QXy , the equation gives,

$$-\frac{1}{y} + \frac{K_o SV}{yQ(K_m + S)} - \frac{K_d V}{yQ} = 0 \tag{32}$$

By substituting $\tau = \theta$, the mass balance equation is obtained as

$$\frac{K_o S \theta}{y(K_m + S)} = \frac{K_d \theta}{y} + \frac{1}{y} \tag{33}$$

Dividing through with Q , and combining the above equations, one can write,

$$\frac{S_o - S}{X} = \frac{K_d \theta}{y} + \frac{1}{y} \tag{34}$$

This is a linear graph function of θ with the straight equation expressed as

$$y = mx + c \tag{35}$$

Where m is the slope and c is the intercept.

Again, by inverting equation (34) and comparing with the equation of a straight line, equation (36) gives a linear graph function of $1/\theta$.

$$\frac{y(K_m + S)}{K_o S} = \frac{X \theta}{S_o - S} \tag{36}$$

$$\frac{yK_m}{K_o S} + \frac{y}{K_o} = \frac{X \theta}{S_o - S} \tag{37}$$

2.5.3 Application of kinetic parameters to the design of bioreactors

The kinetics of bacteria growth is a mathematical model describing bacterial growth. This is generally adopted to predict the performance of biological processes and the qualitative translation observed in the process parameters for the design and operation [30]. For a batch system, the volume of gas (V_g) produced depends on the yield coefficient of substrate (y), the mass of substrate (M), the biogas potential of substrate (B_g), the retention time (θ) and the rate of biogas production (R).

$$V_g = f(yMB_g \theta R) \tag{38}$$

Equation (38) can further be worked out in terms of dimensional analysis as follows:

$$V_g = yM^a B_g^b \theta^c R^d \tag{39}$$

Where M is the mass, L is the length, T is the time, and θ is the temperature.

It is possible to use other secondary dimensions, which are also S.I units, such as kilograms, meters, seconds, and kelvin, or any other system of units. Meanwhile, in terms of dimensional analysis, the individual parameters of the expression of V_g will have the following basic dimensions

$$L^3 = L^3, M = M, B_g = ML^{-3}, R = MT^{-1}L^{-3}, \theta = T \tag{40}$$

Note that y is a dimensionless quantity. Thus, replacing the elements of equation (39) with primary dimensions, one can write,

$$L^3 = (M)^a (ML^{-3})^b (T)^c (MT^{-1}L^{-3})^d \tag{41}$$

$$L^3 = M^a M^b L^{-3b} T^c M^d T^{-d} L^{-3d} \tag{42}$$

For the equation to be homogeneous, the powers of each dimension must be the same on the left and on the right hand side of the equation. If a dimension does not appear at all, then it is implied that it exists only to the power of zero.

Comparing indices;

$$M^o L^3 T^o = M^a M^b L^{-3b} T^c M^d T^{-d} L^{-3d} \quad (43)$$

$$M^o = M^a M^b M^d \quad (44)$$

$$L^3 = L^{-3b} L^{-3d} \quad (45)$$

$$3 = -3b - 3d \quad (46)$$

$$1 = -b - d \quad (47)$$

Considering time,

$$T^0 = T^c T^{-d} \quad (48)$$

$$c = d = \frac{1}{2} \quad (49)$$

Comparing equation (47) and equation (48); then, substituting equation (46) in equation (43),

$$a = 1 \text{ and } b = -\frac{3}{2} \quad (50)$$

Thus, equation (39) becomes;

$$V_g = yM(B_g)^{-3/2} \theta^{1/2} R^{1/2} \quad (51)$$

$$\frac{V_g}{yM} = f \left(\sqrt{\frac{\theta R}{B_g^3}} \right) \quad (52)$$

2.5.4 Experimental validation

All the chemicals and reagents used in this study are of analytical grade as well as products of either SigmaGmb'H, England; Merck Darmstadt, Germany, or May and Baker, England. The parameters monitored were determined using the procedure outlined in [31]. These include the Dissolved Oxygen (DO), Biochemical Oxygen Demand (BOD), Total Solids (TS), Suspended Solids or Non-filterable Solids (SS), and Dissolved Solids or Filterable Solids (DS).

3. Results and discussion

3.1 Experimental Setup for biogas Production and biodegradation

The process flow scheme consists of a bioreactor and other attachments, such as the conveyance beaker, retort stand, magnetic stirrer, sampling beaker, graduated cylinder, and connecting glass rods. The bioreactor is a 10 litre amber coloured bottle deployed to imitate or resemble a reactor. The bioreactor has an outlet connected to a tap for sample collection and an inlet corked airtight with provisions for the conveyance beaker and connecting glass rods. The conveyance beaker is a device used in introducing materials (substrates, water, etc.) into the bioreactor and maintaining the airtight integrity of the system while the system processes (biodegradation) are in progress. The connecting glass rods have been folded to direct the conveyance of biogas and water. The retort stand is used to hold some system attachments in position. The magnetic stirrer is electrically powered to continuously rotate the magnetic rods at several undetermined revolutions per minute. As the magnetic rods that were previously introduced into the digester rotate, they turn the substrates and allow for proper mixing of the sample in the bioreactor. The sampling beaker and the graduated cylinder were used for the collection of samples for biochemical determinations and displaced water (equivalent of biogas produced), respectively.

3.2 Volume of biogas

The volume of biogas produced was recorded daily. In this case, the experiment was monitored as long as the batch process lasted. The cow dung experimental evaluation was monitored throughout 27 days, while that of poultry droppings was monitored throughout 36 days.

The volume of gas produced is shown in Figure 1. It follows that the volume of biogas produced in the poultry dung experiment was evidently higher than that produced in the cow dung experiment. Aye [32] noted that while Chicken manure has a Percentage Lignin Content (PLC) of about 3.4%, the PLC for cow manure ranges from 8.1% to 10.1%, thereby, enhancing the biodegradable fraction for chicken manure (0.73) and its potential to produce gas as against the biodegradable fraction for cow manure (0.60) and its potential to produce gas. In other words, the

higher the lignin content, the lower the biodegradability of the substrate. This has a lot to do with the nature and type of feed being fed to the animal at a particular period. For instance, if a cow is being fed only fibrous grasses/roots, there is a high tendency for its excreta to have a high PLC, unlike the chicken, which will be fed with more biodegradable organic feed with lesser PLC.

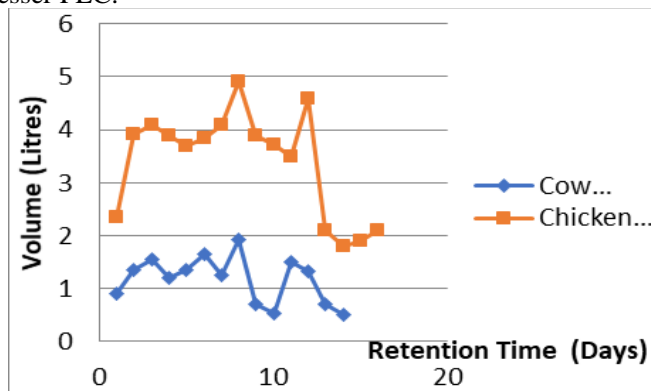


Figure 3. Volume of biogas produced from cow dung and poultry droppings.

Figure 3 depicts that the data obtained followed the same trend as most other bioreactors for biogas production. The variation of volume of biogas produced against time as obtained from laboratory experiment (10 L) interposed with another experiment conducted by [8] at the National Center for Energy Research and Development, University of Nigeria, Nsukka, (150 L) showed similar trends. However, the different volumes of the bioreactors used ensured that different volumes of the biogas were produced. This buttresses the fact that actual biogas with similar characteristics was produced in the laboratory-scale experimental procedure for the determination of biodegradation parameters of animal wastes.

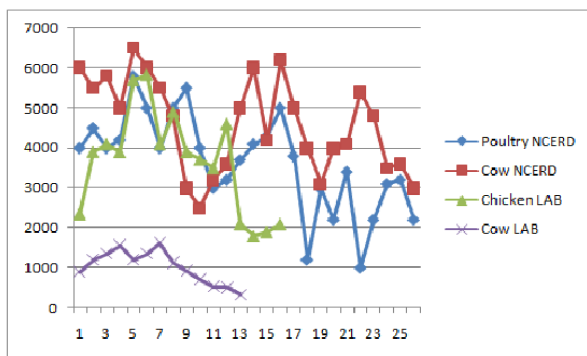


Figure 4. Comparison of biogas production.

The results obtained from both the cow dung and poultry droppings experiments, as well as the comparative analysis with a larger-scale bioreactor, underscore the reliability of laboratory-scale setups in simulating real-world biogas production dynamics. The consistency in production trends across different reactor volumes reinforces the validity of small-scale experiments for evaluating biodegradation parameters. Moreover, the influence of lignin content and feed type on biogas yield highlights the importance of substrate selection and animal diet in optimizing anaerobic digestion processes. These findings not only deepen our understanding of organic waste conversion but also pave the way for more efficient and scalable biogas technologies tailored to local agricultural practices.

3.3 pH

Figure 3 shows the various pH values obtained during the retention time of the experiment. The optimum pH for biogas production for both substrates was found from 6.4 to 7.3. However, satisfactory gas production occurred from 6.5 to 7.3 for cow dung, and from 6.6 to 7.3 for poultry dung. It should be noted that as soon as the pH departs from the optimum ranges, bacterial activity is seriously impaired, resulting in lower gas yields, inferior gas composition (excessive CO₂ content), and obnoxious odour (H₂S – rotten egg smell). The findings from the foregoing give credence to the quality of the experimental analysis.

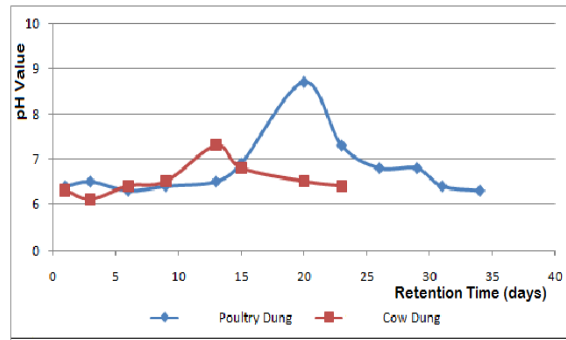


Figure 5. Variation of pH against Retention Time.

3.4 Substrate characteristics for cow dung

The samples, which are representative of substrate (S) and biomass (X), were collected and tested for Biochemical Oxygen Demand (BOD) and suspended solids (SS) during the experiment on cow dung. Figure 4 shows the variation for the determination K_d and Y , and Figure 5 reveals the variation for the determination of K_m and K_o .

A variation of $(S_0 - S_1)$ over X against time was plotted for the determination of endogenous Decay constant (K_d) and yield of biomass (y). These kinetic parameters of cow dung digested were obtained by extrapolating and solving the straight line graph obtained for intercept and slope, which represent the yield of biomass (y) and endogenous decay constant (K_d), respectively. Results revealed that the slope of 1.82 was reached while the endogenous decay constant was found to be 2.0, as well as the yield of biomass obtained was 1.1. In addition, a straight-line graph was obtained between the ratio of the product of time and biomass (θX) against substrate $(S_0 - S_1)$.

By extrapolating the graph in Figure 5 and analyzing both its slope and intercept, key kinetic parameters were determined. These include the maximum substrate utilization rate per unit mass of bacteria (K_m) and the half-velocity coefficient for the substrate (K_o). The extrapolated slope was found to be 375, with a velocity coefficient (K_o) of 1.0, and a maximum utilization rate (K_m) of 1.44. The corresponding intercept (y) was calculated as 260, providing a quantitative basis for evaluating the biodegradation kinetics of the substrate.

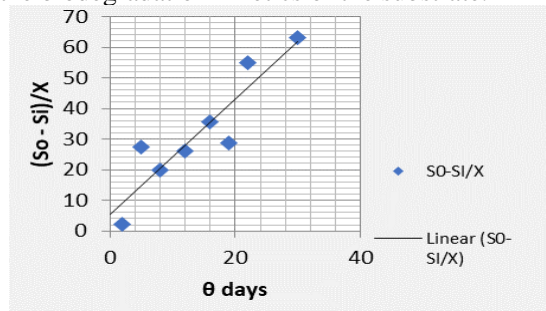


Figure 6. Ratio of substrate to biomass vs time.

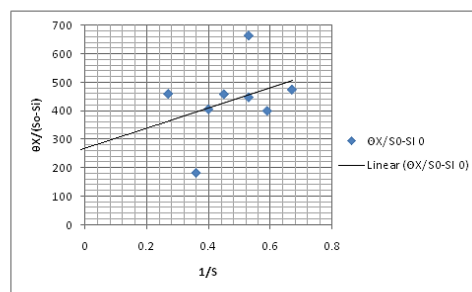


Figure 7. Time ratio product of biomass against substrate.

This experimental investigation into the biodegradation kinetics of cow dung provided valuable insights into microbial activity and substrate utilization. The systematic collection and testing of samples for BOD and suspended solids allowed for a robust evaluation of both substrate (S) and biomass (X) dynamics. The graphical extrapolations in Figures 4 and 5 not only facilitated the determination of key kinetic parameters, such as K_d , Y , K_m , and K_o , but also underscored the reliability of linear modelling in capturing microbial behavior. The consistency in slope and intercept values across different plots reflects the reproducibility of the experimental setup and the effectiveness of cow dung as a viable substrate for microbial digestion. These findings contribute meaningfully to the understanding of organic waste treatment processes.

3.5 Substrate characteristics for chicken droppings

The samples were collected and tested for Biochemical Oxygen Demand (BOD) and Suspended Solids (SS), which are representative of Substrate (S) and Biomass (X). The characteristics of chicken droppings were tabulated to determine the kinetic parameters, such as endogenous Decay constant (K_d), yield coefficient (y), maximum utilization rate for substrate per unit mass of bacteria (K_m), and half velocity coefficient for the substrate (K_o). Figure 6 shows the variation for the determination of K_d and y , while Figure 7 reveals the variation for the determination of K_m and K_o .

A graph of $(S_0 - S_1)$ over X against time was plotted for the determination of endogenous Decay constant (K_d) and yield of biomass (y). These kinetic parameters for chicken droppings digested were obtained by extrapolating and solving the straight line graph obtained for intercept and slope, which represent the yield of biomass (y) and endogenous Decay constant (K_d), respectively. Results revealed that the slope of 3.0 was found, as well as the endogenous Decay constant of 3.0 was obtained. The yield of biomass was given as 1.1, then by extrapolation of points from Figure 7, the slope of the above variation was obtained as 300, with K_m equal to 1.6, K_o was found to be 1.0, and y was obtained as 190.

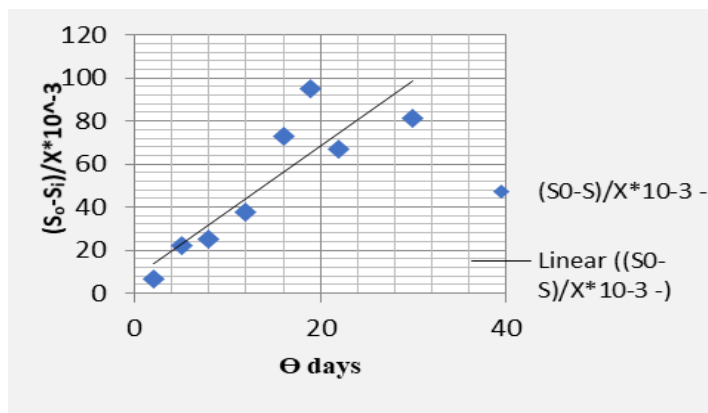


Figure 8. Ratio of substrate to biomass against time.

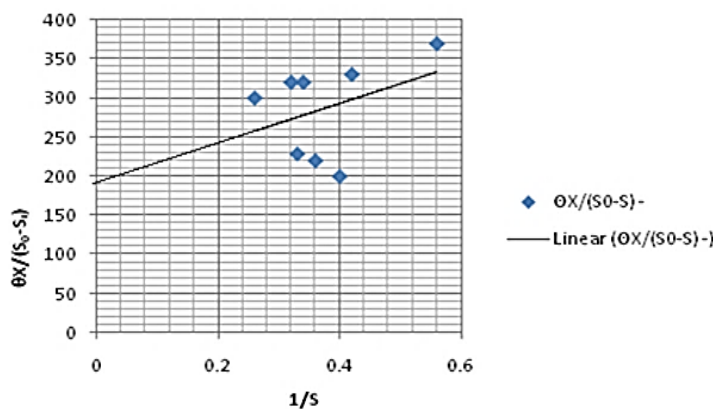


Figure 9. Ratio of the product of time and biomass to substrate against the reciprocal of substrate.

The kinetic evaluation of chicken droppings digestion demonstrated a structured approach to understanding microbial substrate interactions. The consistency in slope and intercept values across Figures 6 and 7 confirms the reliability of the experimental design and the suitability of chicken droppings as a substrate for microbial activity. The derived parameters, known as K_d , Y , K_m , and K_o offer a comprehensive view of the biodegradation potential and microbial efficiency. These findings not only reinforce the applicability of linear modeling in kinetic studies but also highlight the potential of poultry waste in sustainable waste-to-energy conversion systems.

3.6 Kinetic parameters effect on bioreactor design

Substrate characteristics for cow dung and chicken droppings are presented in Tables 1 and 2, respectively. The mathematical expressions of Bacteria growth kinetics were used to describe and characterize the substrate. For different volumes of gas produced, the biogas potential of substrate, the yield coefficient of substrate, and the rate of biogas production were obtained. They were used to predict the performance of biological processes and the translation of qualitative observations into process parameters for the design and operation of the process. The mass M selected for cow dung was 0.85kg, as well as for chicken droppings 0.57 kg. The retention time (θ) for cow dung was found as 16 days, as well as for chicken droppings was obtained as 20 days. These results demonstrate that the volume of a digester should be based on the anticipated mass of substrate and its biogas yield potential.

Table 1. Substrate characteristics for cow dung

V_g (M^3)	yM	$R(Kg_{BOD}/M^3d)$	$(B_g)^3$	V_g/YM	$\sqrt{\theta R}/(B_g)^3$
0.90×10^{-3}	221.00	0.0319	6.4×10^{-5}	4.07×10^{-6}	99.75
1.35×10^{-3}	221.00				
1.55×10^{-3}	221.00				
1.20×10^{-3}	221.00	0.0398	6.4×10^{-5}	5.43×10^{-6}	89.30
1.35×10^{-3}	221.00				
1.65×10^{-3}	221.00				
1.25×10^{-3}	221.00	0.0217	6.4×10^{-5}	5.43×10^{-6}	73.65
0.93×10^{-3}	221.00				
0.71×10^{-3}	221.00				
0.53×10^{-3}	221.00	0.0194	6.4×10^{-5}	2.40×10^{-6}	69.64
0.50×10^{-3}	221.00				
0.33×10^{-3}	221.00				

Table 2. Substrate characteristics for chicken droppings

V_g (M^3)	yM	$R(Kg_{BOD}/M^3d)$	$(B_g)^3$	V_g/YM	$\sqrt{\theta R}/(B_g)^3$
2.35×10^{-3}	108.30	0.0122	3.43×10^{-4}	2.17×10^{-5}	26.6715
3.91×10^{-3}	108.30				
4.10×10^{-3}	108.30				
3.9×10^{-3}	108.30	0.0180	3.43×10^{-4}	5.38×10^{-5}	32.3970
5.7×10^{-3}	108.30				
5.83×10^{-3}	108.30				
4.10×10^{-3}	108.30	0.0169	3.43×10^{-4}	3.23×10^{-5}	31.3914
4.90×10^{-3}	108.30				
3.90×10^{-3}	108.30				
3.71×10^{-3}	108.30				
3.50×10^{-3}	108.30				
4.60×10^{-3}	108.30				
2.10×10^{-3}	108.30				
1.80×10^{-3}	108.30				
1.90×10^{-3}	108.30				
2.10×10^{-3}	108.30				

Using equations (50) and (51), a graph of V_g/yM function of $\sqrt{\frac{\theta R}{(B_g)^3}}$ was plotted to describe the bacterial growth and predict the performance of biological processes that translate the qualitative observations into process parameters

for the design and operation. The bacteria growth kinetic parameters for chicken droppings and cow dung were obtained by extrapolating and solving the straight-line graph obtained. Results depicted in Figures 8 and 9 reveal a slope of 10^{-7} for cow dung as well as 5×10^{-6} for chicken droppings. These slopes reflect the relative efficiency of biogas production, with chicken droppings exhibiting a significantly higher rate, likely due to their richer nutrient profile and faster microbial activity.

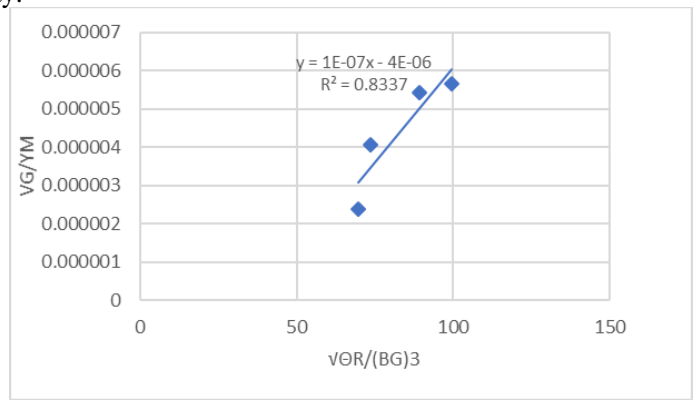


Figure 10. Volume of gas versus bacteria growth for cow dung.

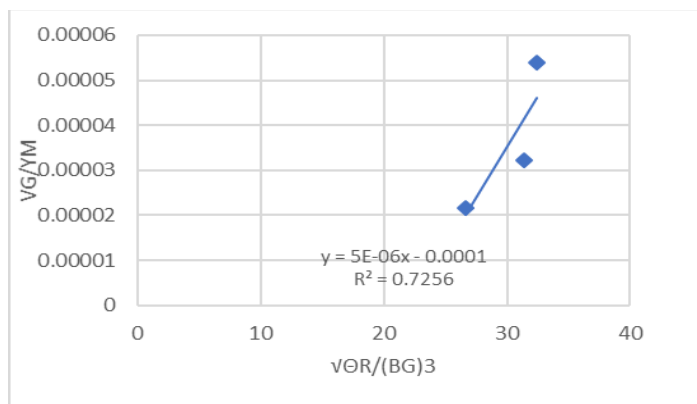


Figure 11. Volume of gas versus bacteria growth for Chicken droppings.

Figure 11 shows how the predicted volume of biogas production from the equations generated aligns with the actual experiment for biogas production. The graph shows a very good correlation, which can serve as a basis for the design of batch anaerobic digesters. By simple proportions, the different substrates revealed the ratios of substrate mix, (2:1) for chicken droppings and (1:1) for cow dung. These ratios exhibit 52 and 12 litres of biogas, respectively. Thus, the volume of a digester depends on the anticipated mass of substrate that will be digested.

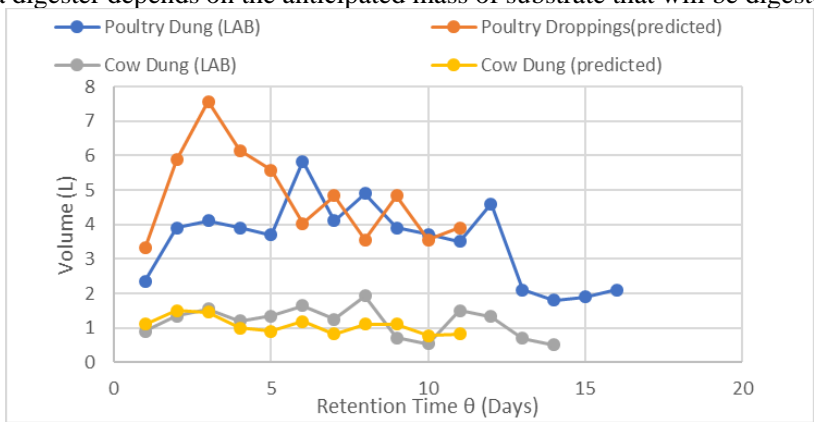


Figure 12. Predicted volume of biogas versus Experimental (actual) volume of biogas produced.

This study highlights the power of kinetic modeling in optimizing biogas production. By linking substrate characteristics to bacterial growth and gas yield, it provides a predictive framework for designing efficient digesters. The strong correlation between predicted and actual values reinforces the reliability of the approach, while the mixing ratios offer practical guidance for maximizing output.

3.7 Discussion

The characterization and kinetic modelling of cow dung and chicken droppings as substrates for biogas production revealed significant insights into their biological behaviour and energy potential. By applying bacterial growth kinetics and analyzing gas yield data, the study successfully translated qualitative substrate observations into quantitative parameters essential for process optimization. The selected substrate masses of 0.85kg for cow dung and 0.57kg for chicken droppings were digested under retention times of 16 days and 20 days, respectively. These values reflect the microbial activity and degradation rates of each substrate. Chicken droppings required a longer retention time, likely due to their higher nitrogen content and slower initial microbial acclimatization, yet ultimately yielded more biogas. This study contributes to the growing body of knowledge on sustainable waste-to-energy systems. By demonstrating the effectiveness of kinetic modeling and substrate optimization, it offers a practical roadmap for rural and urban communities seeking to harness organic waste for renewable energy. The insights gained here can inform policy, guide investment in biogas infrastructure, and promote environmentally responsible waste management practices.

4. Conclusion

A laboratory-scale bioreactor was setup to produce biogas using cow dung and chicken droppings. Biogas produced was collected by water displacement method. During the experiment, samples were collected and tested for some biochemical parameters such as dissolved oxygen (DO), biochemical oxygen demand (BOD), and total suspended solids (TSS). These parameters are representative of substrate characteristics determined in terms of BOD mg/l and SS mg/l, which were evaluated to determine the kinetic characteristics of the substrate. Results revealed that the sample of the cow dung digested had lower biodegradable fraction because of its high percentage of lignin content (PLC) compared with the sample for chicken droppings. About 12 and 58.5 litres of biogas were produced, respectively from the biodegradation of the experimental samples. A mathematical relation developed from anaerobic growth kinetics and further corrected using dimensional analysis was used to estimate the volume of gas produced from the evaluated kinetic parameters. Furthermore, a graph of V_g/yM function of $\sqrt{\frac{\theta R}{(B_0^3)}}$ was plotted to describe the bacterial growth and predict the performance of biological processes that translate the qualitative observations into process parameters for the design and operation. The slope of 10^{-7} was found for cow dung as well as 5×10^{-6} for chicken droppings. This became a predictive tool, used independently to evaluate the daily volume of biogas produced during the experiment. Therefore, it becomes easy to design a bioreactor when the biogas quantity required is known by substituting into the developed formula. These findings greatly reduce the bioreactor size, which is a major determinant in design.

Further research efforts should be geared towards developing other prediction tools for assessing the performance and health of a bioreactor. The kinetic parameters for other substrates can also be determined following the procedure used in this work. This will help determine the functional constants for several other biodegradable substrates peculiar to Nigeria. The by-products of anaerobic digestion, biogas and digestate, can be used in order to create a source of income. Biogas can be upgraded, most of the time by removing the carbon dioxide and water vapour, for the production of electricity and heat. The digestate can be used as a fertilizer or further processed into compost to increase its quality. Finally, future studies could explore comparative analyses with other organic wastes to optimize bioconversion strategies.

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