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Assessing Metacognition in Mathematical Word Problems: A Framework for Teacher Assessment Practices

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Abstract

The metacognition in mathematics assessment on solving word problems, explicitly described the questioning strategies to assess metacognition using the two metacognitive components namely: knowledge of cognition and regulation of cognition. The teachers' knowledge of cognition in mathematics assessment in terms of declarative knowledge was elaborated to reading and sharing of understanding and recognizing elements as major metacognitive assessment strategies. Teachers' procedural knowledge in mathematics assessment was elaborated to demonstrate a thinking process as a major metacognitive assessment strategy. And the teachers' conditional knowledge was elaborated to viewing the other angle of the problem as a major metacognitive assessment strategy. Teachers' regulation of cognition in mathematics assessment in terms of planning was elaborated to managing and visualizing concepts and checking learners as major metacognitive assessment strategies. Teachers' monitoring regulation in mathematics assessment was elaborated to critiquing on their own work and the others' work as major metacognitive assessment strategy. Finally, teachers' evaluating regulation in mathematics assessment was elaborated to reflect on their own work and check students as a major metacognitive assessment strategy.

Keywords

Metacognition; Assessing Metacognition; Mathematical Word Problems

1. Introduction

Metacognitive assessment strategies play a crucial role in word problems because they focus on evaluating and understanding an individual's awareness and control over their cognitive processes. Word problem solving requires more than just the application of mathematical skills; it involves strategic thinking, planning, monitoring, and reflection (Veenman, 2005).

Metacognitive assessment strategies are instrumental in fostering a positive and effective learning environment for students in mathematics. They contribute to the development of essential skills and attitudes that go beyond specific mathematical content, laying the groundwork for lifelong learning, and success in various academic and real-world contexts.

Questions are a useful instrument in evaluating students' intellect and delving into their minds. Good questioning is exhibited when teachers listen to students' answers. Using questions that begin with a solution, teachers can examine the students' mind and inspire them to participate more; for example, if the area of a rectangle is 24 cm^2 and the perimeter is 22 cm, what are its dimensions? Questions that have a wide range of possible methods of solving provide a great impact in student thinking and reasoning (Steinberg et al., 2004; Wiliam, 2007).

Based on the study of Wiliam (2007) about metacognitive assessment, useful feedback is centered on the task rather than on scores and grades; it clarifies why something is right or wrong and determines what next move to do for development. For instance, the feedback, "I want you to go over all of them and write an equal sign in each one" provides the student an instruction to revise her answer. Proficient teachers don't spoon feed the answers to the students who find the subject hard, instead they can guide them, advise them to look for information that can help and use various ways of solving. Replying to a student who says he can't follow, a teacher may reply: "Well, the first part is just like the last problem. Then we add one more variable. See if you can find out what it is. I'll be back in a few minutes." In that way the teacher can encourage the student to do more before she comes back.

On the other hand, Veenman (2005) stated that analysis of think aloud protocols is the most important method in assessing metacognitive skills where learners must keep on talking. The basic principle of think aloud tells that the participants must merely verbalize their ideas during their task performance. According to Pressley and Afflerbach (1995), think aloud protocols can be used for assessing individuals' text characteristics monitoring, difficulties in comprehension, and strategies used to understand the text.

Veenman et al. (2014) conducted a study to assess metacognitive skills and argued that it can be assessed by learning behaviors and impact learning outcome such as students' log-files of computerized tasks. Log- files don't meet the metacognitive consideration for the specific enactments; it was validated against other on-line methods. Participants were given a computerized inductive learning task and added to finish a performance post-test. Through the processes, high convergent validity between log- file indicators and human judgments of learner activities was found in the results.

Turan et al. (2009) argued the importance of metacognitive and self-regulated learning skills. The authors' implemented a different curriculum and used self-regulated learning perception scale and metacognitive awareness inventory in collecting the data of participants. Significant differences among medical school curricular models are also found. Students who study problem-based learning curriculum got higher scores compared to other curricular models.

According to the study of Hodge et al. (2007), tasks show the background of how mathematics is done. Through tasks, students establish thought regarding the background of mathematics and understanding mathematics. Henningsen and Stein (1997) asserts that proficient teachers make sure that tasks guide students to improve their comprehension in a particular topic and participate in high level mathematical thinking.

Anley et al. (2006) claims that having the students to do tasks and learning experiences that give them the opportunities to develop their own ideas about important mathematical concepts and relationships, teachers develop students' productivity in mathematics performances and in learning it. Watson and De Geest (2005) explain giving tasks with a suitable level of mathematical test that will lead to students' growth and the use of upgrading practical activities that improve the students' mathematical thinking and reasoning.

To investigate the metacognition and its relation to achievement goals of Turkish high school students, Sungur and Senler (2009) tested students' metacognition by its preliminary components. The study used Metacognitive Awareness Inventory (MAI), Achievement Goal Questionnaire (AGQ), Competence Expectancy Scale, and the Challenge and Threat Construal. The authors pointed out that respondents had reasonable knowledge about themselves as learners, about strategies, and when and how to apply these strategies after confirming the study.

Onovughe and Hannah (2011) examined secondary school students' awareness and use of metacognitive strategies in comprehending academic materials. Students' awareness and application of some strategies to reading and comprehension was used in collecting data from the participants. Five sets of questions were used in identifying purposes of students for reading, while awareness of reading skills and strategies of the participants were graded on a 2-point scale. In conclusion, a large extent was obtained in each metacognitive strategy in which the students know about metacognitive strategies. However, individuals applied metacognitive strategies in reading and comprehension to a large extent.

In this light, the role of a teacher to select a task for teaching mathematics is crucial in helping learners specifically in solving problems (Polya, 1985). Hence, by utilizing effective tasks, teachers will be able to help learners solve word problems with techniques involving analyzing and understanding problems to obtain correct solutions through planning, monitoring, and evaluating with the help of knowledge of declarative, procedural, and conditional. Finally, metacognitive assessment strategies are important in word problem solving as they provide a window into the cognitive processes involved, allowing for targeted interventions and improvements in problem-solving skills. They contribute to the development of a more effective and efficient problem solver with a deeper understanding of mathematical concepts.

1.1 Statement of Purpose

This qualitative study aimed to propose a model of assessing metacognition in solving mathematical word problems. Specifically, this study sought answers to the statement in pursuit of academic contributions in mathematics education Describe how teachers teach learners' mathematical knowledge and skills in solving word problems in terms of the following components of metacognition: (a) Knowledge of cognition; and (b) Regulation of cognition.

2. Method

This study created and proposed a paradigm for assessing metacognition in the context of solving mathematical word problems using the Multiple Case Study Method. A multiple case study was conducted in order to develop concepts from data and identify relationships and patterns between the process and context that might potentially serve as the basis for the model for metacognitive assessment strategies in the solving of mathematical word problems.

At least 12 master teachers from diverse school clusters, with varying educational backgrounds, victories, and accomplishments from various contests, make up each sample with the maximum variation in case selection. This wide range of illustrations made it easier to understand how teachers use metacognitive assessment strategies to solve mathematical problems.

Multiple observations and in-depth interviews within each case were required for the investigation of the conceptual understanding and classroom practices of the chosen master teachers in order to extract the metacognition assessment process. The primary elements of the data reported in the study were the experiences, beliefs, conditions, interactions, and lessons.

2.1 Population and Sampling Strategy

The research drew from a substantial and well-defined population base. The population for this multiple case study consisted of the 79 junior high school master teachers in the Division of Pampanga, Philippines who were teaching mathematics in Grade 10 during the 2020–2021 academic year. This timeframe positioned the study within the unique context of pandemic-era education, when virtual learning became predominant. Sample size determination was based on qualitative research principles. A minimum of 12 teacher volunteers were needed for this inquiry due to the saturation of data. This number reflects established qualitative research practices where data saturation, rather than statistical power, determines adequate sample size.

The sampling methodology combined multiple approaches to ensure both diversity and quality. Furthermore, a combination of criteria-based and heterogeneous deliberate sampling strategies and methods were used in this study to choose the participants. This dual approach balanced the need for participant variation with specific qualification requirements. The initial sampling phase emphasized diversity. The first step was employing the heterogeneous purposeful sampling technique, also known as maximum variation sampling, to select teacher volunteers with a range of qualities. This approach ensured representation across different teaching backgrounds and experiences. Demographic information was systematically collected through the initial survey. These research selection criteria were included in Part I of the original survey questionnaire, which was exploratory in nature. This part had the following information: (1) a brief biography; (2) a history of schooling; (3) employment history; and (4) achievements.

2.2 Participant Selection Criteria

The study implemented rigorous selection standards to ensure participant quality and relevance. To select teacher participants, the study also used purposeful sampling. The criteria included: (1) being a master teacher in a public junior high school; (2) being a teacher of tenth grade students; (3) having a high score on Turco's (2001) metacognitive skills inventory for teaching problem solving; (4) having the correct answers to the two open-ended questions; and (5) having to conduct online instruction at least once a week. Following initial selection, participants underwent additional evaluation phases. Finally, the selected and qualified participants underwent a series of virtual classroom observations and follow-up interviews. This multi-stage process ensured thorough vetting of participant qualifications.

Expert validation was incorporated to enhance research credibility. Lastly, the researcher considered the opinions, comments, suggestions, and counsel from the many experts in the domains of qualitative research and/or mathematics education to confirm and authenticate the analysis and interpretation of the study. Anonymity protocols were carefully implemented. Based on their numerical values, codes were allocated to the study participants. Participants were asked not to use their real identity during the interview; instead, they should choose a pseudonym or nickname under which they would be addressed.

2.3 Data Collection Instruments

The study employed multiple data collection instruments to ensure comprehensive data capture. An online follow-up interview guide, a virtual observation guide for the virtual classroom teaching and learning process, and an initial survey questionnaire were the four research instruments utilized to collect the data for this study. The initial survey served dual purposes of selection and data collection. Teachers who agreed to participate in the study were selected by means of an initial survey. Virtual classroom observation guides and grand tour online interview guides for specific teachers served as the study's main sources of data. The survey questionnaire was systematically structured and distributed electronically. The questionnaire used to choose the teacher participants was the first piece of research equipment to be distributed using Google Forms. The digital format facilitated efficient data collection during the pandemic period.

The questionnaire contained multiple components addressing different research needs. The first portion contained the invitation letter and permission form explaining the purpose of the study. There were three portions in the second section. Part one's demographic inquiries included questions about background, work history, education, and accomplishments. In part two, Turco (2001) offered a list of metacognitive skills for problem solving instruction, and in part three, there were two open-ended questions.

2.4 Assessment Questions and Observation Protocols

The study incorporated specific questions to assess metacognitive understanding. To assess their general understanding of the notion of metacognition, the teacher participants were given two open-ended questions to answer. These questions served as critical screening tools for participant selection. The assessment questions were strategically designed. One of the questions was: (1) What is metacognition, in your opinion? And (2) How do you use metacognitive thinking strategies to teach problem solving? These questions evaluated both theoretical knowledge and practical application.

Virtual classroom observations were systematically structured. A video recorder was used to record the teacher and students' exact words and facial expressions. As an observer, the researcher filled out the three sections of the virtual classroom observation guide. The observation checklist was comprehensive, and theory based. In contrast, Part B included a problem-solving observation checklist that included 27 statements culled from the metacognitive skills survey. The check list was divided into three sections: four statements following problem-solving, ten statements throughout the process, and thirteen assertions before to problem-solving.

2.5 Data Analysis Methodology

Memoranda written as bracketed notes from the outcomes of the online classroom observations and interviews served as the foundation for interpretations made during the data gathering and process. Memoranda that were pertinent to the study were highlighted in the form of observation protocols and entered into a computer program in order to prepare a systematic interpretation. These were notes that were made spontaneously during the online follow-up interviews or shortly after, such as the research journal entries.

The primary source for the analysis was video clip transcripts from the in-class observations and online interviews with the teacher participants. Secondary documentation, including as field notes and video clips of interviews with selected learner-participants, provide more comprehensive and multifaceted perspectives on teachers' conceptual knowledge and classroom practices. Transcripts were read multiple times to gain a feeling of the entire in order to connect the disparate pieces of data in a meaningful way. As a result, individual interviews were viewed as parts since transcriptions were read and reread with each new interview.

This study used the computer program Microsoft Excel to arrange the data coding. Axial and selective coding were determined by the facts and came from a more inductive standpoint, whereas open coding was based on theoretical and deductive reasoning.

Additionally, a constant comparative method was used to analyze the data. As data from all research participants' online interviews and in-class observations were compared to look for trends and consistency in their responses, codes began to emerge. A comparison of the themes found in the replies from teachers and students as well as the researcher's observations in the classroom were done in order to investigate the study's goal.

In light of the goals of the research, this study adopted and adhered to the three-phase grounded theory analysis methodologies. The phases were described in detail as follows: (1) transcribed responses from teachers and students in each online interview and observation note for analysis; (2) examined observation and interview transcripts to identify preliminary codes; and (3) generated axial and selective codes to identify themes and employ contrast comparison to create patterns between the conceptual understanding of teachers and actual classroom practices in assessing metacognition in mathematics. The researcher triangulated all data sources after completing the three phases of the method to confirm how the themes discovered were supported by the field notes, observation transcripts, and interview transcripts.

3. Results and Discussion

This study examines metacognitive assessment strategies used by master teachers in assessing learners' mathematical problem-solving skills, specifically in solving word problems. The research framework is based on two main metacognitive components: knowledge of cognition (declarative, procedural, and conditional) and regulation of cognition (planning, monitoring, and evaluating). The findings reveal that teachers employ various questioning techniques to evaluate how learners understand problems and regulate solutions verbally through strategic assessment approaches.

3.1 Declarative Knowledge Assessment

3.1.1 Reading and Sharing of Understanding

The primary strategy for assessing declarative knowledge involves reading and sharing of understanding, which encompasses specific techniques designed to check learners' comprehension of content, terminologies, keywords, and phrases in word problems.

3.1.1.1 Reading Aloud and Corrective Feedback

Reading aloud serves as a fundamental assessment technique that allows teachers to monitor learners' comprehension and provide immediate corrective feedback. As one teacher (T1) explained: *"I asked them to read the problem first and afterwards asked them if they understand it. Then, I called on a student to restate the problem to see if he/she understood the problem I posed on the screen correctly. For me, it is there that you'd figure out if what the learner says is correct and if it is not, it is then that I will do something to correct it."*

A notable classroom interaction demonstrates this strategy:

T3: S3.1 would you please read the given word problem?

S3.1: In a circle J, angle YZX is an inscribed angle which measures 35 raised to zero [35^0^] and the intercepted arc of angle ZYX measures 160 raised to zero [160^0^]. Find the measurement of angle ZXY.

T3: Please repeat this [underlining 35^0^ and 160^0^]

S3.1: 35 raised to zero and 160 raised to zero po

T3: Okay S3.1 those are 35 degrees and 160 degrees

This interaction illustrates how corrective feedback addresses misconceptions immediately, as T3 clarified the student's confusion between degree symbols and exponents.

3.1.1.2 Verbal Jigsaw and Verbal Cloze

These techniques help teachers assess learners' understanding of mathematical vocabulary and contextual comprehension. T1 demonstrated verbal jigsaw questioning during his virtual classroom observation about distance between two points:

T1: Okay, what do you mean by this [(highlighted the phrase) you decided to run from your home at point E (10, 3) through the park at point C (7,7)]

S1.3: Well, sir, it said that you, the one training for the marathon, ran from your house to point E where x equals ten and y equals three then you went through the park at point C where x equals seven and y equals seven too.

T1 emphasized the importance of this approach: *"We know that Mathematics is taught in the English language so sometimes, there are students who find it difficult to understand the problem. If that gap can be bridged even in the simplest or smallest way, it will help them in understanding the problem."* Verbal cloze techniques involve leaving blanks in questions to assess comprehension, as demonstrated by T7:

T7: So, the center is located at the blank points of the central angle.

S7.2: At point (3,2) ma'am.

T7: Good! The problem states that point (5,4) lies on a circle, that's why it's at the blank of the circle.

S7.2: On the circumference of the circle, ma'am.

3.1.2 Recognizing Elements

This strategy focuses on assessing learners' ability to identify components within word problems, utilizing convergent questioning, background knowledge prompts, and pumping techniques.

3.1.2.1 Convergent and Background Knowledge Prompts.

Teachers use these techniques to assess students' recall of previously learned concepts. T1 demonstrated this approach when discussing distance formulas:

T1: S1.4, please tell me when we will use the horizontal line formula in finding the distance between two points?

S1.4: Sir, as far as I remember, you know if horizontal line formula is to be used if the two points are y-coordinates.

T1: [follow-up] correct! Ahmmm... how about the vertical line, when will we need to use it? Yes S1.1

S1.1: Sir, if they are both x-coordinates, sir.

T1 explained his purpose: *"Well, my purpose, sir, is to help them think of their use in solving problems and when to use them. At the same time, {I do it} to check if they really know or if they really listened to the discussion."*

3.1.2.2 Pumping Questioning

This technique helps teachers assess learners' ability to identify required information and leading questions in problems. T11 demonstrated extensive use of pumping questions:

T11: S11.1 please tell me more about the problem posted in the screen.

S11.1: Well, ma'am, it is about the distance formula, madam.

T11: How did you know that?

S11.1: Uh, because there are points that are in the problem.

T11: So, if there are points, does that mean it's already distance formula?

T11 articulated her philosophy: *"I would always ask my students a series of questions to explain the problem itself, {to explain} their understanding of the problem, what the given was in the problem, etc., and what they don't understand about it. For me, you need to pump learners to determine how they understand the problem/what they do understand in the problem."*

3.2 Procedural Knowledge Assessment

3.2.1 Demonstrating Thinking Process

This strategy assesses how learners demonstrate their cognitive processes through question-based outlines and focus questioning sequences.

3.2.1.1 Question-Based Outline

Teachers use processing guide questions to assess students' understanding of their solution processes. T7's interaction exemplifies this approach:

T7: Alright, tell me how did you come up with your answer? What is the first thing you did in your solution?

S7.9: Madam, I substituted the given in the problem with the formula about distance between two points, madam.
T7: So, you used substitution, right? So, can you explain what you did in the substitution part?
S7.9: The given x sub 2 equals 7, x sub 1 equals 7, y sub 2 equals 10, and y sub 1 equals 3 [stop for a while]
T7: [follow-up] Alright, so what did you do next?

T1 explained the rationale: *"You can see there if students really understand what the process they are doing regarding the problem. So, if the learners can explain their work correctly, then I think that they really understand the lesson."*

3.2.1.2 Focus Questioning Sequence

T2 used focusing question sequences to advance students' thinking:

T2: You used to have a distance formula in solving the two points (7, 7) and (10, 3). Where did you get negative three?
S2.3: If you subtract 7 and 10.
T2: Why is there a need to subtract 7 and 10?
S2.3: [long pause] so as to get the distance of the horizontal line.
T2: Why are 7 and 3 also subtracted?
S2.5: So as to get the vertical line measurement.

T2 explained her purpose: *"The reason why I do it is to help the learners figure out the process of the problem for them to learn, for them to understand the content of the problem, and for them to be able to apply it in different things/areas/aspects."*

3.3 Conditional Knowledge Assessment

3.3.1 Viewing the Other Angle of the Problem

This strategy emphasizes assessing learners' ability to consider alternative approaches and build awareness toward problem-solving tasks.

3.3.1.1 Multi-Pronged Questioning

Teachers use this technique to help students view problems from multiple perspectives. T5 demonstrated this approach:

T5: Okay S5.3, your answer in the problem is 5 miles using the distance formula, right? Do you think we can use another way to solve the problem?

S5.3: Perhaps ma'am, when I saw the figure, I was able to form from the problem, I thought of using distance formula, but I also thought of Pythagorean theorem, madam. [recalls his grade 9 topic]

T5: Why did you think of Pythagorean theorem?

S5.3: Because it is the same as distance formula. It also appears that way, except that a and b have no value but if I am to get the horizontal and vertical line, it can represent the value for a and b.

T5 emphasized the importance of support questions: *"Support questions are very important so the learners themselves can generate questions from the problem and for you to know if they are able to do so. As such, you have to support them if they tend to get off track in doing this."*

3.3.1.2 Divergent Questioning

This technique encourages critical thinking and self-reflection. T8's interaction demonstrates this approach:

T8: S8.5 your answer about the given problem was $5\sqrt{2}$. Did you ask yourself after solving the problem?

S8.5: Madam, [I asked myself] what to do first because the problem was challenging but I knew the formula since I understand the lesson about distance between two points.

T8: So, what were your questions before solving the problem?

S8.5: I thought at first whether to draw and if I actually did, would I see the formula I was looking for? But I still drew.

3.4 Planning Regulation Assessment

3.4.1 Managing and Visualizing Concepts

This strategy assesses learners' ability to organize content and visualize problems effectively.

3.4.1.1 Reflective Toss and Redirect Classroom

Teachers use these techniques to guide students in organizing materials needed for problem-solving. T4's extensive interaction illustrates this approach:

T4: Given circle L, the measurement of angle GLK is x+138 degrees and the measure of arc KJ is x+48 degrees, while angle GLJ is 180 degrees. Tell me how to get the value of x.

S4.3: Sir, based on what I remember from your discussion of other similar problems, I have to plot angle GLK and arc KJ onto the circle to see how I will go from there.

T4: So, after you've put the angles onto the circle, what will you do next?

S4.1: I have to see what figure it'll come up with inside the circle, whether it is a semicircle or quarter circle or whole circle to help me decide how many degrees they will all become equal.

T4 explained her assessment purpose: *"I do it to know how they analyze the problem so I asked them questions like, 'how did you come up with this figure given this data?' You would then see it in their explanation."*

3.4.1.2 Process Prompt

This technique assesses how learners visualize concepts through illustration and labeling. T2's interaction demonstrates this approach:

T2: Given pt. E (10.3), pt. C (7,7), and pt. D (7,3), what quadrant do they belong? Yes S2.2?

S2.2: Madam, in quadrant 1, madam.

T2: Why do you think so?

S2.2: Because everything is positive, the sign of the integers that is, and so I thought it's in quadrant 1.

T2: Okay, you're right, but if you will plot and graph these points what figure will you create?

S2.2: [graphed the points] Madam, here it is, I came up with a triangle when I connected the points, ma'am.

T2 reflected on this strategy: *"So, I asked them a good way to understand the problem. So, he/she plotted it onto a graph, so from there, I saw that {ah} he/she understands the meaning of the problem."*

3.5 Monitoring Regulation Assessment

3.5.1 Critiquing Own and Others' Work

This strategy emphasizes assessing learners' ability to evaluate their own work and recognize errors in others' solutions.

3.5.1.1 Agreeable Learning

Teachers use this technique to assess students' listening skills, judgment, and decision-making abilities. T5 explained her approach: *"Usually, after a certain student explains his/her answer, I would ask another student, 'Oh, do you agree with what your classmate has said? If yes, please tell me your reasons.' If the other student does not agree with it, I would ask him/her to explain it again because he/she might have thought of a different way to solve {the problem} or might have a better idea {in solving it}."*

T2 elaborated on her purpose: *"To test the learner's decision-making on why he/she chose such answer or why he/she agrees with his/her classmate's answer and at the same to be able to explain his/her own reasons. It is by such that you can see if the learner really uses his/her critical thinking even when it's through online class."*

3.5.1.2 Zooming

This technique helps students recognize errors through detailed examination of solutions. T4 demonstrated this by posting problems with solutions containing errors for students to identify. She explained: *"I provided them with an example bearing a solution and an answer. Well, I did it to see if they really understood the two examples I gave.] I asked them to look at the problem carefully, to investigate if there was an error, and to explain why thereafter. By doing that, you would see how learners think and if they were really listening attentively during the discussion."*

T12 reflected on his use of zooming: *"Ah, is it called zooming technique when you ask the learners to look for errors? Okay, I used that because I want to help them understand the process and at the same time, it is for them to become attentive and participative in my class. Also, whenever I do it, I see that they are being aware of what they are doing and it becomes their attitude to review their work from time to time."*

3.5.1.3 Repetition and Rephrasing

Teachers use these techniques to help students recognize familiar patterns and monitor their solutions. T2 explained:

"For the learner to practice what he/she has learned and to become familiar with the steps. Ask the student to explain his/her work. If he/she knows how to explain his/her work, it only means that he/she monitors his/her actions. You will see it when he/she is confident in explaining his/her work. {follow up} Yes, you can pose some questions related to the previous thought of the learner for you to know how strong/how solid his/her argument is regarding the explanation."

3.6 Evaluating Regulation Assessment

3.6.1 Reflecting on Work

This strategy assesses learners' ability to generalize and justify their solutions through comprehensive reflection.

3.6.1.1 Question-Based Summary

Teachers use this technique to assess students' ability to synthesize their problem-solving processes. T1's extensive interaction demonstrates this approach:

T1: S1.7 please tell me how you came up with your answer? Can you tell me your whole process to solve the problem?

S1.7: Yes, sir. So first, the problem refers to the distance between two points, sir, that is why I used horizontal, vertical, and slant formula.

T1: And then?

S1.7: I understood what to look for, sir. After I identified it, I plotted a graph to see how the figure would look like, sir.

T1: So, what was the appearance of your figure?

S1.7: Well, sir, I was able to form a triangle, sir. Based on what you taught us, we would know what formula to use based on the figure we would get.

T1 explained his assessment purpose: *"For me, when they know how to generalize the problem process, it means that they also know how to evaluate the whole process {follow-up} yes, so they will attain realization regarding what they learned after solving the problem."*

3.6.1.2 Constructive-Challenge Questioning

This technique tests students' ability to defend their thinking processes. T4's interaction exemplifies this approach:

T4: In a circle Q, with angle YZX measure 35 degrees and intercepted arc of angle ZYX measures 160 degrees. Find the measure of angle ZXY. That's the problem, right. So tell me S4.8 why did you get an answer of 245 degrees?

S4.8: Ma'am, when I added 35 degrees and 80 degrees, the answer I got is 115 degrees and I subtracted it from 360 degrees.

T4: Okay S4.8 I get your point, by the way, how did you get 80?

S4.8: Because in the given, madam, the arc of angle ZYX is 160 degrees, and in the theorem that you said, it comprises half of it, and I thought that it is inscribed since Q has no element.

T4 reflected on this technique: *"If they can really defend their answer against my argument, it means that they really know how to evaluate their own solution. I also used this questioning technique to make the student would talk about his solution. You should let the student think instead of just straightforwardly telling him/her, 'you're wrong.'"*

3.7 Theoretical Framework and Implications

The study's findings align with established metacognitive theory. As Schraw et al. (2006) noted, declarative knowledge refers to an individual's conceptions and beliefs about their personal abilities, task structures, and cognitive goals. The importance of declarative knowledge in learning, as recognized by Presley et al. (1987), is significantly related to reading and sharing of understanding and recognizing elements of given problems.

Procedural knowledge, defined as knowledge about the "how's" of things, concepts, terms, definitions, and execution of procedural skills to perform cognitive tasks, reflects the process or individual strategies in solving particular problems (Schraw et al., 2006). Individuals with procedural skills possess a repertoire of strategies in thinking and learning, and those with higher degrees of procedural knowledge tend to perform tasks with larger repertoires of effective strategies (Presley et al., 1987).

Conditional knowledge, described as knowledge about when and where to use concepts, terms, things, definitions, and why to apply such cognitive actions (Young & Fry, 2008), reflects understanding of the limitations of procedural knowledge (Schraw et al., 2006). Reynolds (1992) emphasized that conditional knowledge is crucial for learners to select particular strategies and significantly allocate their resources under different conditions.

3.8 Metacognitive Assessment Strategies in Mathematics

This study elaborated that the metacognitive assessment process in solving word problems is the teachers' knowledge of cognition on declarative, procedural, and conditional, and teachers' regulation of cognition on planning, monitoring, and evaluating, which are undertaken by the metacognitive assessment strategies. For declarative, there is two metacognitive strategies which are reading and sharing of understanding and recognizing elements, while demonstrating thinking process for procedural and viewing the other angle of the problem for conditional. For planning, there is one metacognitive strategy which is managing and visualizing concepts while critiquing own and other's work for monitoring and reflecting in own work for evaluating.

Table 1. Model of Metacognition in Mathematics Assessment

Reading and Sharing of Understanding & Recognizing Elements	Demonstrating Thinking Process	Viewing the Other Angle of the Problem	Managing and Visualizing Concepts	Critiquing Own and Others' Work	Reflecting on Own Work
Declarative	**Procedural**	**Conditional**	**Planning**	**Monitoring**	**Evaluating**
	Knowledge of Cognition	⟷		**Regulation of Cognition**	

As to the **knowledge of cognition**, the **teacher's declarative knowledge assessment technique** emerged from the teachers' practices was *reading and sharing of understanding and recognizing elements* which include specific metacognitive assessment strategies such as reading and translating, terminologies, keywords, and phrases, recalling previous concepts, and diagnosing the problem's specifics through assessment techniques such as reading aloud, corrective feedback, verbal jigsaw, and verbal cloze techniques.

While **procedural knowledge assessment strategy** manifests *demonstrating thinking process* which include specific metacognitive assessment strategies such as pushing their thinking forward through question-based outline and focus questioning sequence.

Moreover, a **teacher's conditional knowledge assessment technique** includes *viewing the other angle of the problem*, which include specific and elaborated metacognitive assessment strategies such as to view the other angle of the problem and build their awareness using techniques such as multi-pronged questioning and divergent questioning.

As to **regulation of cognition**, **teacher's planning regulation assessment technique** presents *managing and visualizing concepts* which include specific and elaborated metacognitive assessment strategies such as organizing the content and visualizing the problem using reflective toss questioning, redirect classroom technique, and process prompt questioning.

In a similar manner, **teacher's monitoring regulation assessment technique** reflects *critiquing on their own work and the others' work*, which include specific metacognitive assessment strategies such as critiquing the work of others and recognizing errors using agreeable learning technique, zooming, and repetition and rephrasing feedback.

Finally, **teacher's evaluating regulation metacognitive assessment strategy** manifests on *reflecting on one's own work* which include specific metacognitive assessment strategies that elaborate concepts on generalization and justification using questioning-based summary and constructive-challenge questioning techniques.

The rounded arrow in the figure above shows that the metacognitive assessment process is a parallel process which suggests that teachers' metacognitive strategies in assessment depends on the condition needed in the word problem. Metacognitive assessment is positively related to mathematical problem-solving ability (Aminah & Sabandar, 2011) and plays an important role during each level of problem solving (Goos *et al.*, 2000). Hence, this should be employed explicitly to develop mathematical problem-solving skills, otherwise, its failure corresponds to mathematical thinking and problem-solving inadequacy (Desoete, 2007). If students are assessed metacognitively, this ensures congruency to application of learned strategies to new platforms. Effective use of metacognitive assessment strategies could be beneficial for students to learn and develop problem solving skills which are associated with successful outcomes (Stillman & Mevarech, 2010).

3.9 Principles of Metacognitive Assessment

Based on the collective understanding and classroom practices of participating teachers, the study presents seven principles that underpin metacognitive assessment strategies:

1. Metacognitive assessment strategies help learners acquire mathematical knowledge and skills by reading and sharing of understanding through reading aloud, corrective feedback, verbal jigsaw, and verbal cloze.
2. Metacognitive assessment strategies help learners acquire mathematical knowledge and skills by recognizing the elements of the problem through convergent questioning, background knowledge prompt, and pumping techniques.
3. Metacognitive assessment strategies help learners acquire mathematical knowledge and skills by demonstrating a thinking process through outline-based questioning and focus questioning sequence.
4. Metacognitive assessment strategies help learners acquire mathematical knowledge and skills by viewing the other angle of the problem through multi-pronged and divergent questioning.
5. Metacognitive assessment strategies help learners acquire mathematical knowledge and skills by managing and visualizing concepts through process prompt, reflective toss, and redirect classroom techniques.
6. Metacognitive assessment strategies help learners acquire mathematical knowledge and skills by critiquing their own work and the work of others through agreeable learning, zooming, repetition, and rephrasing feedback techniques.
7. Metacognitive assessment strategies help learners acquire mathematical knowledge and skills by reflecting on their own work through question-based summary and constructive-challenge questioning.

4. Conclusion

The study's conclusions support the idea that metacognitive assessment strategies improve students' word problem-solving and mathematical comprehension. Teachers who use a range of metacognitive assessment strategies not only help students perform better mathematically, but they also help them build their metacognitive knowledge and regulation. This puts students in a stronger position to cultivate their own learning and create the environments that support and encourage meaningful learning.

The use of metacognition to education creates new categories for both the specific and advanced metacognitive strategies used by math teachers and the assessment strategies that students absorb from mathematical discourse. Creating links between these specific metacognitive assessment strategies through the lens of education might promote more insightful and productive classroom discussions.

When metacognitive assessment strategies are integrated and cohesive, math problem solving becomes more important. In the model that is being described, selecting metacognitive approaches for assessment is not a sequential or linear process. Teachers can therefore use these strategies as they see fit, considering their unique situation, subject-matter competence, and students' level of mathematical understanding and knowledge. Most importantly, to use metacognitive assessment techniques, math teachers need to have a solid understanding of mathematical concepts and procedures.

4.1 Ethical Approval and Consent to Participate

This study complies with the Declaration of Helsinki and received a Certificate of Exception from the Philippine Normal University Research Ethics Committee (REC Code: 03022022-014). All participants provided informed written consent prior to their involvement in the research. The study adhered to ethical guidelines for educational research, ensuring voluntary participation, confidentiality of data, and the right to withdraw from the study at any time without penalty. Participants were fully informed about the study's purpose, procedures, potential risks and benefits, and data handling practices. All data were anonymized and securely stored in accordance with institutional data protection policies.

4.2 Consent for Publication

All participants provided written consent for publication of their anonymized data and findings. Identifying information was removed to ensure confidentiality and protect participant anonymity.

4.3 Competing Interests

The authors declare no competing financial interests or personal conflict of interest.

4.4 Funding Information

The authors do not receive financial means while collecting and publishing this study.

4.5 Authors' Contributions

This journal uses Contributor Roles Taxonomy (CRediT) to recognize individual author contributions, reduce authorship disputes, and facilitate collaboration. Dr. Joseph D. Reyes served as the lead researcher, handling conceptualization, methodology, investigation, analysis, original writing, supervision, and project administration across all research phases. While Dr. Zenaida Q. Reyes, contributed significantly to conceptualization, methodology, validation, formal analysis, editing, and visualization as a key co-researcher.

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References

Aminah, M., & Sabandar, J. (2011). The potency of metacognitive learning to foster mathematical logical thinking. Department of Mathematics Education, Yogyakarta State University. https://eprints.uny.ac.id/1345/1/P%20-%2035.pdf

Blakey, E. M., & Spence, S. (1990). Developing metacognition. ERIC Clearinghouse. https://files.eric.ed.gov/fulltext/ED327218.pdf

Costa, A. L. (1984). Mediating the metacognitive. *Educational Leadership*, *11*, 57-62. https://files.ascd.org/staticfiles/ascd/pdf/journals/ed_lead/el_198411_costa.pdf

Delclos, V. R., & Harrington, C. (1991). Effects of strategy monitoring and proactive instruction on children's problem-solving performance. *Journal of Educational Psychology*, *83*(1), 35-42. https://psycnet.apa.org/record/1991-19728-001

Desoete, A. (2007). Evaluating and improving the mathematics teaching-learning process through metacognition. *Electronic Journal of Research in Educational Psychology*, *5*(3), 705-730. https://repositorio.ual.es/bitstream/handle/10835/612/Art_13_186.pdf?sequenc

Flavell, J. H. (1987). Speculations about the nature and development of metacognition. In F. E. Weinert & R. H. Kluwe (Eds.), *Metacognition, motivation and understanding* (pp. 21-29). Lawrence Erlbaum Associates. https://lincs.ed.gov/sites/default/files/4_TEAL_Metacognitive.pdf

Goos, M., Galbraith, P., & Renshaw, P. (2000). A money problem: A source of insight into problem solving action. *International Journal for Mathematics Teaching and Learning*. https://www.cimt.org.uk/journal/pgmoney.pdf

Ignacio, A. G., Jr., & Reyes, J. D. (2017). Exploring mathematics achievement goals using Kolb's learning style model. *Asia Pacific Journal of Multidisciplinary Research*, *5*(1), 19-24.

Muijs, D., & Reynolds, D. (2005). The effectiveness of the use of learning support assistants in improving the mathematics achievement of low achieving pupils in primary school. *Educational Research*, *45*(3), 219-230. https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=b1bb79855238bbb7e7d15728dea1ce6731ed1bc6

Onovughe, G., & Hannah, A. (2011). Assessing ESL students' awareness and application of metacognitive strategies in comprehending academic materials. *Journal of Emerging Trends in Educational Research and Policy Studies*, *2*(5), 343-346. https://dergipark.org.tr/tr/download/article-file/283972

Ornstein, A. C., & Hunkins, F. P. (1998). Curriculum: Foundations, principles and issues (3rd ed.).

Allyn and Bacon. Pintrich, P. R. (2002). The role of metacognitive knowledge in learning, teaching, and assessing. *Theory into Practice*, *41*(4), 219-225. http://www.jstor.org/stable/1477406

Polya, G. (1985). Mathematical discovery: On understanding, learning and teaching problem solving (Combined ed.). Wiley. https://www.isinj.com/mt-usamo/Mathematical%20Discovery%20Polya%20(1981,%20Wiley).pdf

Pressley, M., Borkowski, J. G., & Schneider, W. (1987). Cognitive strategies: Good strategy users coordinate metacognition and knowledge. In R. Vasta & G. J. Whitehurst (Eds.), *Annals of child development* (Vol. 4, pp. 80-129). JAI Press.

Pressley, M., & Afflerbach, P. (1995). Verbal protocols of reading: The nature of constructively responsive reading. Lawrence Erlbaum Associates.

Ratner, C. (1991). Vygotsky's sociohistorical psychology and its contemporary applications. Plenum Press. https://doi.org/10.1007/978-1-4899-2614-2

Reyes, J. D. (2019). Mathematics anxiety and self-efficacy: A phenomenological dimension. *Journal of Humanities and Education Development*, *1*(1), 22-34. https://mail.theshillonga.com/index.php/jhed/article/view/9/4

Reyes, J. D., Insorio, A. O., Ingreso, M. L. V., Holario, F. F., & Gutierres, C. R. (2019). Conception and application of contextualization in mathematics education. *International Journal of Educational Studies in Mathematics*, *6*(1), 1-18. https://dergipark.org.tr/en/download/article-file/664590

Reyes, J. D., & Reyes, Z. D. (2024). Metacognitive learning in solving mathematical word problems. *Psychology and Education*, *22*(2), 217-253. https://doi.org/10.5281/zenodo.12751160

Reyes, J. D., Cabrera, G. A., & Ocampo, N. D. (2021). Curricular perspectives of selected mathematics: Descriptive cross-sectional non-experimental research. *Eurasian Journal of Teacher Education*, *2*(3), 160-176. https://dergipark.org.tr/en/pub/ejte/issue/66119/982757

Reynolds, R. E. (1992). Selective attention and prose learning: Theoretical and empirical research. *Educational Psychology Review*, *4*(4), 345-391. https://doi.org/10.1007/BF01332144

Schraw, G., & Moshman, D. (1995). Metacognitive theories. *Educational Psychology Review*, *7*(4), 351-371. https://doi.org/10.1007/BF02212307

Schraw, G., Crippen, K. J., & Hartley, K. (2006). Promoting self-regulation in science education: Metacognition as part of a broader perspective on learning. *Research in Science Education*, *36*(1-2), 111-139. https://doi.org/10.1007/s11165-005-3917-8

Steinberg, R. M., Empson, S. B., & Carpenter, T. P. (2004). Inquiry into children's mathematical thinking as a means to teacher change. *Journal of Mathematics Teacher Education*, *7*(3), 237-267. https://doi.org/10.1023/B:JMTE.0000033083.04005.d3

Stillman, G., & Mevarech, Z. (2010). Metacognition research in mathematics education: From hot topic to mature field. *ZDM Mathematics Education*, *42*(2), 145-148. https://doi.org/10.1007/s11858-010-0245-x

Sungur, S., & Senler, B. (2009). An analysis of Turkish high school students' metacognition and motivation. *Educational Research and Evaluation*, *15*(1), 45-62. https://doi.org/10.1080/13803610802591667

Turan, S., Demirel, O., & Sayek, I. (2009). Metacognitive awareness and self-regulated learning skills of medical students in different medical curricula. *Medical Teacher*, *31*(10), e477-e483. https://doi.org/10.3109/01421590903193521

Veenman, M. V. J. (2005). The assessment of metacognitive skills. In B. Moschner & C. Artelt (Eds.), *Lernstrategien und Metakognition: Implikationen für Forschung und Praxis* (pp. 75-97).

Waxmann. Veenman, M. V. J., Bavelaar, L., De Wolf, L., & Van Haaren, M. G. P. (2014). The on-line assessment of metacognitive skills in a computerized learning environment. *Learning and Individual Differences*, *29*, 123-130. https://doi.org/10.1016/j.lindif.2013.01.003

Watson, A., & De Geest, E. (2005). Principled teaching for deep progress: Improving mathematical learning beyond methods and material. *Educational Studies in Mathematics*, *58*(2), 209-234. https://doi.org/10.1007/s10649-005-2756-x

Wiliam, D. (2007). Keeping learning on track: Formative assessment and the regulation of learning. In F. K. Lester, Jr. (Ed.), *Second handbook of mathematics teaching and learning* (pp. 1053-1098). Information Age Publishing. https://www.dylanwiliam.org/Dylan_Wiliams_website/Papers_files/IAEA%2004%20paper.pdf

Young, A., & Fry, J. D. (2008). Metacognitive awareness and academic achievement in college students. *Journal of the Scholarship of Teaching and Learning*, *8*(2), 1-10. https://files.eric.ed.gov/fulltext/EJ854832.pdf